A Further Examination of Insurance Pricing
And
Underwriting Cycles

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Madsen, Chris K.
GE Insurance Solutions
Østbanegade 135, 2100 Copenhagen Ø, Denmark
Phone: +45 33 97 95 02, Fax: +45 33 97 94 41
E-Mail: chris.madsen@ge.com

Haastrup, Svend
GE Insurance Solutions
Østbanegade 135, 2100 Copenhagen Ø, Denmark
Phone: +45 33 18 95 75, Fax: +45 33 97 94 41
E-Mail: svend.haastrup@ge.com

Pedersen, Hal W.
Warren Centre for Actuarial Studies and Research
Faculty of Management
University of Manitoba
Winnipeg, Manitoba R3X 1J8, Canada
Phone: +1 204 474 9529
E-Mail: pedersn@ms.umanitoba.ca

Abstract

This paper is a follow-up to the paper "An Examination of Insurance Pricing and Underwriting Cycles". In that paper, Madsen and Pedersen laid out a theory of a "price of risk" and suggested that this price links all risky financial transactions. In particular, the paper detailed insurance pricing in terms of this price of risk and supported this with an analysis of the performance of the property and casualty insurance industry for the past fifty years.

In this paper, we follow up on some of the questions raised as feedback to the original paper. In addition, we offer some more specific examples based on actual treaty data. We also extend the use of this to a broader framework and put forth a plausible explanation for why our industry has been relatively slow to develop a complete and coherent pricing theory. We discuss that the lack of a coherent pricing theory may in fact be amplifying the underwriting cycle due to consistent mis-pricing.

We recognize that some of our hypotheses are controversial, but we believe a healthy debate – supported with actual data and underlying pricing theory - is necessary to move our industry and its theory of price forward.

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Introduction

Insurance needs a coherent pricing theory. Current actuarial pricing does not account for the optionality offered by insurance contracts. It is loss estimation rather than pricing and will only generate acceptable results under elusive conditions. Promoting a common understanding of the difference between these two terms is essential in moving our industry forward.

The views expressed in this paper are those of the authors. We know they are controversial, but we believe that a healthy debate is necessary to accelerate the development of a coherent pricing theory in insurance.

This paper is a follow-up to the paper by Madsen and Pedersen, 2003. In this paper, we follow up with some more concrete examples and explore further some of the feedback we have received in the past two years.

Some of the questions that continually haunt the property casualty industry include the following:

- ❑ Why are property casualty insurers' returns on equity continually so dismal?
- ❑ Why do property casualty insurers keep entertaining so many perspectives on risk management and pricing?
- ❑ Why do property casualty insurers have so many actuaries?
- ❑ Why do many property casualty insurers think their business is more art than science?
- ❑ Why is the underwriting cycle such a mystery and why is it so amplified?

We suggest that the answer to these questions is, at least in part, the lack of a coherent pricing theory.

Background and Review

As suggested in "An Examination of Insurance Pricing", selling insurance is like selling call options. We suggested then that the underwriting cycle was a direct result of insurers lack of pricing for the inherent optionality in insurance portfolios.

Insurance is ripe with options. Consider the following:

Simple Example

An insurer, A Insurance, buys reinsurance from a reinsurer, B Re. It is a stop-loss treaty covering property claims arising during the next year. At the moment of underwriting, B Re does not know the amount of premium that A Insurance will

underwrite nor is there anything other than a general indication of the price that A Insurance will charge. Thus, Be Re is granting A Insurance several powerful options:

- ❑ The very nature of insurance is for the insurer to call (assume) the losses if they exceed a given strike level (attachment point). This is the exact definition of an option.

But the optionality does not end there.

- ❑ A Insurance can change their pricing thus producing more claims that may flow through the cover. In other words, knowing that a given cover is in place may change the behavior of A Insurance, and – as a direct result – the behavior of the reinsured losses.
- ❑ A Insurance can also, if some calamity pierces the coverage during the year, write new policies below cost, as A Insurance no longer has any cost associated with the policies.

These are powerful and valuable options related to volume and price, and B Re ignores the value of these at its own peril.

The ladder two points are completely disregarded by B Re in its pricing, and we can argue about how well B Re is handling the first point.

In the investment world, this disregard simply would not be possible. If A Insurance were hedging currency risk rather than insurance risk, A Insurance would be forced to estimate how much currency it would have to hedge at the end of the year. If A Insurance wanted to be completely hedged, it would have to continuously add or subtract from its coverage. In other words, A Insurance can hedge, but retains the risk that it has more or less to hedge than it predicted. In the insurance world, B Re takes on this risk, but does not charge for it.

In the simple example above, the disregard for optionality means that B Re rarely charges enough. One might conclude that this drains profitability somewhat. In fact, it is a bit worse than this. When B Re charges too little, B Re sells more because pricing is lower than the true market equilibrium at the time and B Re is allowing A Insurance to arbitrage. This causes severe adverse losses over time, so B Re raises prices to another unnatural state – one that is too high. Now, A Insurance does not renew and leaves to place its business with an (often new) competitor. B Re's volume drops and B Re needs two good years with the new price for every bad one to break even. But, as people in the industry will attest to, one is fortunate to get one good year for each bad. Thus, the poor results will outweigh the good depressing long-term returns on equity.

In the following sections, we will explore the first of the three bullets mentioned above. The other two are related and no less important, but we believe these should be addressed directly through contract terms and conditions.

What is Insurance?

Insurance is a financial transaction on losses. At the time a policy is written, the insurer has a certain estimate of what the expected loss is (ultimate estimate or reserve). Over time, this estimate develops as more information becomes available. Thus, the expected loss (ultimate estimate or reserve) develops over time much like the price of a stock develops over time.

Buying insurance is equivalent to buying a call option on losses[i]. Conversely, selling insurance is the equivalent of selling a call option on losses. The insured has an option on losses that pays only if losses exceed a certain specified threshold. The insurer is generally short naked calls (naked because the insurer does not have an underlying position that offsets the cash flow of the call).

The value of the call option is the present value of the payoff using the risk-free rate and risk-neutral probabilities as shown in Equation 1. For multi-period examples, the lattice in Figure 1 can be extended, and the calculation in Equation 1 is then completed at every node[ii].

Equation 1: Price of Call Option

$$C = \frac{1}{R}\left[qC_u + (1-q)C_d\right],$$

where

$R = 1 + r$, where r is the risk-free rate,

q is the risk-neutral probability and is calculated as $q = \dfrac{R-d}{u-d}$,

$C_u = \max(u \cdot L - K, 0)$,

$C_d = \max(d \cdot L - K, 0)$,

u is the "up-move" factor,

d is the "down-move" factor, and

K is the attachment point

Note, that in the case, where $u = \dfrac{1}{d}$, the ultimate lattice produces a log-normal distribution and the Black-Scholes result[iii]. The volatility of this distribution is described as $\sigma = \dfrac{\ln(u)}{\sqrt{\Delta t}}$. As such, we can define the risk-neutral probability directly in terms of volatility, which will be more familiar to people used to work with Black-Scholes, and to people more familiar with fitting loss distributions. As such, we adopt the following:

$$u = e^{\sigma\sqrt{\Delta t}}$$
$$d = e^{-\sigma\sqrt{\Delta t}}$$

As a result, our price becomes a function of volatility and the rate of interest – not surprising given the fact that this produces the Black-Scholes result and that these are the Black-Scholes parameters.

Viewed graphically, we can construct a basic binomial lattice similar to what we do with stock options.

Figure 1: Insurance
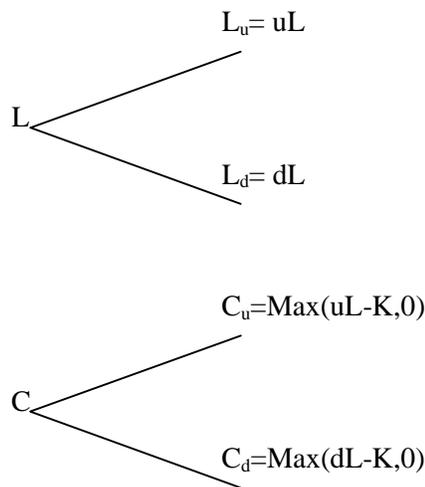
In Figure 1, L is the insurer's estimate of the ultimate underlying loss and C is the price of the call option (insurance) of a stop-loss treaty with attachment point K. The ultimate underlying loss estimate develops over time much like the price of a stock develops as more information becomes available. In fact, we can make analogies to stock options as in the following table:

Table 1: Comparing Equity Options with Loss Options

Stock/Equity Options		Loss Options (Insurance)	
Parameter	Interpretation	Parameter	Interpretation
r	Risk-free rate of interest	r	Risk-free rate of interest
σ	Volatility of equity returns (note this is equivalent to the volatility of equity	σ	Volatility of ultimate loss relative to initial estimate

	prices divided by the beginning equity price)		
S_0	Equity price at time 0	L_0	Discounted ultimate loss estimate at time 0
S_t	Equity price at time t	L_t	Ultimate loss estimate at time t
K	Strike price	K	Attachment point

We suggest that basic options theory can be used favorably to price insurance policies. In fact, the fact that insurers do not do this explains much of what we have observed in the re-insurance market since they were deregulated.

Consider the following very simple Example 1. In Example 1, we are looking at a line of business with fairly low overall volatility of 9.5% [iv]. This is the standard deviation of ultimate loss estimate relative to the initial estimate and represents the uncertainty we associate with this estimate. Note, that the option price produces a price that is substantially higher than the insurance price. The latter is generally based on the discounted expected payoff of the reinsurance contract.

Example 1: Option Pricing versus Traditional Insurance Pricing

Option pricing

$$C = \mathrm{E}^Q\left[e^{-rt}\left(L_t - K\right)^+\right] = \frac{1}{R}\left(q \cdot C_u + (1-q) \cdot C_d\right)$$

Traditional insurance pricing

$$P = \mathrm{E}^P\left[e^{-rt}\left(L_t - K\right)^+\right] = \frac{1}{R}\left(p \cdot C_u + (1-p) \cdot C_d\right)$$

The notation is described below. Note, that u is a function of lambda as well as sigma. We will explain this later. For now, we introduce that there is a term lambda(t) that scales the volatility. In the following example, we have set this term to 1, so it has no consequence for the purpose of this example.

<u>Known at the time of pricing</u>

Ultimate loss estimate at time 0		1,000
Cover Terms	Attachment point	1,000
	xol	1,000
sigma (st. dev. of changes in ultimate loss estimate)		9.5%
r (risk-free rate of interest)		5.0%
lambda(t)		1.00

<u>Pricing</u>

u = exp(sigma(t)*sqrt(dt)) = exp(sigma*lambda(t)*sqrt(dt))	1.100
d = 1/u	0.909
q = (1+r-d)/(u-d)	73.8%

		<u>Value</u>	<u>Probability</u>	<u>Payoff</u>
Loss value	L	1,000		1,100 L_u / 909 L_d
Option value	C	70	73.8% / 26.2%	100 C_u / - C_d
Insurance Pricing	P	48	50.0% / 50.0%	100 C_u / - C_d
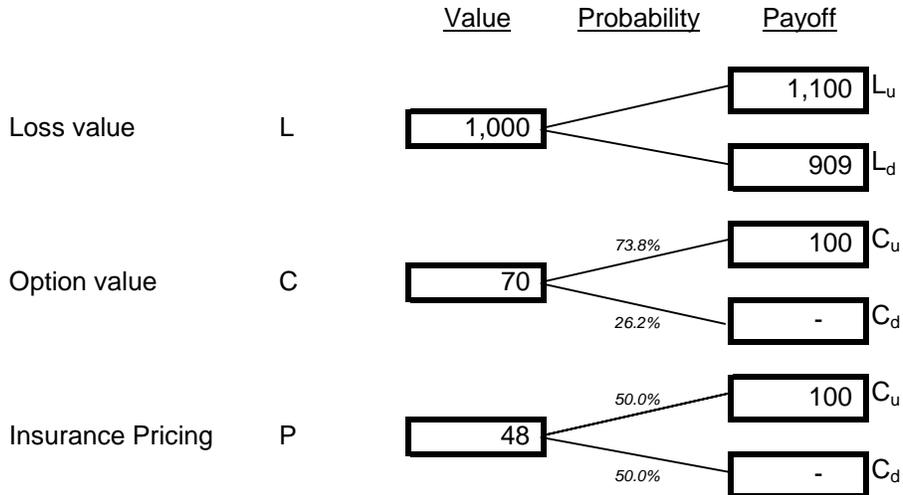
It is generally the case that the option price will exceed the traditional insurance price. To demonstrate this and the magnitude of it, we show how the prices behave relative to each other as we change some of the key underlying parameters. In the graphic below, we hold the rate of interest constant at 5% and vary only the volatility. The graphic shows the ratio of option pricing divided by insurance pricing as shown in Example 1.

Figure 2: Comparing the Price/Volatility Trade-offs Under Different Pricing Methods
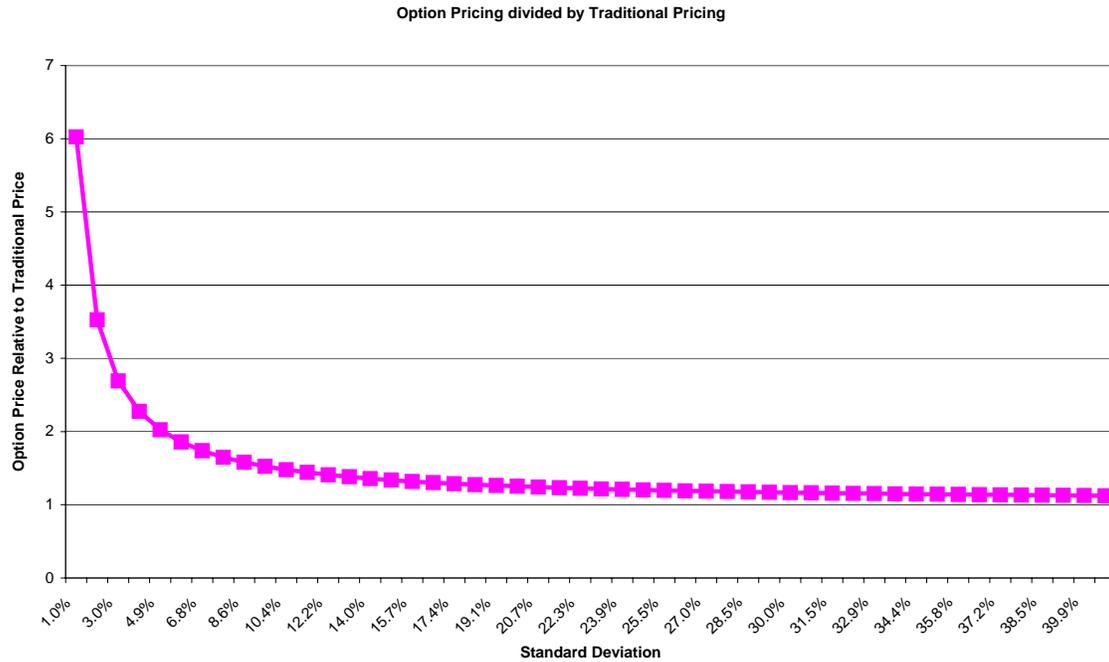
It is important to note, that this result comes from the fact that ultimate losses tend to develop with factors u and d=1/u (similar to stock prices). If the underlying losses developed exactly as expected at the time of pricing, we would observe factors u and d=2(1+r)-u [v], and in this case the risk-neutral probabilities would equal the real probabilities and thus, the option price would equal the discounted expected payoff. But, the latter is not what we observe, and this simple example suggests that in general, reinsurers tend to underprice low-medium volatility lines and price high volatility lines more fairly. This is consistent with available market data, where the cat lines, though volatile, are generally fairly profitable, whereas lower volatility lines are more marginal. In fact, return on *risk-adjusted* capital for low-volatility lines has historically been substantially below the level one would expect based on the underlying risk, whereas it has been much more attractive on catastrophe insurance, which is highly volatile.

In Figure 3, we have varied the interest rate and picked a level of volatility consistent with that of an average reinsurer with a traditional mix of business.

Figure 3: Comparing Price/Interest Rate Trade-offs Under Different Pricing Methods

Option Pricing divided by Traditional Pricing
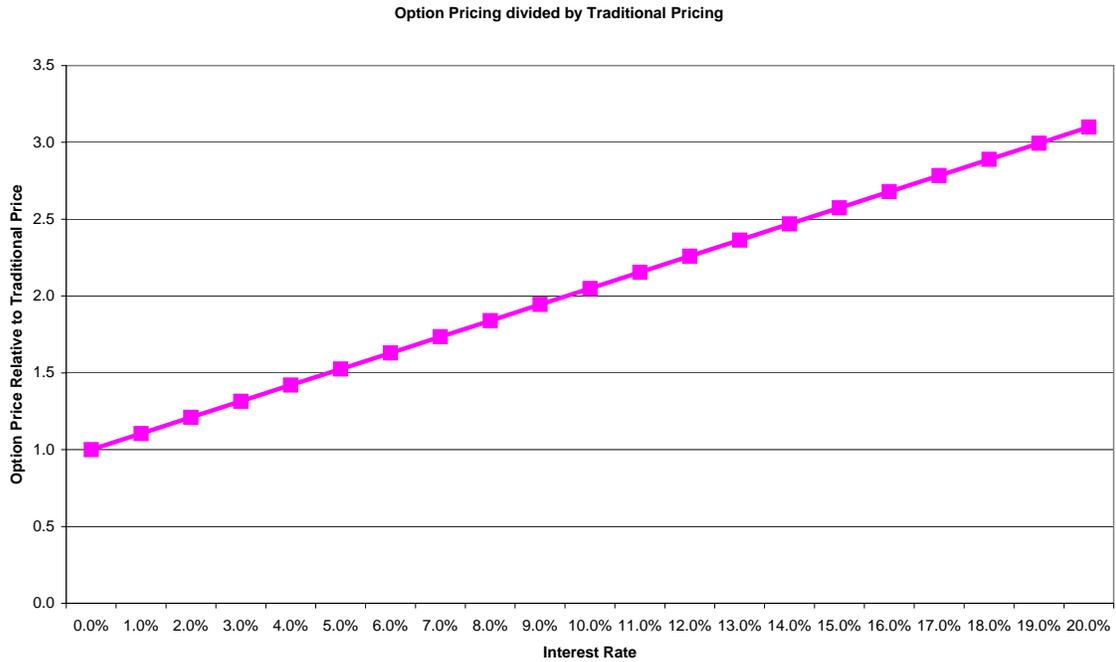
This suggests that the lower the rate of interest the closer the option price is to the discounted expected price. As such, we should not be surprised that recent lower rates of interest are coinciding with better reinsurer results.

Simple Example (Continued)

B Re's pricing approach (discounting expected payoffs) will work as long as B Re's actuaries have correctly assessed the final distribution. Even small fluctuations from the true distribution can cost B Re its profit. Consider the following case with an interest rate of 5%:

		Su	**1.10**
S	0.9567		
		Sd	**0.91**

Here, the underlying insurance (ground-up loss) can develop over time to two scenarios: The upper case and the lower case. A Insurance's actuaries have correctly assessed this and discounted the losses, and B Re is aware of this.

Now, Be Re's actuaries go to work. They study the payout of an attachment point of 1.00 and discount the expected payout associated payout associated with this.

This is referred to as the price under the "P" measure, as they use real probabilities.

Under P Measure			
		Cu	0.10
C	0.0476		
		Cd	-

Thus, B Re's actuaries arrive at a price of 0.0476, which is correct in this case. It is identical to the risk-neutral "Q" measure, since the final distribution was correctly assessed. Stated differently, the d=2(1+r)-u, which we mentioned earlier.

Under Q Measure			
		Cu	0.10
C	0.0476		
		Cd	-

The correct price will always be the one under the Q measure. Otherwise, A Insurance will have an arbitrage opportunity. If B Re's price is too low, A Insurance will sell more insurance, buy a risk-free asset and reinsurance and have an arbitrage opportunity.

Sell Insurance	Buy Bond	Buy Call	Net Cash Outlay		Payoff	Pay Insurance	Sell Bond	Get Call Payoff	Net Payoff
0.5011	(0.4535)	(0.0476)	-		u	(0.5762)	0.4762	0.1000	-
					d	(0.4762)	0.4762	-	-

A Insurance can replicate the reinsurance payoff by selling .5011 of insurance and buying .4535 of the risk-free asset and spending .0476 on reinsurance. This requires no cash outlay and A Insurance is now completely hedged, as we can see. The net payoff is zero in both the upper and lower case.

But B Re is under pressure to write business and while B Re's actuaries think they know the distribution, they cannot be completely sure, so press the price. This can be illustrated a couple of ways (all of which lead to the same point). We will say that B Re assumes the total loss will be slightly better than estimated (a little under 5% to be exact). Now, the Q and P measure prices no longer match and A Insurance has an arbitrage opportunity.

B Re assumes that the underlying price for A Insurance is given, but that the ultimate will develop just a little more favorably than originally estimated.

		Su	**1.05**
S	0.9567		
		Sd	**0.87**

and now

				Cu	0.05
C	0.0249				
				Cd	-

A Insurance happily accepts the reinsurance contract at 0.0249 and writes more direct insurance effectively arbitraging:

Under P Measure

Sell Insurance	Buy Bond	Buy Call	Net Cash Outlay		Payoff	Pay Insurance	Sell Bond	Get Call Payoff	Net Payoff
0.5011	(0.4535)	(0.0249)	0.0227		u	(0.5762)	0.4762	0.1000	-
					d	(0.4762)	0.4762	-	-

A Insurance now generates .0227 of risk-free profit per .5011 of direct insurance A Insurance writes. A Insurance is completely hedged as long as B Re wants to accept the business.

In reality, A Insurance does not need to worry. B Re has already committed and made no requirements as to underlying volume. As such, A Insurance can arbitrage as much as they want and B Re is exposed.

The subtle point made here that the consequences of under-pricing can be as dire in the insurance industry as they are in the financial markets. In other words, pricing below "fair value" is allowing arbitrage and since there are no real contractual limits to volume, the arbitrage can be exploited. In the end, this depresses profits and increases the volatility of the underwriting cycle. We will discuss this further, but first take a look at our suggested pricing framework, which we will then apply to two real-case examples.

Pricing Framework

The price of a call option is given by the following:

$$C = \mathrm{E}^{Q}\left[e^{-rt}\left(L_t - K\right)^{+}\right] = \frac{1}{R}\left(q \cdot C_u + (1-q) \cdot C_d\right), \text{ where}$$

R represents the risk-free rate plus one,
q the risk-neutral probability,
C_u the call option payoff in the "up" case, and
C_d the call option payoff in the "down" case.

As we saw in the simple example above, this must necessarily be so. Otherwise, there is an opportunity for arbitrage.

We apply this to the pricing of an excess of loss contract, but make one additional change. Namely, we propose that the value of a cash flow is the same whether the source

is insurance, equities or some other financial asset. In other words, we recognize that there is a single price of risk that permeates all financial transactions. This price of risk can be described by the implied option volatility and we have observable market prices for this through the Chicago Board Options Exchange (CBOE) "Vix", which trade at the average implied volatility of several S&P 500 options. When the Vix are high, the price of risk is high as the markets perceive greater risk. We submit that this translates to transactions beyond the equity markets[vi].

$$q = \frac{R-d}{u-d} = \frac{R - e^{-\sigma_t \sqrt{\Delta t}}}{e^{\sigma_t \sqrt{\Delta t}} - e^{-\sigma_t \sqrt{\Delta t}}} = \frac{R - e^{-\sigma \lambda_t^* \sqrt{\Delta t}}}{e^{\sigma \lambda_t^* \sqrt{\Delta t}} - e^{-\sigma \lambda_t^* \sqrt{\Delta t}}} \text{, where}$$

$$\lambda_t^* = \frac{vix_t}{\overline{vix}} \text{, } \overline{vix} = \frac{\sum_{t=1}^{N} vix_t}{N} \text{,}$$

σ is the long-term standard deviation of the underlying ultimate loss estimate divided by the initial loss estimate,

σ_t is the standard deviation at time t reflecting the market price of risk (estimated implied volatility) of the underlying ultimate loss estimate divided by the initial loss estimate,

Δt is the time increment during which the option is in effect, and

$R = 1 + r$, where r is the risk-free rate.

One can rewrite the Black-Scholes formula with this formulation. Black-Scholes will stay intact as scaling the long-term volatility to the volatility of today is equivalent to using today's volatility thus producing today's prices.

Thus, we have made two important changes to insurance pricing:

❑ We price under the *Q* measure
❑ We create the implied volatility of an insurance contract and use this in our pricing

Two Real-Case Examples

The Industry Combined Ratio and General Profitability

In "An Examination of Insurance Pricing", we demonstrated that a "price of risk" observed through market prices can be adapted to insurance prices. In other words, there is a single price of risk in the financial markets (including insurance) that permeates all financial transactions. In an incomplete market such as insurance, prices may differ from this true "arbitrage-free" or risk-neutral price, but they do so at the peril of market

participants. We will review later why such a discrepancy can exist in an incomplete market for extended periods of time.

In "An Examination of Insurance Pricing", we used the "Vix[vii]" option prices from the Chicago Board Options Exchange (CBOE) to get at the price of risk.

We used this information to show that

❑ The underwriting cycle moves exactly the opposite of what one would expect based on option pricing. This is because actuarial pricing techniques use basic expected discounting rather than option pricing. In option pricing, one raises the price when interest rates increase. Most insurers do exactly the opposite failing to recognize that a higher rate of interest increases the probability last a layer will be pierced.

❑ The turning points in the underwriting cycle can be identified by creating an option price index and a traditional price index. Whenever the two methods cross, the cycle turns. Stated differently, sometimes insurers charge too much and sometimes too little. Sometimes they are just right, but this occurs by sheer chance.

We have updated the key graphic below.

Figure 4: Systematic Over- and Under-pricing
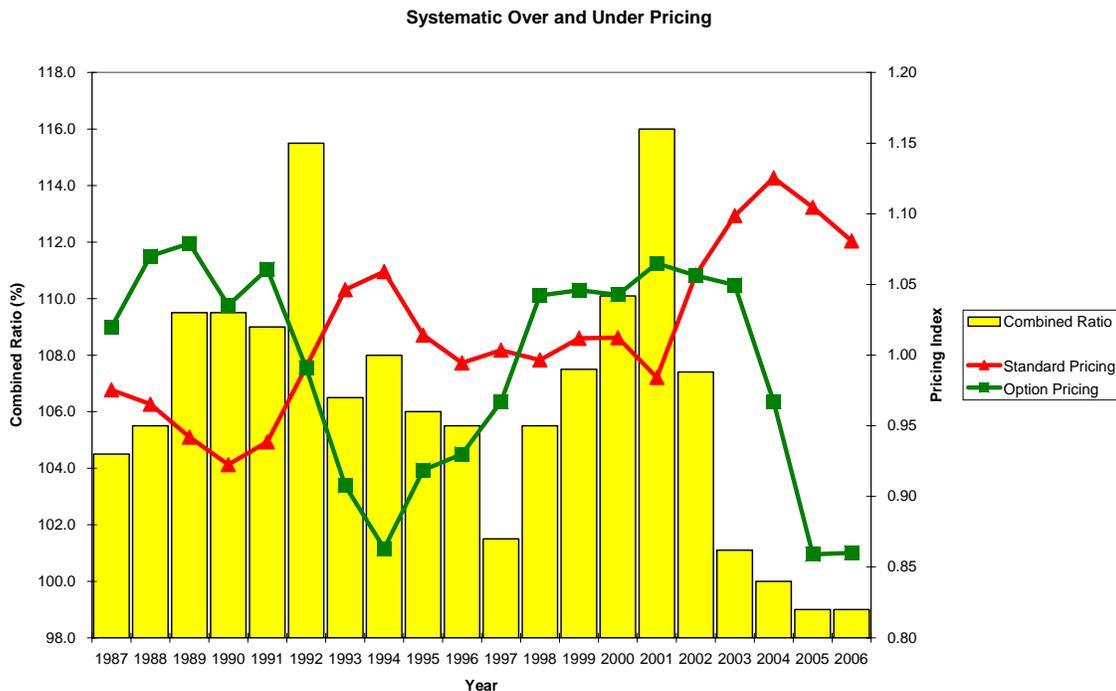
As can be seen, the graphic looks one year ahead and suggests that the property and casualty insurance industry will have at least one more "good" year in terms of financial performance. This is because traditional pricing (red triangles) suggests a higher price than the option price (green squares) thus improving financial results.

This graphic is interesting and thought-provoking, but the generality of it makes it difficult to demonstrate the value of our proposition without a doubt:

- ❑ It is based on insurer's reported results with the inherent lag in reporting and possible reserving bias.
- ❑ The indexes do not propose a correct price per se. Rather they are based on a general model of the industry.

To address these and other questions, we reviewed actual data to develop a specific example of how an insurer would use this in practice.

Reviewing Actual Data from a Low-volatility Line of Business

This example is from a real book of business, so the lags and reporting issues that one encounters above are not present in this data to the same extent. In addition, the data is a homogeneous portfolio, which should facilitate the explanation of results.

The line of business is excess of loss treaty business on a low to medium volatility book. The combined ratio through time is shown below, where we show underwriting years on the x-axis and the combined ratio on the y-axis.

Figure 5: Actual Segment Combined Ratio

Combined Ratio
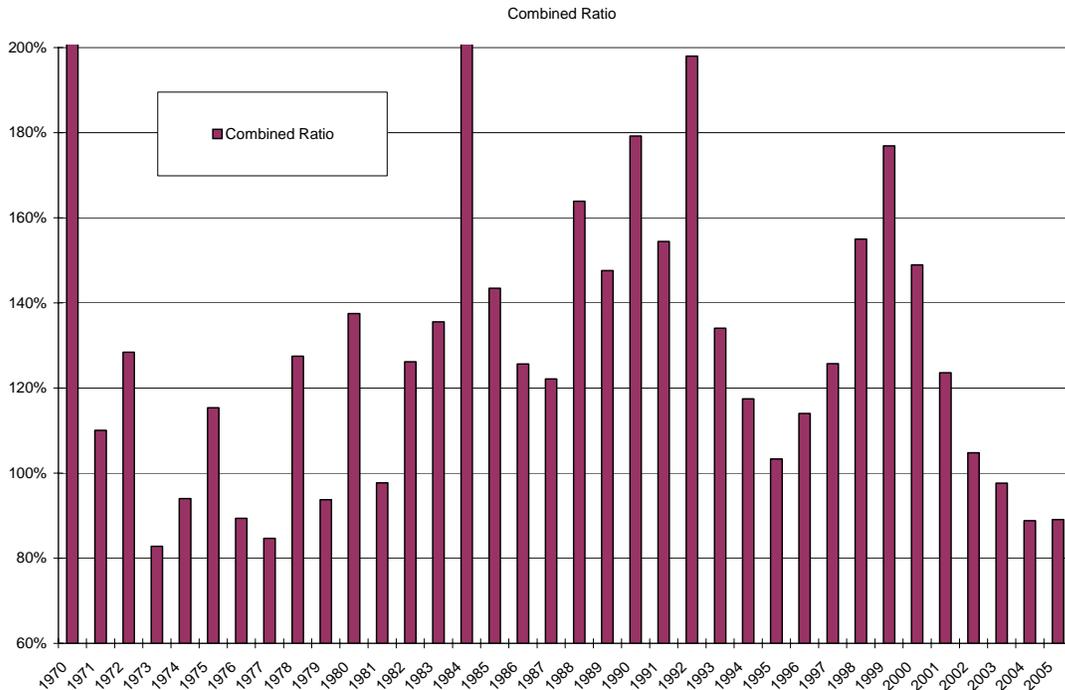
Our first goal is to try to explain this behavior. We need to be sensitive to the fact that the reinsurance market in Europe was heavily regulated until about 1994. Deregulation had been in progress for about ten years up to this point. As such, we can separate the above into three segments:

- Regulated market place (~1983 and prior)
    - Average combined ratio of 109.5% (excluding 1970)
    - Standard deviation of combined ratio of 20.0%
    - Prices are set centrally and jointly. Reinsurers are very profitable and there is little competition on price. At this time, pricing did not matter because prices were unnaturally high.
- Market place in transition (~1984 through ~1993)
    - The market begins to change in some locations allowing price to fluctuate. Pricing begins to matter and more actuaries are hired.
    - Average combined ratio of 163.1%
    - Standard deviation of combined ratio of 42.2%
- Deregulated market place (~1994 and later)
    - Market prices fall where they may. Actuaries price deals extensively, but no coherent pricing theory exists and there is always uncertainty about the "right" price allowing parameters to be easily modified to get the price one wants.
    - Average combined ratio of 120.4%
    - Standard deviation of combined ratio of 27.5%

Our focus will be on the latter two periods, as they are more indicative of the future.

To set an option price structure for this segment, we need to estimate ground-up losses. Ideally, the ground-up distribution would be readily available, but in most cases, the current environment does not offer one. To develop a ground-up loss distribution, we define a few parameters that relate the ground-up loss distribution to the reinsurance distribution (which is readily available). Based on these parameters, the ground-up distribution will be entirely described.

$$a_1 = \frac{E(GL)}{E(RL)}$$

$$a_2 = \frac{AP}{E(GL)}$$

$$f_{GroundUp} = \frac{Stdev(GL)\big/E(GL)}{StDev(RL)\big/E(RL)}$$

Here,

GL is the ground-up loss,
RL is the reinsurance loss, and
AP is the attachment point.

In addition, we set

$CR_{i,0} = \dfrac{E(RL_i)_0}{RP_i} = 90\%$, where CR is the combined ratio, RL the reinsurance loss and RP

the reinsurance premium. The subscript i denotes the underwriting year.

$\Delta t = 4.4\,years$, which is the duration of the payout pattern.

Note, that for simplicity we have included expenses in our ultimate loss estimate. This simplifies notation and has no further implications.

The last two assumptions can be reviewed directly from the data. The first is the initial combined ratio estimate. In other words, how was the business viewed at the time it was underwritten? This may be directly available if the reinsurance data is complete from the date of inception. In our case, we have set it to 90% for all underwriting years consistent with the initial indications, though they will vary from underwriting year to underwriting

year. If the data is readily available by underwriting year, one can simply use it to get a more accurate result.

The second is the duration of the business based on the claims payment pattern. This can be estimated directly from the payout pattern. This gives us an indication of how long it will take to pay out this business on average. We expect to make adjustments to our ultimate loss estimate for the same period of time on average.

Given the mean and standard deviation of the *ground-up* log-normal distribution as well as the layer that is reinsured, one can determine the *reinsurance* expected loss and standard deviation (see Appendix). We can reverse this, so we find the ground-up mean and standard deviation given the reinsurance information.

For any underwriting year, *i*, we have

$$\mathrm{E}(RL_i) = CR_0 \cdot RP_i$$
$$\mathrm{E}(GL_i) = a_1 \cdot \mathrm{E}(RL_i)$$
$$AP_i = a_2 \cdot \mathrm{E}(GL_i) = a_2 \cdot a_1 \cdot \mathrm{E}(RL_i)$$
$$\frac{StDev(GL)}{\mathrm{E}(GL)} = f_{GroundUp} \cdot \frac{StDev(RL)}{\mathrm{E}(RL)}$$

Thus, we now have to estimate 3 parameters to fully describe the value of the option: a_1, a_2, and $f_{GroundUp}$. These can be estimated in a variety of ways.

Any distributional assumption can be made at this point (unless the three parameters are directly available from existing data in which case one can avoid the restrictions of distributional assumptions), and the parameters found either through algebraic manipulation or through simulation.

For our purposes, in order to estimate the parameters, we assume the ground-up loss is log-normal. This is consistent with the binomial lattice framework and the log-normal is commonly used in the insurance industry as an estimate of aggregate losses. Once the three parameters are given in addition to the reinsurance data, our ground-up distribution is entirely described.

Once the parameters, and through them the ground-up distribution, have been found, pricing the option is a straight-forward exercise as laid out in our previous examples. But, it is important to recognize our previous work: The price of risk differs over time. As such, for pricing purposes, there is not a constant volatility. It fluctuates and we can use our price of risk concept to recognize this.

In our original paper, we defined a price of risk as volatility relative to return. The required us to estimate a coefficient of variation for insurance, which includes the

estimation of a return (e.g. "drift" in reserve estimates) in insurance. There are two problems with this:

- ❑ The results are very sensitive to the mean return
- ❑ We know from options theory that the return does not matter

Accordingly, we will now modify our concept of the price of risk. It is important to note, that is leaves our original work intact albeit with different parameters. In other words, the graph in the previous section (Figure 4) can be reproduced in the following framework. To avoid confusion, we introduce our new price of risk as a "Price of Risk Factor" or λ^*. It is simply a scaling factor of volatility measured by the current value of vix relative to its trailing historical average.

$$\lambda_t^* = \frac{vix_t}{\overline{vix}},$$

where $\overline{vix} = \dfrac{\sum\limits_{t=1}^{N} vix_t}{N}$

Whatever volatility we get for the ground-up loss, we will rescale it based on our "Price of Risk Factor".

$\sigma_{GroundUp,t} = \sigma_{GroundUp} \cdot \lambda_t^*$, where $\sigma_{GroundUp}$ is the actuarial estimate of volatility of the ground-up loss.

To test our theory further, we find the parameters a_1, a_2, and $f_{GroundUp}$ through a minimum sum of squared errors approach. Stated differently, since we are trying to understand the underwriting cycle, we simply let our history of combined ratios tell us what reasonable assumptions for our three parameters would be. We assume the parameters are the same for each underwriting year. This makes sense since this is a fairly homogeneous portfolio. Given complete data and an individual treaty, the parameters would be explicit by treaty by underwriting year.

Hypothesis
Traditional insurance pricing does not properly account for the optionality inherent in insurance contracts. As such, insurance contracts can be arbitraged thus extending losses when prices are low relative to the arbitrage-free price. Thus, we should be able to show that the combined ratio tends to be extended when the arbitrage-free price is high relative to the traditional (actual) price.

Test
$Minimize \ \sum\limits_{i=1}^{I} \left(CR_i - \dfrac{C_i}{P_i} \right)^2$, where

- ❑ We allow losses to be rescaled to get the best fit possible
- ❑ The adjusted option price is the option price incorporating our Price of Risk Factor

Here,

CR_i is the reinsurance combined ratio for underwriting year i,

C_i is the call option (reinsurance) price as laid out in our pricing framework (using the ground-up log-normal distribution as described by parameters a_1, a_2, and $f_{GroundUp}$), and

P_i is the actual reinsurance premium for underwriting year i.

In our case, we get the following results:

$$a_1 = 9.5\%$$
$$a_2 = 90.5\%$$
$$f_{GroundUp} = 9.6\%$$

Reviewing these for reasonability, they do not seem unreasonable. One could argue that they do not even have to be reasonable per se, as they are akin to state variable pricing factors or simply the parameter values that best describe the available data. As such, they do not have to be observable.

The graph below summarizes the results.

Figure 6: Combined Ratio versus Pricing Methods

Combined Ratio Versus Pricing Methods
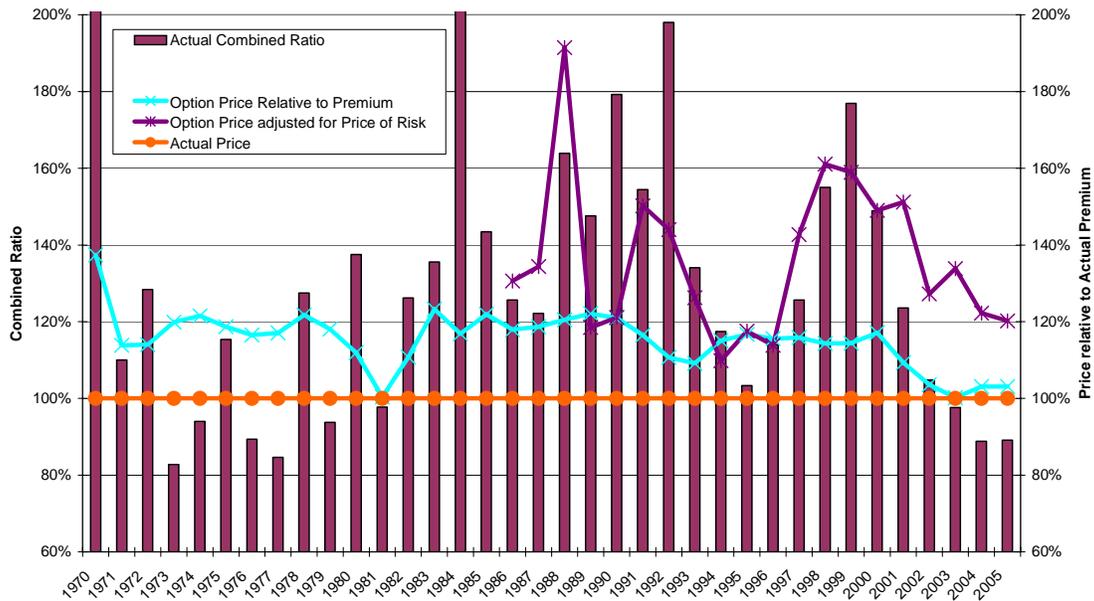
On the graph, we see the combined ratio as before in the columns. The orange (diamond) line is the actual price relative to premium, which naturally is 100% and the price is the actual premium. The blue (crosses) line is the unadjusted option price using a constant volatility relative to premium. The purple (stars) line is the adjusted option price using the Price of Risk Factor relative to premium.

As is visually obvious, the adjusted options price has a correlation coefficient of 79.2% with the actual combined ratio. The main deviations are 1989 and 1990 as well as 2002 and later.

The "Burns' Day Storm" hit in late January, 1990, causing severe losses in northern Europe. It was one of the strongest storms on record. Such deviation from our price would be expected. This is why insurance exists and it is the eventuality we price for.

As for 2002 and later, we suggest that the ultimate here is not done developing. In other words, we would not be surprised if losses here continue to accumulate and get closer to our price. In fact, the drift term of reserves would suggest this to be true.

There are many interesting observations that extend this result. For example, one can calculate the associated return on equity assuming a capital factor of 15% of reserves per annum.

$$ROE = \frac{\sum_{t=1}^{T} PVCF_t}{\sum_{t=1}^{T} PVCapital_i}$$

That is, we sum the present value of results for each underwriting year at the time of underwriting and divide by the present value of the capital requirement. This gives an indication of the return on equity. Based on this, we develop the following graphic.

Figure 7: Risk/Reward Trade-offs of Pricing Alternatives

Risk/Reward Trade-Offs of Price Alternatives
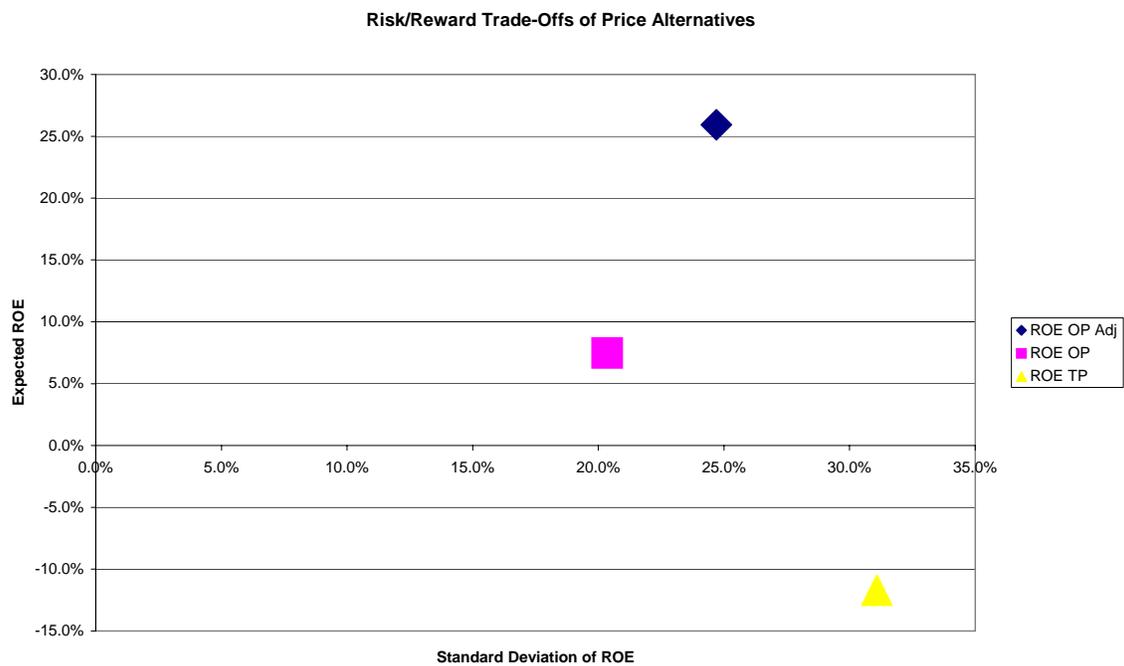
Which business would you prefer to be in? Industry members will recognize that the situation with the yellow (triangle) marker is more commonly observed than the other two. This is the actual case. The "OP" (square) marker is the option price but not yet adjusted for the price of risk. The "OP Adj" (diamond) marker is based on the option price of our framework.

Taking a different view on this, we can restate what the market would have looked like under our options prices. It is very interesting that the following dynamics exist.

Table 2: Combined Ratios and Volatility Under Different Pricing Methods

	Actual CR	Adj CR	Adj CR incl PoR
Expected Value	133.5%	119.9%	101.4%
Standard Error	31.1%	25.4%	20.2%

Here, we have looked at the combined ratio with our implied volatility adjustment as well as with it. "PoR" denotes price of risk..

We suggest that a third of the volatility underwriting cycle be eliminated if option pricing was adopted. In fact, it might be more significant than this. This is based on the simple notion that consumers intuitively know when they are getting a good deal. When prices are below the fair price, they buy more insurance or press terms and conditions. In other words, reinsurers implicitly sell more insurance when they do so at a loss and this is costly when the attachment point is fixed. Conversely, insurers sell less when they sell at a premium. The extension of this is that volatility increases. To the extent that one would charge the fair price at all times, one would expect to remove at least part of this phenomenon.

The following graph shows the combined ratio as it has been and what it would have been under the various prices.

Figure 8: Combined Ratio under Different Pricing Assumptions
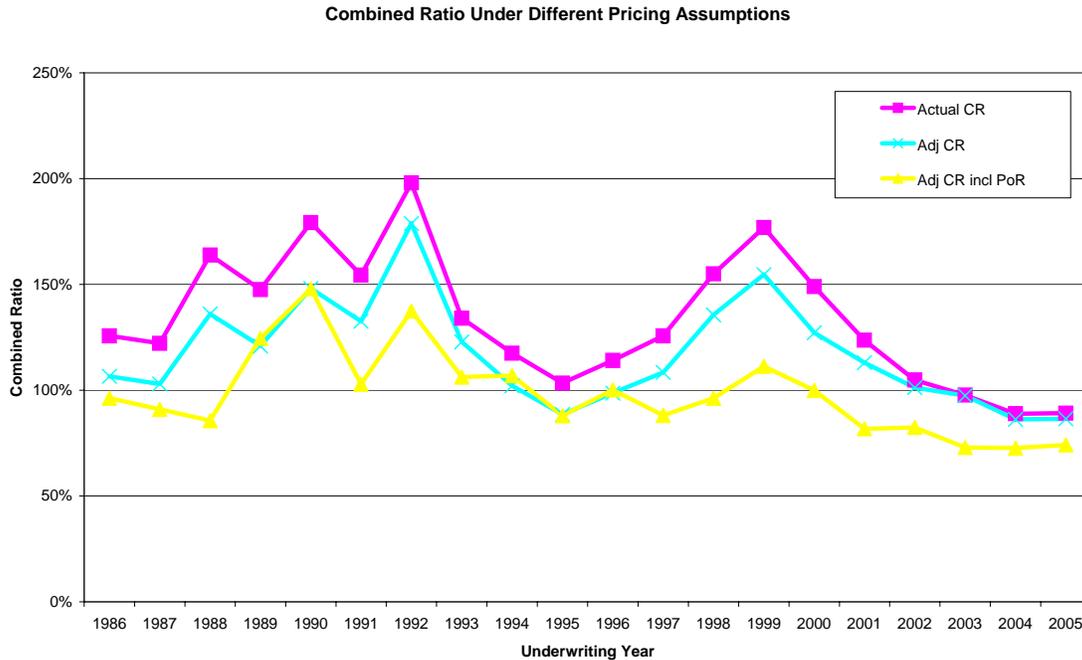
Some may argue that given that the option price is consistently above the actual price, one would have written nothing if one has stuck to our pricing framework. This is realistically correct and for most, it would have been a good decision. Why would anyone want to write a line at prices that generate an ROE of –11% with high volatility? Especially when arbitrage is readily available for the same reasons.

But there are lines, where the opposite is true. In particular, in the current environment with low interest rates, any medium to high volatility line has a good chance of providing segments where the market price is close to the option price. Following our pricing structure by segment, one would naturally shift into the proper segments at the right time with favorable results. In other words, following our framework in the current market-place, one would tend to be closer to the market in the high volatility segments and thus end up writing a lot of this type of business. Conversely, one would largely exit the low volatility markets as the market would be lower.

Exiting a business does not imply that the insurer is not willing to write the business. It simply means that the reinsurer's price is above the market price and, as such, one would naturally expect to lose the more opportunistic customers.

Why?

Observers of the industry may question how such a situation can develop. This situation has perplexed the reinsurance industry for as long as it has been deregulated and there are hardly any industry conferences where the underwriting cycle is not discussed. It has always baffled the authors of this paper that an industry can operate with such little understanding of the fundamentals that drive its business.

But the reason the cycle exists to the extent it does, is that the source is deeply rooted and that it did not use to matter. Reinsurers "suffer" from a long and profitable history until the industry was deregulated. In an environment of heavy regulation where everybody was making money, it did not matter if prices were set right. They were always high. Now, in a period of deregulation, prices have been forced to unnaturally low levels because actuarial techniques do not recognize optionality in general, and most actuaries use the same techniques for pricing. In other words, while companies differ in the prices, the source of those prices is usually the same (often erroneous) pricing framework.

The reinsurance market is an incomplete market. This means that all risks cannot be readily hedged with available market derivatives, though as we have seen, insurers can replicate reinsurance payoffs and generate arbitrage profits. But there is no easy way to arbitrage insurance ad infinitum to eliminate discrepancies, because it is not a continuous market. Thus, arbitrage can exist, but it does not move prices as it would in a continuous and complete market. This arbitrage is a direct cost to reinsurers and depresses profits.

How can this exist? In an incomplete market, participants are cursed (or blessed depending on your perspective) by not knowing the truth. As such, they can make up their own truth and our survival instinct usually makes such truth one that is favorable to oneself. This is not a new concept.

In an inefficient and incomplete market, participants can delude themselves. A recent study by McKinsey & Co. found that 80% of insurance CEOs believed that they would do the right thing in the next down market. But only 20% believed that other participants would… This is reminiscent of the classic introductory psychology course scenario where 80% of a given pool of participants think they are above average, though – or rather because - they have nothing to base it on.

Similarly, a behavorial study by Professor Vernon Smith compared aspects of complete and incomplete markets and found that incomplete markets stay incomplete while complete markets quickly converge to efficiency (even with few participants). It is important to note his definitions of a complete and incomplete market:

Complete: Continuous two-sided bid-ask system such as electronic exchanges.
Incomplete: Bilateral markets, where buyers and sellers find each other and make
               direct contact.

What really was interesting to Prof. Smith was the fact that given the choice, incomplete market participants chose to stay with the incomplete market. Not knowing any better, they simply had "lots of confidence to just go it alone. We have a long way to go to understand this choice." He calls this "survival economics" rather than "maximization of profits". Though he never specifically mentions the reinsurance market, the authors find the parallels eerie.

So, there is a human comfort in being able to make up the truth such that it suits oneself. But, it is not very profitable.

Change will necessarily come at some point, but given that we are in an incomplete market and that this scenario is itself fairly stable, we need a pivotal event. A pricing theory and framework such as what we are proposing here could go a long way. Also, change of the market structure could facilitate. As more existing (run-off) insurance losses are traded, it is plausible that a bid-ask exchange could be developed that would spill over to the current market place.

The parallels here between the insurance market of today and the options market of 30 years ago are interesting. Prior to the emergence of the Chicago Board Options Exchange (CBOE) and the Black-Scholes option theory, options pricing was hardly as organized and straight-forward as it is today. A similar exchange for insurance risk combined with a coherent theory of pricing could do wonders to the viability of the industry in the long-term.

Summary and Conclusion

We have described in detail the underwriting cycle of the industry in general as well as a specific segment. We have shown that our hypothesis explains more than 60% of the variation in the underwriting cycle and that the industry would generate returns closer to what one would expect given the level of risk, if it were to follow our framework.

We have showed that any reinsurer or insurer would be well served to price assuming that underlying loss estimates will develop like a lattice with factor u and d=1/u. This would force a price rigor and structure that would at the same time make arbitrage against the insurer more difficult. The main parameter to estimate is the volatility and the result is not as sensitive to changes in this parameter as traditional pricing is to changes in other distributional parameters.

We have also shown that reinsurance can be replicated with direct insurance, and that this means that insurance can actually be arbitraged. But the market remains incomplete and we have explained this.

The reinsurance market needs to be more complete. Further product development is needed for this to develop, but a coherent pricing theory is missing to set the structure around the pricing of the financial products. We have shown that option theory can be applied successfully to accomplish this and hope to see more products introduced with this in mind.

Appendix

The Expected Payoff of a Reinsurance Contract Given an Underlying Log-Normal Distribution

$$Y = \min\left\{(b-a),(X-a)^+\right\}$$

First Moment

We have

$$E(Y) = \int_a^b (x-a)f(x)dx + \int_b^\infty (b-a)f(x)dx \text{, hence}$$

$$E(Y) = \sigma^2\left[f(a)-f(b)\right] + (\mu-a)\left[F(b)-F(a)\right] + (b-a)\left[1-F(b)\right]$$

Second Moment

We have

$$E(Y^2) = \int_a^b (x-a)^2 f(x)dx + \int_b^\infty (b-a)^2 f(x)dx \text{, hence}$$

$$E(Y) =$$
$$(\mu-a)\sigma^2\left[f(a)-f(b)\right]$$
$$-(b-a)\sigma^2 f(b) + \sigma^2\left[F(b)-F(a)\right]$$
$$+(\mu-a)^2\left[F(b)-F(a)+(b-a)^2(1-F(b))\right]$$

Here,

$$f(x) = \frac{1}{\sigma\cdot\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$F(x) = \Phi(x)$, which is the cumulative standard normal distribution

a is the attachment point
b is the upper limit

X denotes the ground-up loss random variable and Y the reinsurance loss random variable.

References

A.M. Best Company, "Discipline and Specialized Distribution Are Keys to Program Underwriters' Success", February 1, 1999 Special Report

Arnold, Tom and Crack, Timothy: "Using the WACC to Value Real Options", Financial Analysts Journal, Vol. 60, No. 6, November/December 2004

Black, Fischer and Scholes, "The Pricing of Options and Corporate Liabilities", Journal of Political Economy, 81:3, 1973, pp. 637-654

Ciezadlo, Greg, "Market Cycle Update, Personal Lines", CAS Spring Meeting, 2002

Commerce Department's National Institute of Standards (NIST) and International Sematech (IST), "E-Handbook of Statistical Methods"

Embrechts, Paul: "Actuarial versus financial pricing of insurance", Swiss Institute of Technology, Switzerland

Fenn, Paul: "Round and Round – some of the theoretical explanations of the underwriting cycle", www.globalreinsurance.com

Gerber and Pafumi: "Utility Functions: From Risk Theory to Finance", North American Actuarial Journal, 1998, Volume 2, Number 3.

Harrington, Cynthia, "Beyond Belief – Pioneering Insights Into Behavioral Finance", CFA Magazine, September/October 2003

Hull, John C., "Options, Futures, and Other Derivatives", Prentice Hall, 1997

Insurance Information Institute

Insurance Journal, ""Combined Ratio for 2001 on Track to Hit Record High, ISO President Says", Property and Casualty Magazine, April 2001

James, Julian, "The challenge for change", Speech at ILICA conference, 29 September 2004

James, Kaye D. and Oakley, David, "Underwriting Cycles by Kaye D. James - Discussion by David J. Oakley"

Luenberger, David, "Investment Science", Oxford University Press, 1998

Mack, Thomas, "Distribution-free calculation of the standard error of chain-ladder

reserve estimates", *ASTIN Bulletin,* 23, 1993, pg. 213-225

Madsen, Chris and Pedersen, Hal: "An Examination of Insurance Pricing and Underwriting Cycles", AFIR Colloquium, September, 2003, pg 769-794

Mitchell, Roger, "A Model Mind – Robert C. Merton on Putting Theory into Practice", CFA Magazine, July-August, 2004

Moore, James, "Hard Markets", RiskIndustry.com, August 25, 2001

Sepp, Thomas and Bate, Oliver, "The Underwriting Cycle – a phenomenon to be controlled, not endured", McKinsey & Company, November 27, 2003

Sherris, Michael: "Solvency, Capital Allocation and Fair Rate of Return in Insurance", UNSW, Sydney, Australia, January, 2004

Skurnick, David, "The Underwriting Cycle", CAS Underwriting Cycle Seminar, April, 1993

Swiss Re, "Deregulation and liberalisation of market access: the European insurance industry on the threshold of a new era in competition", sigma No. 7/1996

Swiss Re, "Profitability of the non-life insurance industry: it's back-to-basics time", Sigma, No. 5/2001

Swiss Re, "Reinsurance – a systematic risk?", Sigma, No. 5/2003

Tarashev, Nikola et al: "Investors' attitude towards risk: what can we learn from options?", BIS Quarterly Review, June 2003

Verall, R.J.: "Obtaining Predictive Distributions for Reserves which Incorporate Expert Opinion", CAS Forum, 2004

Wang, Shaun: "A Universal Framework for Pricing Financial and Insurance Risks"

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[i] To see this, graph the payoff function of an excess of loss contract. It is exactly the payoff of a call option.

[ii] In continuous time, this becomes Brownian motion, which can be described as

$$\frac{dL}{L} = \mu \cdot dt + \sigma \cdot dz \, .$$

[iii] Note, that many distributions can be explained by changing u's and d's in the binomial lattice.

[iv] Volatility of ultimate reserves will generally range from 0% to 60+% depending on line of business.

[v] If p=50% and $L_0 = \mathrm{E}^P\left(\dfrac{L_1}{R}\right)$, then d=2R-u.

[vi] For more detail on this, please see Madsen and Pedersen 2003.

[vii] The CBOE Volatility Index - more commonly referred to as "VIX" - is an up-to-the-minute market estimate of expected volatility that is calculated by using real-time S&P 500 (SPX) index option bid/ask quotes. VIX uses nearby and second nearby options with at least 8 days left to expiration and then weights them to yield a constant, 30-day measure of the expected volatility of the S&P 500 Index. The underlying for options is an "Increased-Value" Volatility Index (VXB), which is calculated at 10 times the value of VIX. For example, when the level of VIX is 12.81, VXB would be 128.10.