A MILLIMAN GLOBAL FIRM

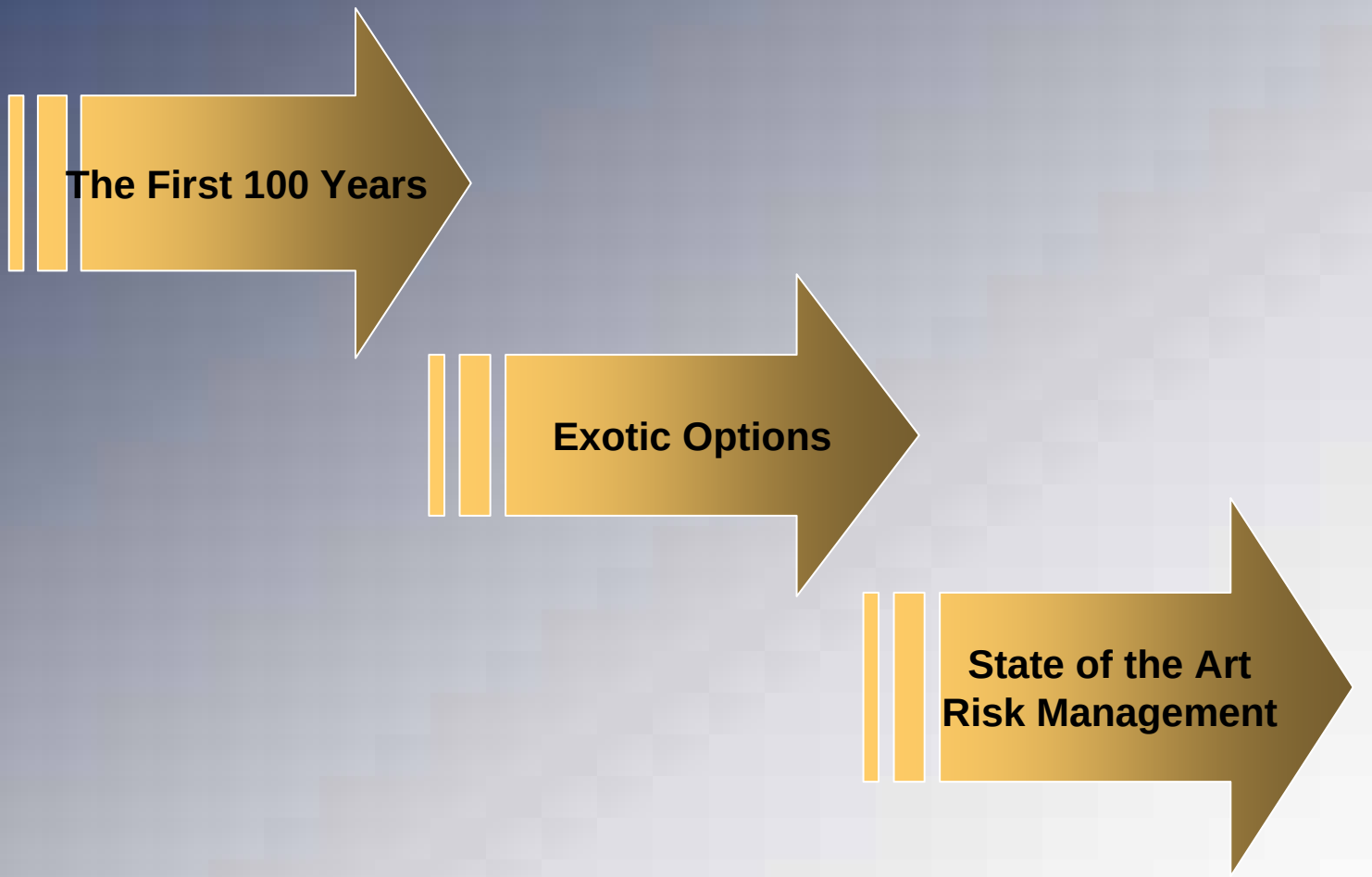
Milliman

Consultants and Actuaries

Exotic Options

Through 100 Years to Current State of the Art Risk Management of Investment Guarantees

Contents


The First 100 Years
Exotic Options
State of the Art
Risk Management


100 Years

The First 75 Years

- **Bachelier (1900)**
- **Levy (1925)**
- **Ito (1951)**
- **Markowitz (1952)**
- **Reddington (1952)**
- **Sprenkle (1958)**
- **Samuelson (1965)**
- **Thorpe (1969)**
- **Black & Scholes (73)**

Black & Scholes

- **Black & Scholes (1973)**
- **Merton (1973)**
- **Harrison & Kreps (1977)**

Exotic Options

Exotic Options

- **Margrabe (1978)**

$$PO_T = \max(Q_1 S_1(T) - Q_2 S_2(T), 0)$$

- **Stulz (1982) & Rubinstein (1991)**

$$PO_T = \max(\max(S_1, S_2) - X, 0)$$

- **Turnbull & Wakeman (1991), Curran (1992, 1994), Yor(1993), Zhang (1994).**

Deferred Annuities

- **Margrabe (1978)**

$$PO_T = \max(Q_1 S_1(T) - Q_2 S_2(T), 0)$$

Deferred Annuities

- **Margrabe (1978)**

$$PO_T = \max(Q_1 S_1(T) - Q_2 S_2(T), 0)$$

$$GC_T = \max(gb_T a_{x+T,i(T)} - AS_T, 0)$$

Compound Options

- **Stulz (1982) & Rubinstein (1991)**

$$PO_T = \max(\max(S_1, S_2) - X, 0)$$

Compound Options

- **Stulz (1982) & Rubinstein (1991)**

$$PO_T = \max(\max(S_1, S_2) - X, 0)$$

- **Useful for valuing benefits of the form**

$$\max(S_1, S_2, X)$$
$$= X + \max(\max(S_1, S_2) - X, 0)$$

- **Generalises**

$$\max(S_1, S_2, S_3, X)$$
$$= \max(S_1, \max(S_2, S_3, X, 0), 0)$$

Guaranteed Annuity Options

- **Stulz (1982) & Rubinstein (1991)**

$$PO_T = \max(\max(S_1, S_2) - X, 0)$$

$$Benefit = \max(AS_T, GB_T, GB_T g_T a_{x+T,i(T)})$$

$$GC_T = \max(AS_T, GB_T, GB_T g_T a_{x+T,i(T)}) - AS_T$$

$$= GB_T + \max(\max(AS_T, GB_T g_T a_{x+T,i(T)}) - GB_T, 0) - AS_T$$

$$ULBenefit = \max(AS_T, AS_T g_T a_{x+T,i(T)})$$

$$CommuteGC_T = \max(AS_T, gb_T a_{x+T,i(T)}, gb_T h_T) - AS_T$$

Regular Premium Business

- **Asian Options**

  Options based on the average rather than the terminal value of the underlying asset/ index

- **Regular Premium**

  Policyholder has an option to pay future premium instalments -- office

$$AS_T = (1-w)^T (1-k) P \left[\frac{I_T}{I_0} + \frac{I_T}{I_1} + \cdots + \frac{I_T}{I_{T-1}} \right]$$

$$\begin{aligned} AS_T &= (1-w)^T (1-k) P [S_1 + S_2 + \cdots S_T] \\ &= (1-w)^T (1-k) PT\bar{S}_T \end{aligned}$$

$$\begin{aligned} GC_T &= \max(GB_T - AS_T, 0) \\ &= \max(GB_T - (1-w)^T (1-k) TP\bar{S}_T, 0) \end{aligned}$$

State of the Art Risk Management

Economic Balance Sheet
Unhedged

Starting Position	Total Liabs	Total Assets	Equity	Bonds	Derivatives
Asset Backing	100	100	50	50	
Value of Guarantee	20	20	10	10	
Balance	5	5	2.5	2.5	
Total	125	125	62.5	62.5	

Equities down 20%	Total Liabs	Total Assets	Equity	Bonds	Derivatives
Asset Backing	90	90	40	50	
Value of Guarantee	25.6	18	8	10	
Balance	-3.1	4.5	2	2.5	
Total	112.5	112.5	50	62.5	

- Scenario: equities down 20%, bonds unchanged
- 20% equity fall → value of guarantee increases to 25.6
- ALM capital impact: £7.6. Difference between new value of guarantees and re-valued assets that support guarantees (= 25.6–18)
- Capital injection = £3.1 to restore solvency and £7.6 to restore financial strength to perfectly matched position

Economic Balance Sheet
OTC Hedge

Starting Position	Total Liabs	Total Assets	Equity	Bonds	Derivatives
Asset Share	100	100	50	50	
Value of Guarantee	20	20			20
Balance	5	5	2.5	2.5	
Total	125	125	52.5	52.5	20

Equities down 20%	Total Liabs	Total Assets	Equity	Bonds	Derivatives
Asset Share	90	90	40	50	
Value of Guarantee	25.6	25.6			25.6
Balance	4.5	4.5	2	2.5	
Total	120.1	120.1	42	52.5	25.6

- Buy an OTC put option to match the liability guarantee
- In theory a perfect match: no ALM impact or capital injection; but:
  - Pricing & Cash Flow
  - At mercy of market: pay implied volatility of market / counterparty
  - Other assumptions needed to make hedge perfect
  - Periodic rebalancing required, incurs high transactions costs

Replication

Put Option = (Short Position in Underlying)
+ Long Position in Risk Free

+

Position in Volatility Sensitive Asset
+ Position in Interest Sensitive Asset

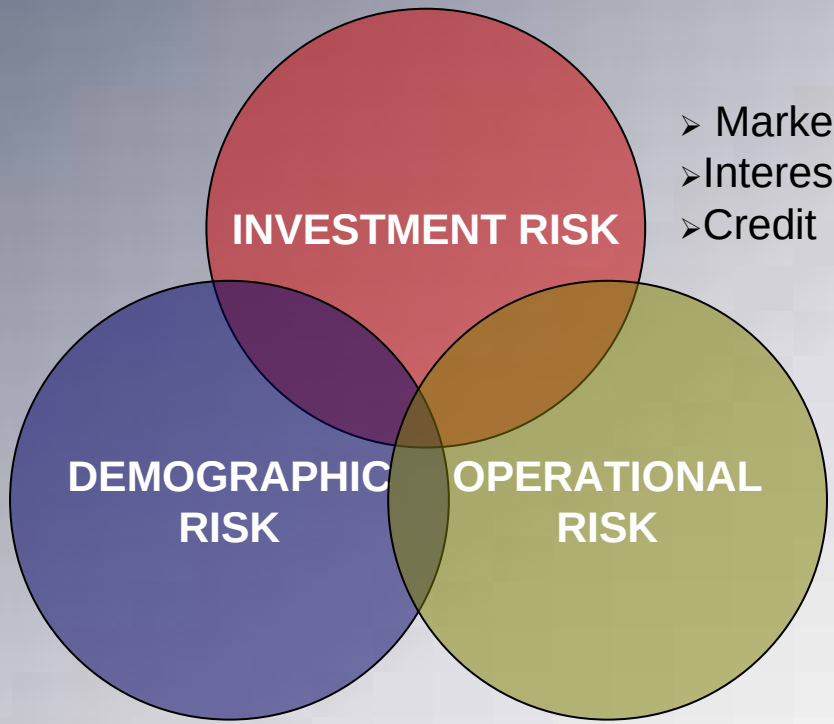
- Delta: → Protects against small immediate changes in underlying
- Gamma: → Protects against small immediate changes in delta
- Vega: → Protects against small immediate changes in Vol
- Rho: → Protects against small immediate changes in interest

Dynamic Hedge in the Balance Sheet

			Equity	Bonds	Risk Free	Options	Swaps
Asset Backing							
Value of Guarantee	20		(24)	(24)	60	5	3
Balance							

- "Manufacture" the OTC derivative
- Protects against risks of:
  - Underlying assets falling
  - Volatility spiking
  - Interest rates changing
- Dynamic ➔ Technology requirements

Risk Inventory



- Market
- Interest Rate
- Credit

- Underwriting
- Experience Statistics
- Reinsurance

- Procedures and Controls
- Mis-selling

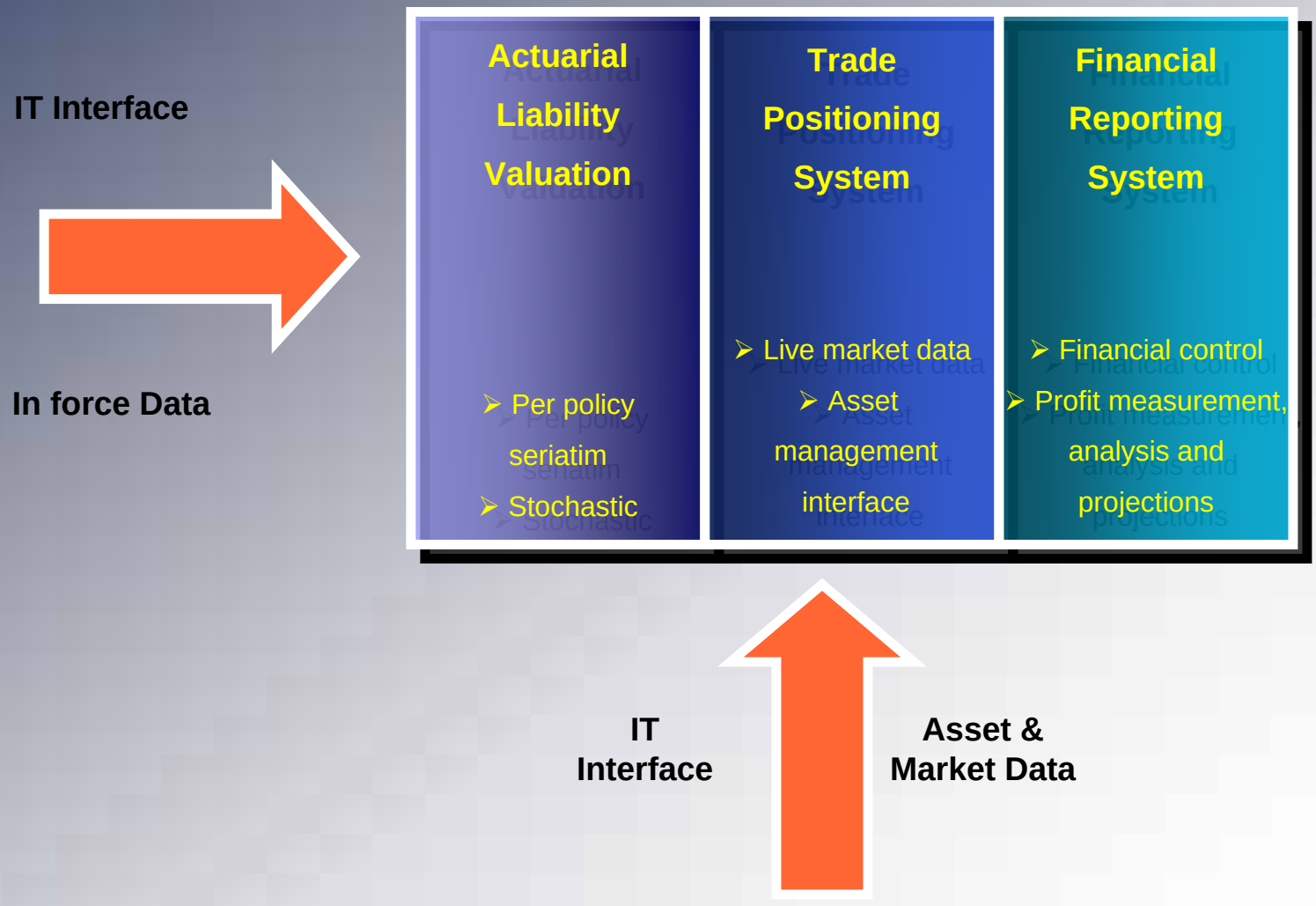
Dynamic Hedging: Requirements

- Market consistent stochastic valuation of liability guarantees on a policy-by-policy and daily basis
- Capital markets expertise to identify hedge portfolio
- Systems capability to dynamically monitor position frequently
- Effective control systems
  - Performance and risk attribution through analysis of profit movements
  - Financial reporting requirements
  - Financial projection analysis – back testing of strategy to demonstrate effectiveness of the hedge

- The bottom line: a very complex task requiring specialist actuarial, capital market and systems expertise / investment

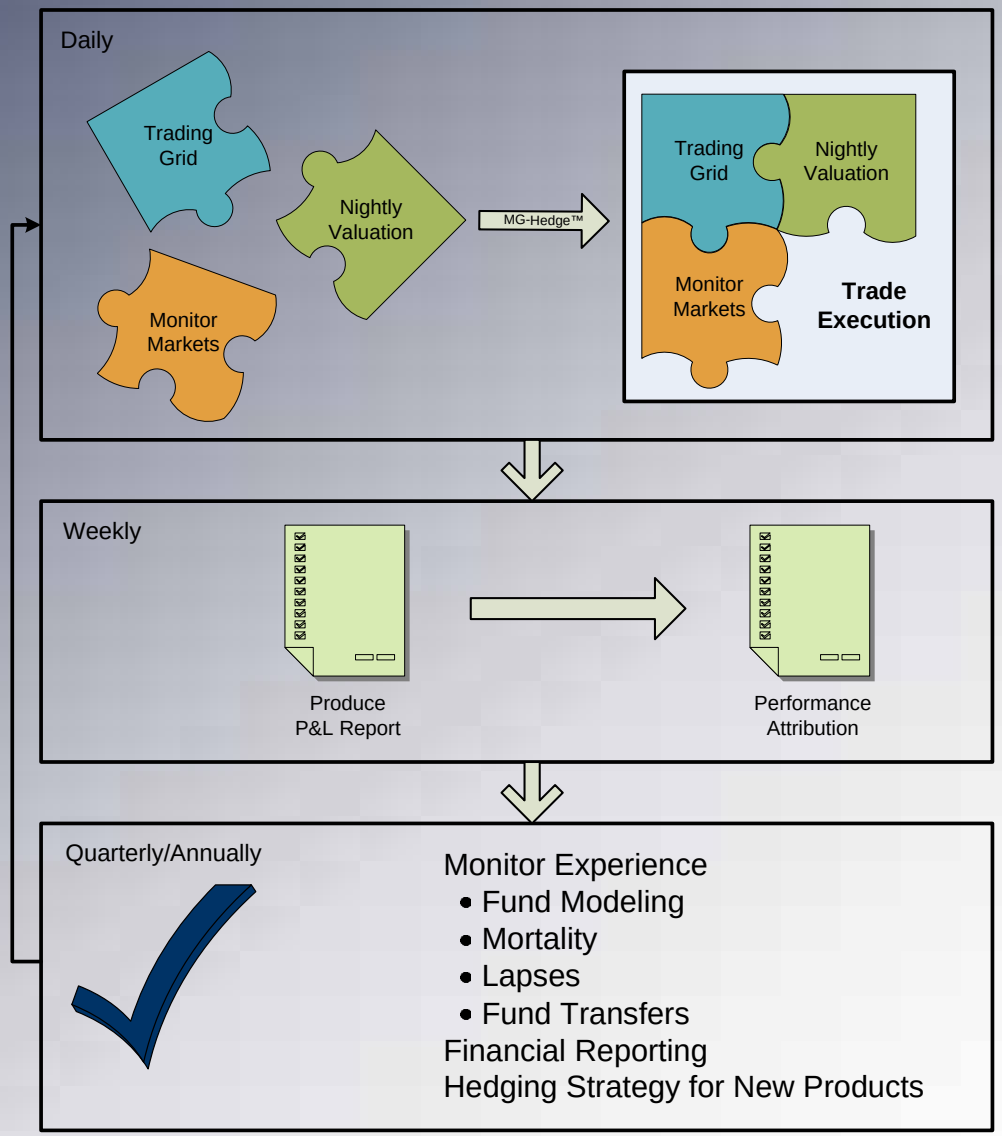
MG-Hedge System

structure reflects market risk management framework



Overview of Hedging Program

for Insurance Guarantees



Goal of Hedging

Replicate Embedded Option So That:

Beginning of Period Guarantee Value	Beginning of Period Hedge Asset Value
+ Interest	+ Gains on Hedge Assets due to market movements
- Claims	
+ Guarantee Premiums	- Losses on Hedge Assets due to market movements
+ Changes due to market movements	
End of Period Guarantee Value	**End of Period Hedge Asset Value**

EOP Guarantee Value

–

EOP Hedge Asset Value

minimize ***Net Gain (Loss)***

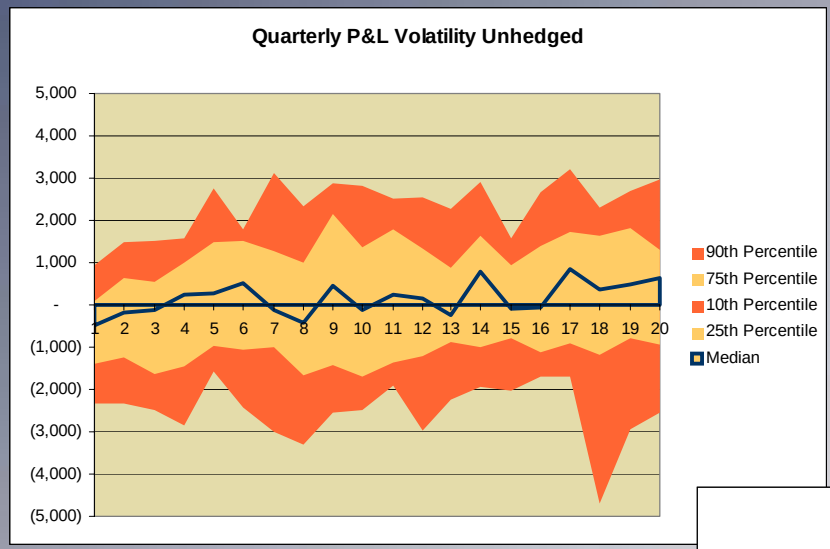
minimize ***Balance Sheet Volatility***

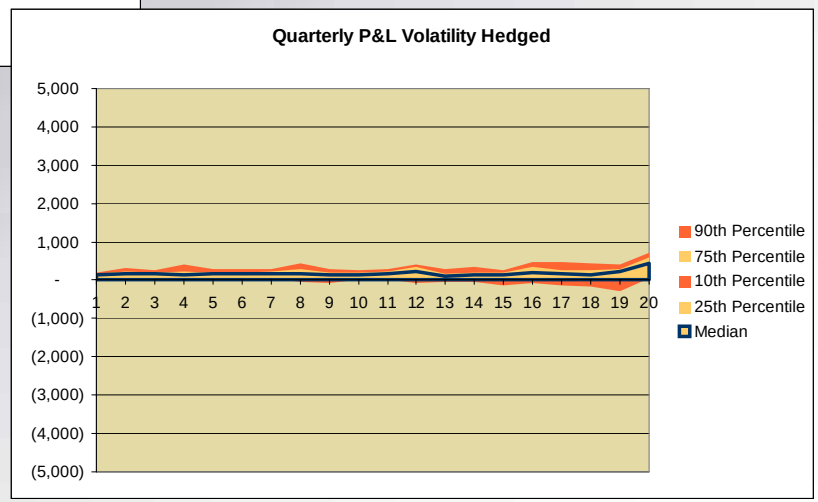
Allowing for Demographic & Path Dependencies

Hedge Report
Income Statements & Projections

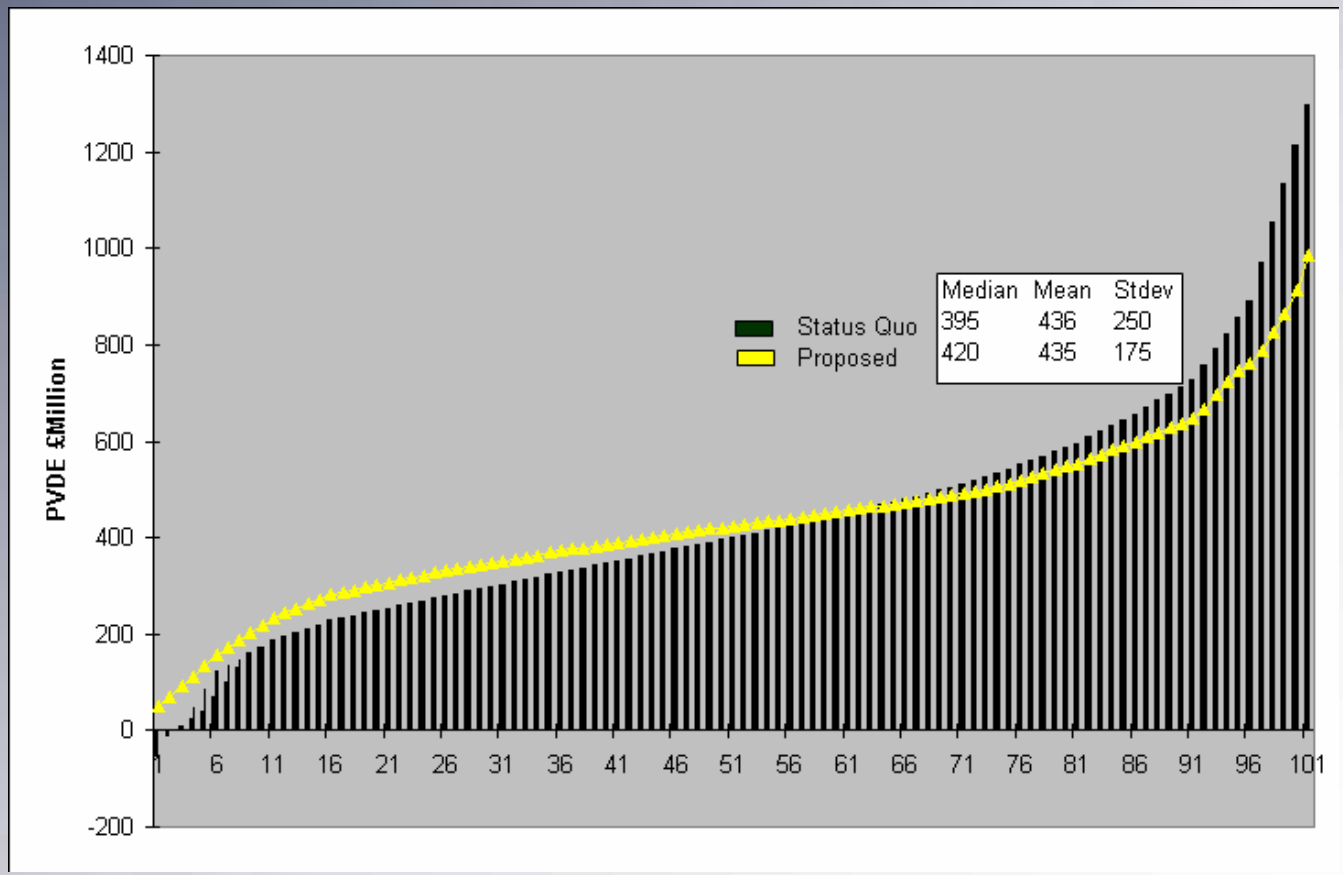
Income Statement Projection					
	Projection Year				
	1	2	3	4	5
Total Income	$ 12,274.0	$ 46,955.1	$ 98,128.8	$ 124,681.3	$ 7,796.7
Premium Income	$ 668.4	$ 1,728.0	$ 3,461.3	$ 2,901.5	$ 2,608.6
Investment Income	$ 11,605.6	$ 45,227.0	$ 94,667.5	$ 121,779.8	$ 5,188.1
Fixed Income Portfolio	$ 244.0	$ 1,470.2	$ 4,729.2	$ 10,155.3	$ 13,586.5
Futures	$ 1,864.6	$ 6,156.0	$ 8,421.2	$ 6,260.8	$ 333.8
Options & Swaps	$ 9,497.0	$ 37,600.8	$ 81,517.1	$ 105,363.7	$ (8,732.2)
Total Expenses	$ 9,826.8	$ 39,441.3	$ 91,189.1	$ 125,981.0	$ 11,270.7
Increase in Fair Value Liability	$ 9,758.1	$ 39,291.6	$ 91,067.3	$ 125,977.3	$ 11,270.7
Interest on Debt	$ 68.6	$ 149.7	$ 121.8	$ 3.7	$ -
Pre-Tax Income	$ 2,447.2	$ 7,513.8	$ 6,939.7	$ (1,299.7)	$ (3,474.0)
Equity Market Return	-25%	-25%	-25%	-25%	15%
10 Year Interest Rate	5%	5%	4%	4%	3%

Hedge Reporting
P&L - Unhedged vs. Hedged





Embedded Value Reporting



Replication

Reddington Immunisation	Black Scholes Dynamic Hedging
Immunises Small immediate changes	Immunises Small Immediate Changes
Frequent Rebalancing	Frequent Rebalancing
Duration Matching (rate of change wrt underlying interest)	Delta (rate of change of value with respect to underlying)
Convexity Matching (second derivative)	Gamma (second derivative)
	ADD: Vega, Rho etc

Contacts

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