

A MILLIMAN GLOBAL FIRM



Milliman

Consultants and Actuaries

Exotic Options

Through 100 Years to Current State of the Art
Risk Management of Investment Guarantees

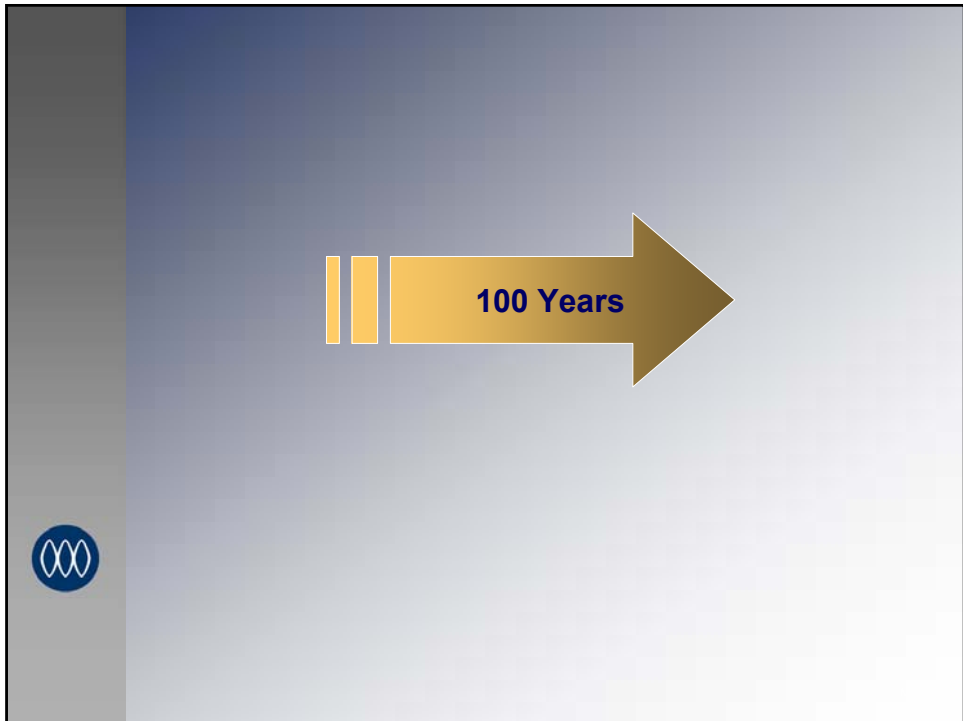
Contents

The First 100 Years

Exotic Options

State of the Art
Risk Management





The First 75 Years

- Bachelier (1900)
- Levy (1925)
- Ito (1951)
- Markowitz (1952)
- Reddington (1952)
- Sprenkle (1958)
- Samuelson (1965)
- Thorpe (1969)
- Black & Scholes (73)

Black & Scholes

- **Black & Scholes (1973)**
- **Merton (1973)**
- **Harrison & Kreps (1977)**



Exotic Options

- **Margrabe (1978)**

$$PO_T = \max(Q_1 S_1(T) - Q_2 S_2(T), 0)$$

- **Stulz (1982) & Rubinstein (1991)**

$$PO_T = \max(\max(S_1, S_2) - X, 0)$$

- **Turnbull & Wakeman (1991),
Curran (1992, 1994), Yor(1993),
Zhang (1994).**



Deferred Annuities

- **Margrabe (1978)**

$$PO_T = \max(Q_1 S_1(T) - Q_2 S_2(T), 0)$$



Deferred Annuities

- **Margrabe (1978)**

$$PO_T = \max(Q_1 S_1(T) - Q_2 S_2(T), 0)$$

$$GC_T = \max(gb_T a_{x+T, i(T)} - AS_T, 0)$$



Compound Options

- **Stulz (1982) & Rubinstein (1991)**

$$PO_T = \max(\max(S_1, S_2) - X, 0)$$



Compound Options

- **Stulz (1982) & Rubinstein (1991)**

$$PO_T = \max(\max(S_1, S_2) - X, 0)$$

- **Useful for valuing benefits of the form**

$$\begin{aligned} &\max(S_1, S_2, X) \\ &= X + \max(\max(S_1, S_2) - X, 0) \end{aligned}$$

- **Generalises**

$$\begin{aligned} &\max(S_1, S_2, S_3, X) \\ &= \max(S_1, \max(S_2, S_3, X), 0) \end{aligned}$$



Guaranteed Annuity Options

- **Stulz (1982) & Rubinstein (1991)**

$$PO_T = \max(\max(S_1, S_2) - X, 0)$$

$$Benefit = \max(AS_T, GB_T, GB_T g_T a_{x+T, i(T)})$$

$$\begin{aligned} GC_T &= \max(AS_T, GB_T, GB_T g_T a_{x+T, i(T)}) - AS_T \\ &= GB_T + \max(\max(AS_T, GB_T g_T a_{x+T, i(T)}) - GB_T, 0) - AS_T \end{aligned}$$

$$ULBenefit = \max(AS_T, AS_T g_T a_{x+T, i(T)})$$

$$CommuteGC_T = \max(AS_T, gb_T a_{x+T, i(T)}, gb_T h_T) - AS_T$$



Regular Premium Business

- **Asian Options**

Options based on the average rather than the terminal value of the underlying asset/ index

- **Regular Premium**

Policyholder has an option to pay future premium instalments -- office

$$AS_T = (1-w)^T (1-k)P \left[\frac{I_T}{I_0} + \frac{I_T}{I_1} + \dots + \frac{I_T}{I_{T-1}} \right]$$

$$\begin{aligned} AS_T &= (1-w)^T (1-k)P[S_1 + S_2 + \dots + S_T] \\ &= (1-w)^T (1-k)PT\bar{S}_T \end{aligned}$$

$$\begin{aligned} GC_T &= \max(GB_T - AS_T, 0) \\ &= \max(GB_T - (1-w)^T (1-k)TP\bar{S}_T, 0) \end{aligned}$$



**State of the Art
Risk Management**



Economic Balance Sheet Unhedged

Starting Position	Total Liabs	Total Assets	Equity	Bonds	Derivatives	Equities down 20%	Total Liabs	Total Assets	Equity	Bonds	Derivatives
Asset Backing	100	100	50	50		Asset Backing	90	90	40	50	
Value of Guarantee	20	20	10	10		Value of Guarantee	25.6	18	8	10	
Balance	5	5	2.5	2.5		Balance	-3.1	4.5	2	2.5	
Total	125	125	62.5	62.5		Total	112.5	112.5	50	62.5	

- Scenario: equities down 20%, bonds unchanged
- 20% equity fall → value of guarantee increases to 25.6
- ALM capital impact: £7.6. Difference between new value of guarantees and re-valued assets that support guarantees (= 25.6–18)
- Capital injection = £3.1 to restore solvency and £7.6 to restore financial strength to perfectly matched position



Economic Balance Sheet OTC Hedge

Starting Position	Total Liabs	Total Assets	Equity	Bonds	Derivatives	Equities down 20%	Total Liabs	Total Assets	Equity	Bonds	Derivatives
Asset Share	100	100	50	50		Asset Share	90	90	40	50	
Value of Guarantee	20	20			20	Value of Guarantee	25.6	25.6			25.6
Balance	5	5	2.5	2.5		Balance	4.5	4.5	2	2.5	
Total	125	125	52.5	52.5	20	Total	120.1	120.1	42	52.5	25.6

- Buy an OTC put option to match the liability guarantee
- In theory a perfect match: no ALM impact or capital injection; but:
 - Pricing & Cash Flow
 - At mercy of market: pay implied volatility of market / counterparty
 - Other assumptions needed to make hedge perfect
 - Periodic rebalancing required, incurs high transactions costs



Replication

Put Option = (Short Position in Underlying)
+ Long Position in Risk Free

+

Position in Volatility Sensitive Asset
+ Position in Interest Sensitive Asset

- Delta: → Protects against small immediate changes in underlying
- Gamma: → Protects against small immediate changes in delta
- Vega: → Protects against small immediate changes in Vol
- Rho: → Protects against small immediate changes in interest



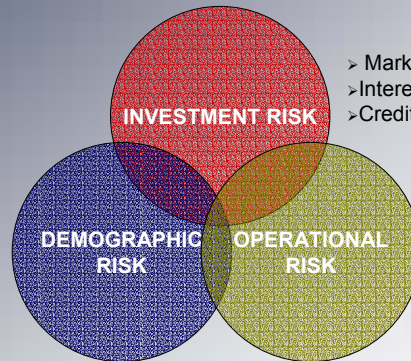
Dynamic Hedge in the Balance Sheet

		Equity	Bonds	Risk Free	Options	Swaps
Asset Backing						
Value of Guarantee	20	(24)	(24)	60	5	3
Balance						

- “Manufacture” the OTC derivative
- Protects against risks of:
 - Underlying assets falling
 - Volatility spiking
 - Interest rates changing
- Dynamic → Technology requirements



Risk Inventory



- Market
- Interest Rate
- Credit

- Underwriting
- Experience Statistics
- Reinsurance

- Procedures and Controls
- Mis-selling



Dynamic Hedging: Requirements

- Market consistent stochastic valuation of liability guarantees on a policy-by-policy and daily basis
- Capital markets expertise to identify hedge portfolio
- Systems capability to dynamically monitor position frequently
- Effective control systems
 - Performance and risk attribution through analysis of profit movements
 - Financial reporting requirements
 - Financial projection analysis – back testing of strategy to demonstrate effectiveness of the hedge
- **The bottom line:** a very complex task requiring specialist actuarial, capital market and systems expertise / investment



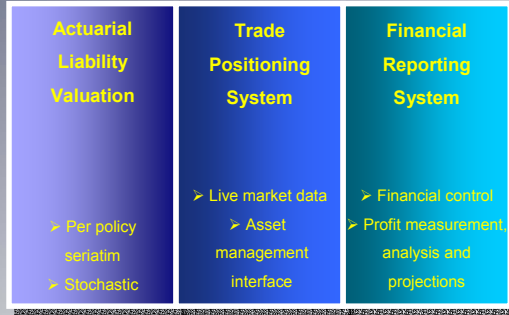
MG-Hedge System

structure reflects market risk management framework

IT Interface



In force Data



IT Interface

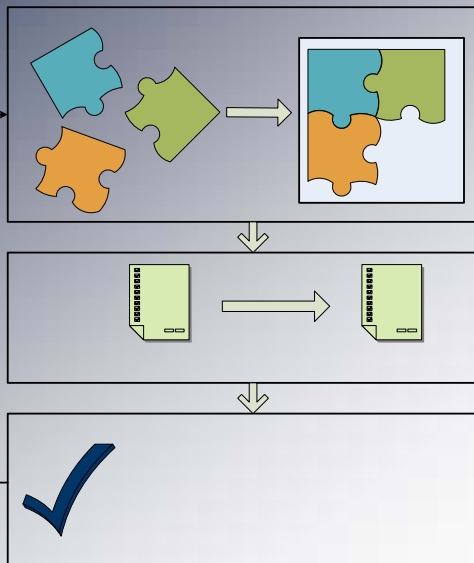


Asset & Market Data



Overview of Hedging Program

for Insurance Guarantees



Goal of Hedging

Replicate Embedded Option So That:

Beginning of Period Guarantee Value + Interest - Claims + Guarantee Premiums + Changes due to market movements		Beginning of Period Hedge Asset Value + Gains on Hedge Assets due to market movements - Losses on Hedge Assets due to market movements
End of Period Guarantee Value		End of Period Hedge Asset Value

EOP Guarantee Value

-

EOP Hedge Asset Value

Net Gain (Loss)

minimize →

minimize →

Balance Sheet Volatility

Allowing for Demographic & Path Dependencies



Hedge Report

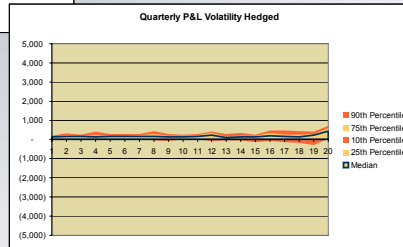
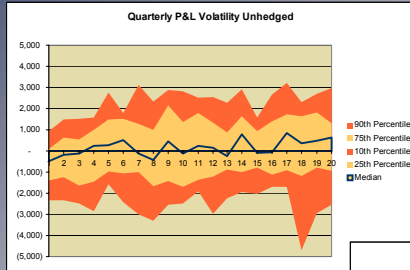
Income Statements & Projections

Income Statement Projection					
	Projection Year				
	1	2	3	4	5
Total Income	\$ 12,274.0	\$ 46,955.1	\$ 98,128.8	\$ 124,681.3	\$ 7,796.7
Premium Income	\$ 668.4	\$ 1,728.0	\$ 3,461.3	\$ 2,901.5	\$ 2,608.6
Investment Income	\$ 11,605.6	\$ 45,227.0	\$ 94,667.5	\$ 121,779.8	\$ 5,188.1
Fixed Income Portfolio	\$ 244.0	\$ 1,470.2	\$ 4,729.2	\$ 10,155.3	\$ 13,586.5
Futures	\$ 1,864.6	\$ 6,156.0	\$ 8,421.2	\$ 6,260.8	\$ 333.8
Options & Swaps	\$ 9,497.0	\$ 37,600.8	\$ 81,517.1	\$ 105,363.7	\$ (8,732.2)
Total Expenses	\$ 9,826.8	\$ 39,441.3	\$ 91,189.1	\$ 125,981.0	\$ 11,270.7
Increase in Fair Value Liability	\$ 9,758.1	\$ 39,291.6	\$ 91,067.3	\$ 125,977.3	\$ 11,270.7
Interest on Debt	\$ 68.6	\$ 149.7	\$ 121.8	\$ 3.7	\$ -
Pre-Tax Income	\$ 2,447.2	\$ 7,513.8	\$ 6,939.7	\$ (1,299.7)	\$ (3,474.0)
Equity Market Return	-25%	-25%	-25%	-25%	15%
10 Year Interest Rate	5%	5%	4%	4%	3%

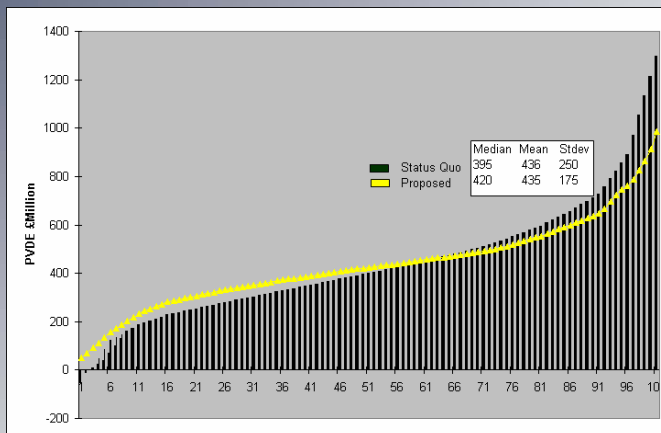


Hedge Reporting

P&L - Unhedged vs. Hedged



Embedded Value Reporting



Replication

Reddington Immunitisation	Black Scholes Dynamic Hedging
Immunises Small immediate changes	Immunises Small Immediate Changes
Frequent Rebalancing	Frequent Rebalancing
Duration Matching (rate of change wrt underlying interest)	Delta (rate of change of value with respect to underlying)
Convexity Matching (second derivative)	Gamma (second derivative)
	ADD: Vega, Rho etc



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