DOWNSIDE RISK – ECONOMETRIC MODELS AND FINANCIAL IMPLICATIONS

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RISK AND RETURN

- THE TRADE-OFF BETWEEN RISK AND RETURN IS THE CENTRAL PARADIGM OF FINANCE.
- HOW MUCH RISK AM I TAKING?
- HOW SHOULD I RESPOND TO RISKS THAT VARY OVER TIME?
- HOW SHOULD I RESPOND TO RISKS OF VARIOUS MATURITIES?

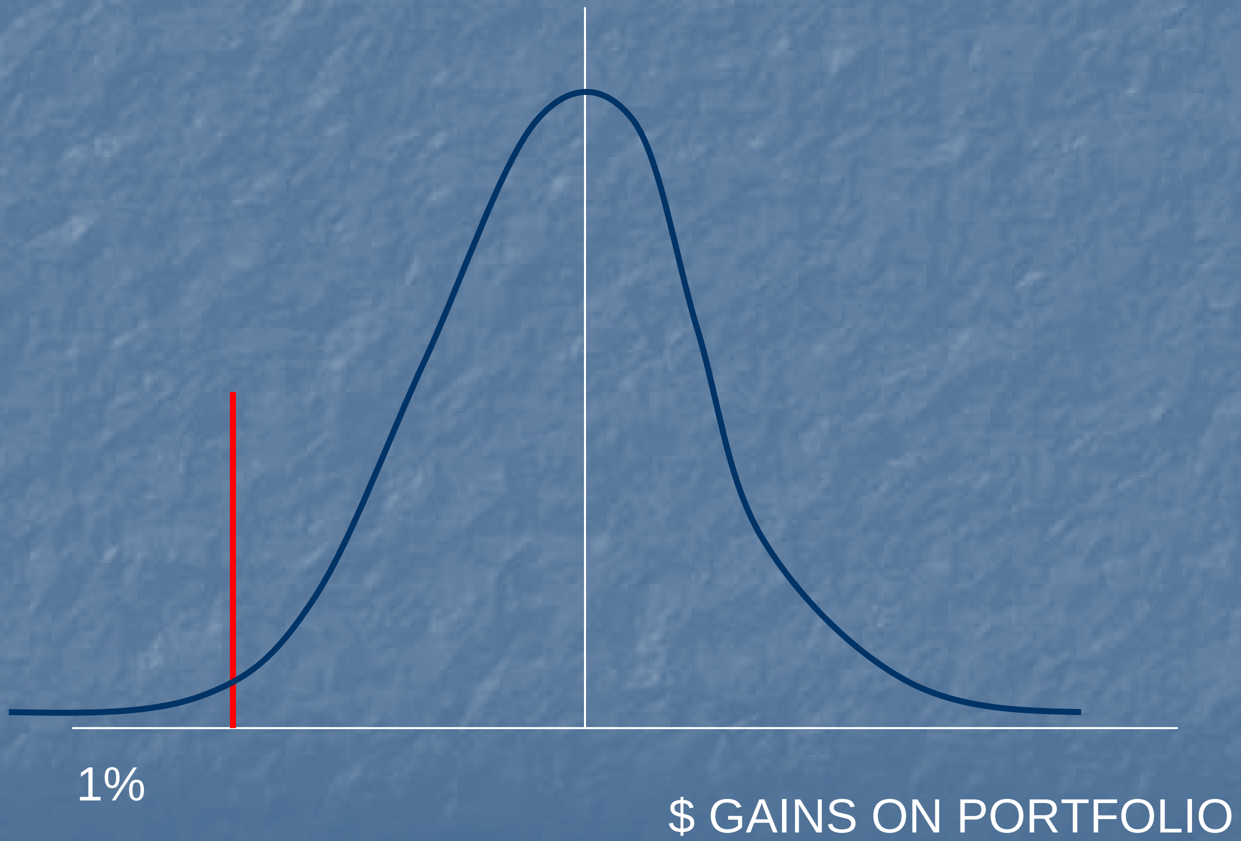
DOWNSIDE RISK

- The risk of a portfolio is that its value will decline, hence DOWNSIDE RISK is a natural measure of risk.
- Many theories and models assume symmetry: c.f. MARKOWITZ, TOBIN, SHARPE AND BLACK, SCHOLES, MERTON and Volatility based risk management systems.
- Do we miss anything important?

MEASURING DOWNSIDE RISK

- Many measures have been proposed. Let *r* be the one period continuously compounded return with distribution *f(r)* and mean zero. Let x be a threshold.
  - $\text{Skewness}=E\left(r^3\right)/E\left(r^2\right)^{3/2}$
  - $\text{Probability of loss} = P\left(r<x\right),$
  - $\text{Expected Shortfall} = E\left(r-x \middle| r<x\right)$
  - $\text{x is the } \alpha \text{ Value at risk if } P\left(r<-x\right)=\alpha$

PREDICTIVE DISTRIBUTION OF PORTFOLIO GAINS



MULTIVARIATE DOWNSIDE RISK

- WHAT IS THE LIKELIHOOD THAT A COLLECTION OF ASSETS WILL ALL DECLINE?
- THIS DEPENDS PARTLY ON CORRELATIONS
- FOR EXTREME MOVES, OTHER MEASURES ARE IMPORTANT TOO.

MULTIVARIATE DOWNSIDE

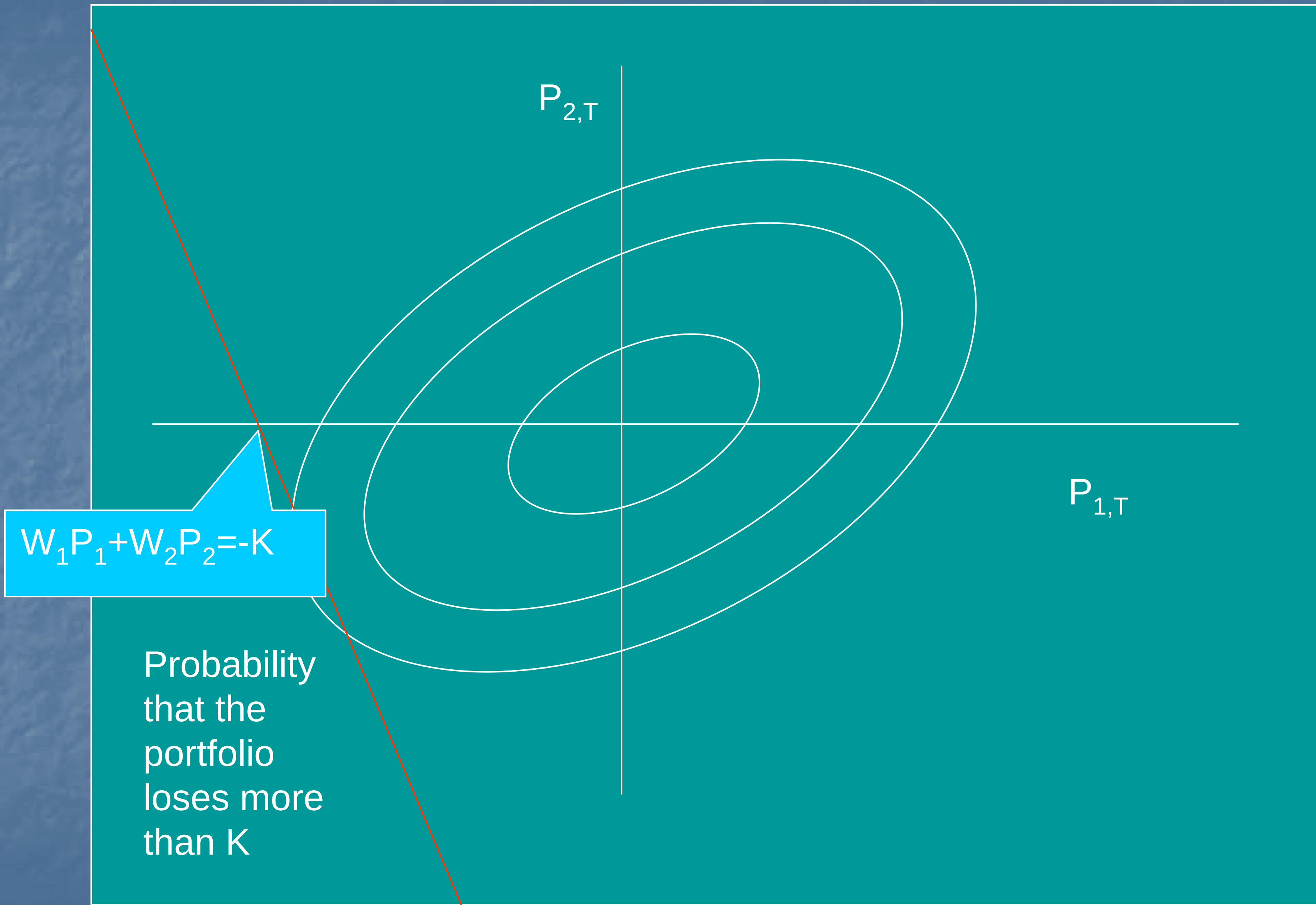
- “Where are my correlations when I need them?” – a portfolio manager’s lament.
- When country equity markets decline together more than can be expected from the normal correlation pattern, it is called CONTAGION.
- Correlations and volatilities appear to move together.

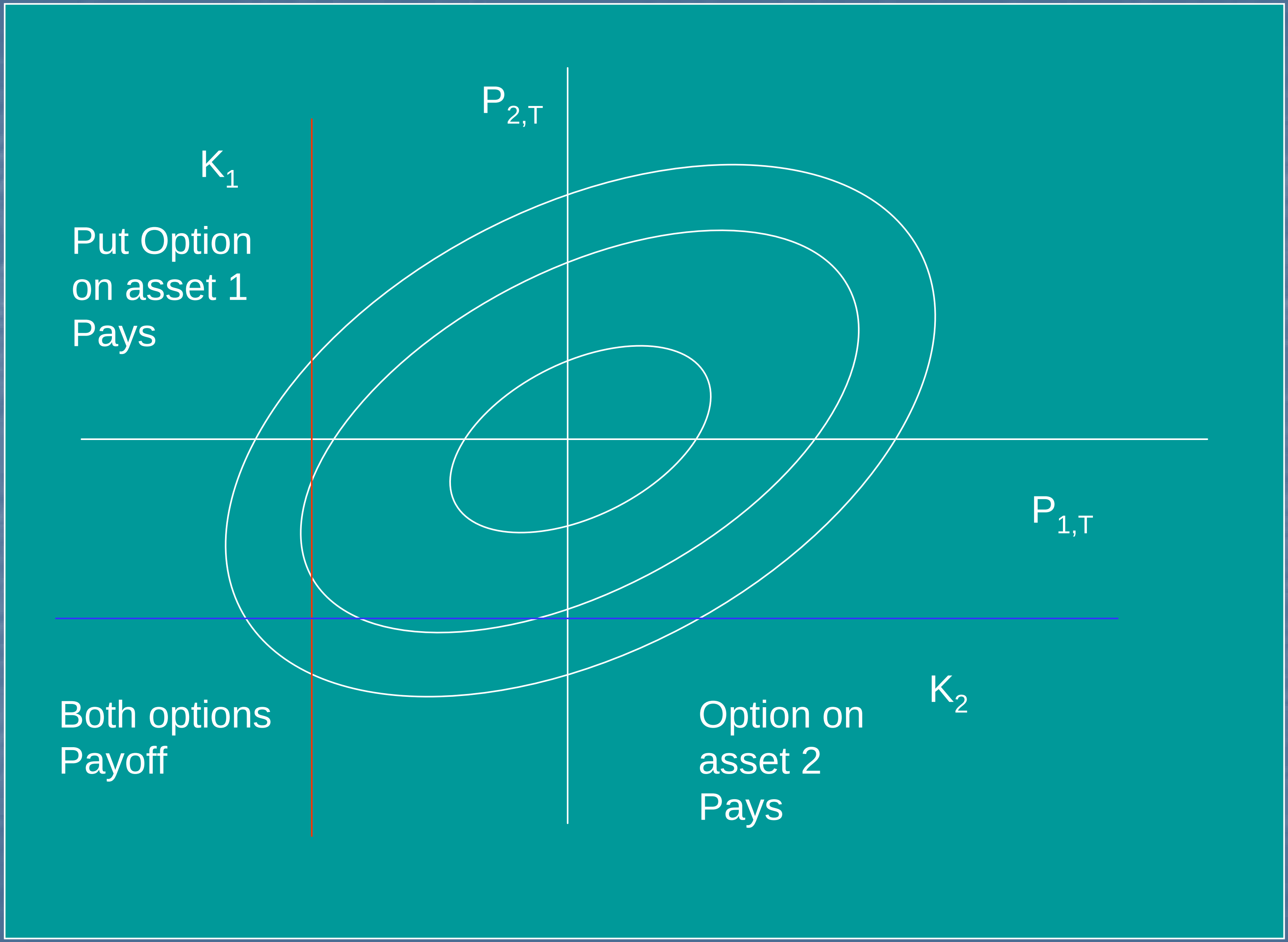
MEASURING JOINT DOWNSIDE RISK

- Let y_i be the return on asset i
- Find x such that $\alpha = P\left(y_i < x_i\right)$
- $$\lambda_\alpha = P\left(y_i < x_i \middle| y_j < x_j\right) = P\left(y_j < x_j \middle| y_i < x_i\right)$$
-
- *Tail dependence* (lower tail dependence) is defined as the limit as this probability goes to zero. What is the probability that one asset has an extreme down move when another has an extreme down move?

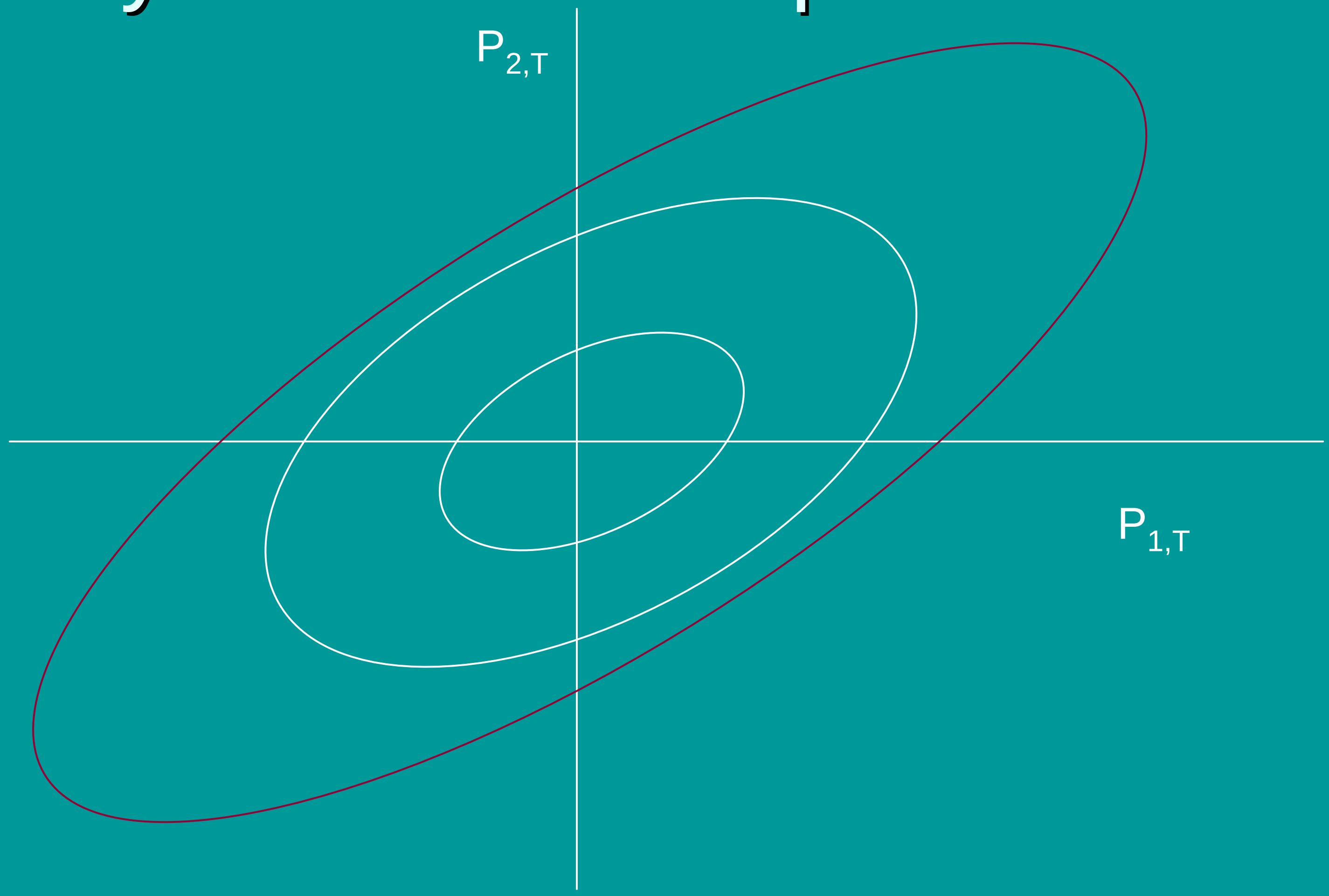
DEFAULT CORRELATIONS

- Define an indicator for default and measure the correlation between these indicators
- $$\rho_{i,j}^{D} = Corr\left(I_{\{y_i<x_i\}}, I_{\{y_j<x_j\}}\right)$$
  $$= \frac{P\left(I_{\{y_i<x_i\}} I_{\{y_j<x_j\}}\right) - \alpha^2}{\alpha\left(1-\alpha\right)}$$
  $$= \left(\lambda_\alpha - \alpha\right) / \left(1-\alpha\right)$$
- For extremes, the default correlation is the same as the tail dependence.

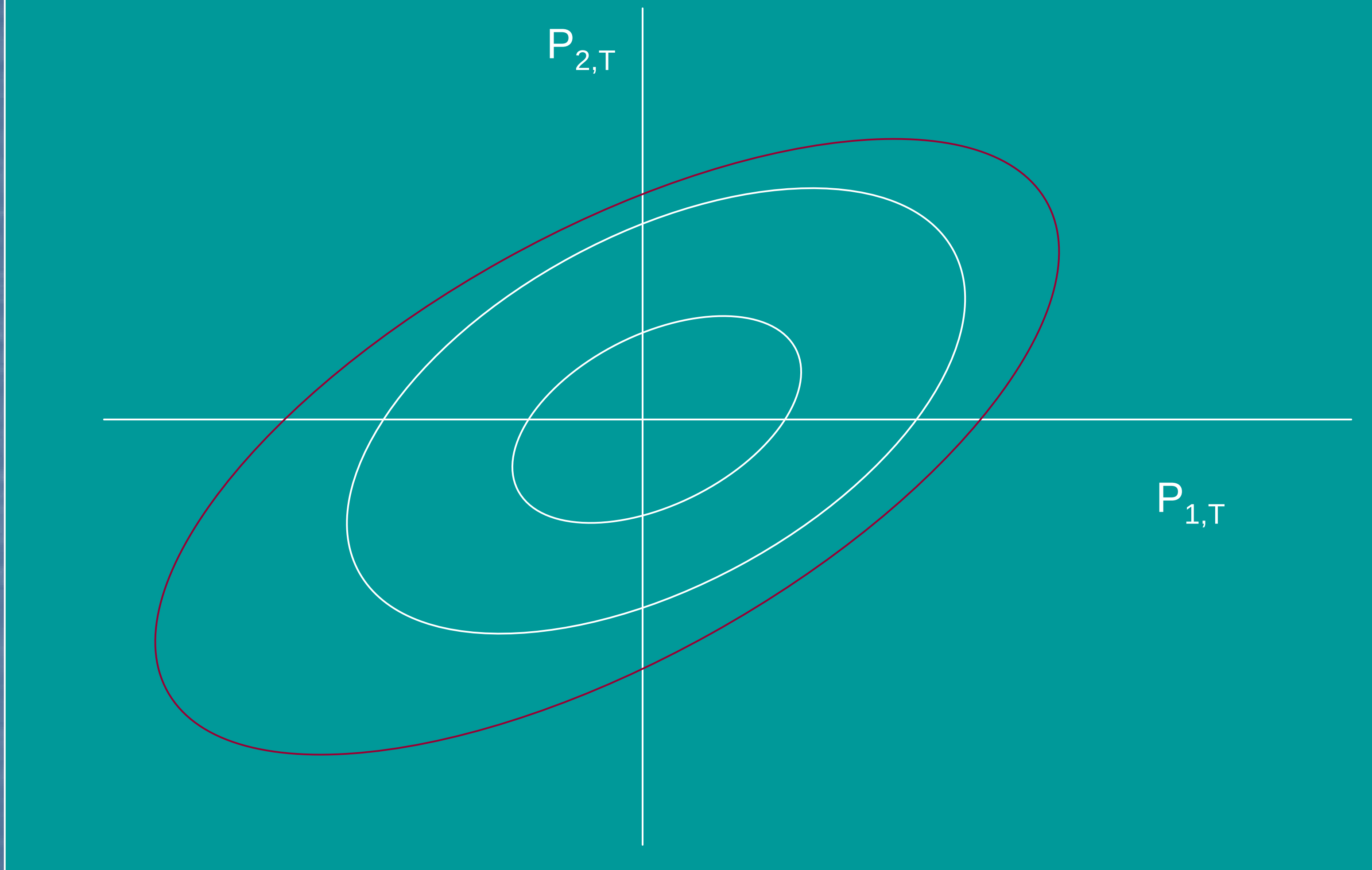

$P_{2,T}$
$P_{1,T}$
$W_1P_1+W_2P_2=-K$
Probability that the portfolio loses more than K

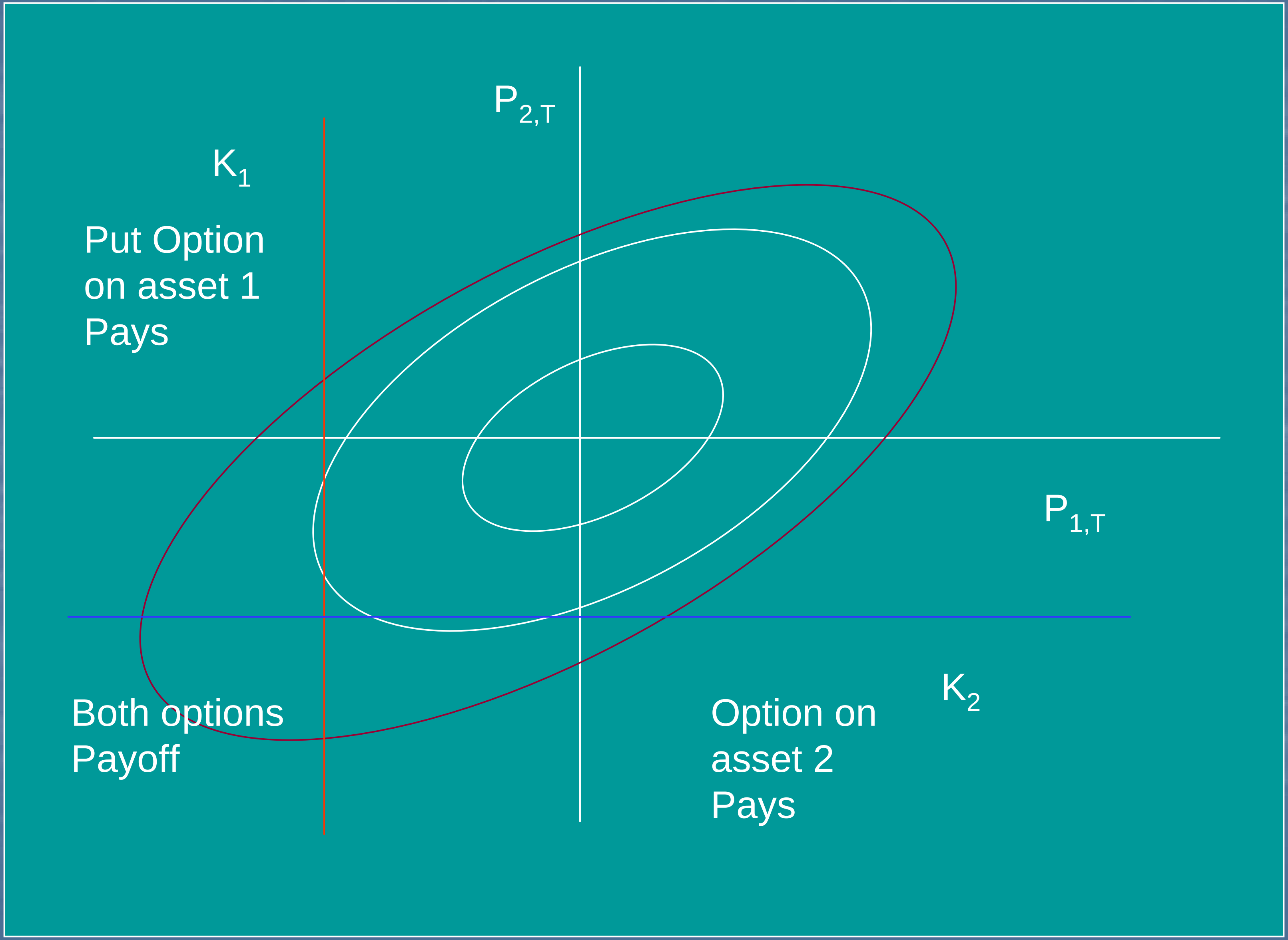

$P_{2,T}$
K_1
Put Option on asset 1 Pays
$P_{1,T}$
K_2
Both options Payoff
Option on asset 2 Pays

Symmetric Tail Dependence



Lower Tail Dependence




$P_{2,T}$
K_1
Put Option
on asset 1
Pays
$P_{1,T}$
K_2
Both options
Payoff
Option on
asset 2
Pays

CREDIT DERIVATIVES

- IT IS WELL DOCUMENTED THAT THE MULTIVARIATE NORMAL DENSITY UNDERPRICES JOINT EXTREME EVENTS SUCH AS DEFAULTS.
- INDUSTRY HAS ADOPTED A T-COPULA TO PRICE CREDIT BASKETS and CDO tranches.
- TAIL DEPENDENCE IS ESSENTIAL IN THESE MODELS.

THE PURPOSE OF MY TALK TODAY

TIME SERIES ANALYSIS OF DOWNSIDE RISK

PURPOSE OF MY TALK TODAY

- TO SHOW HOW DOWNSIDE RISK CAN BE MODELED AS A TIME SERIES PROCESS
- USING SIMPLY TIME AGGREGATION OF STANDARD TIME SERIES MODELS
- *CONSEQUENTLY*
- DOWNSIDE RISK CAN BE PREDICTED
- DYNAMIC HEDGING AND DYNAMIC PORTFOLIO STRATEGIES CAN BE IMPLEMENTED.

AN ECONOMETRIC FRAMEWORK

- MODEL THE ONE PERIOD RETURN AND CALCULATE THE MULTI-PERIOD DISTRIBUTION
- RETURN FROM t UNTIL t + T IS:

$$R_{t+T} = \sum_{j=t+1}^{T+t} r_j$$

ALL MEASURES CAN BE DERIVED FROM THE ONE PERIOD DENSITY

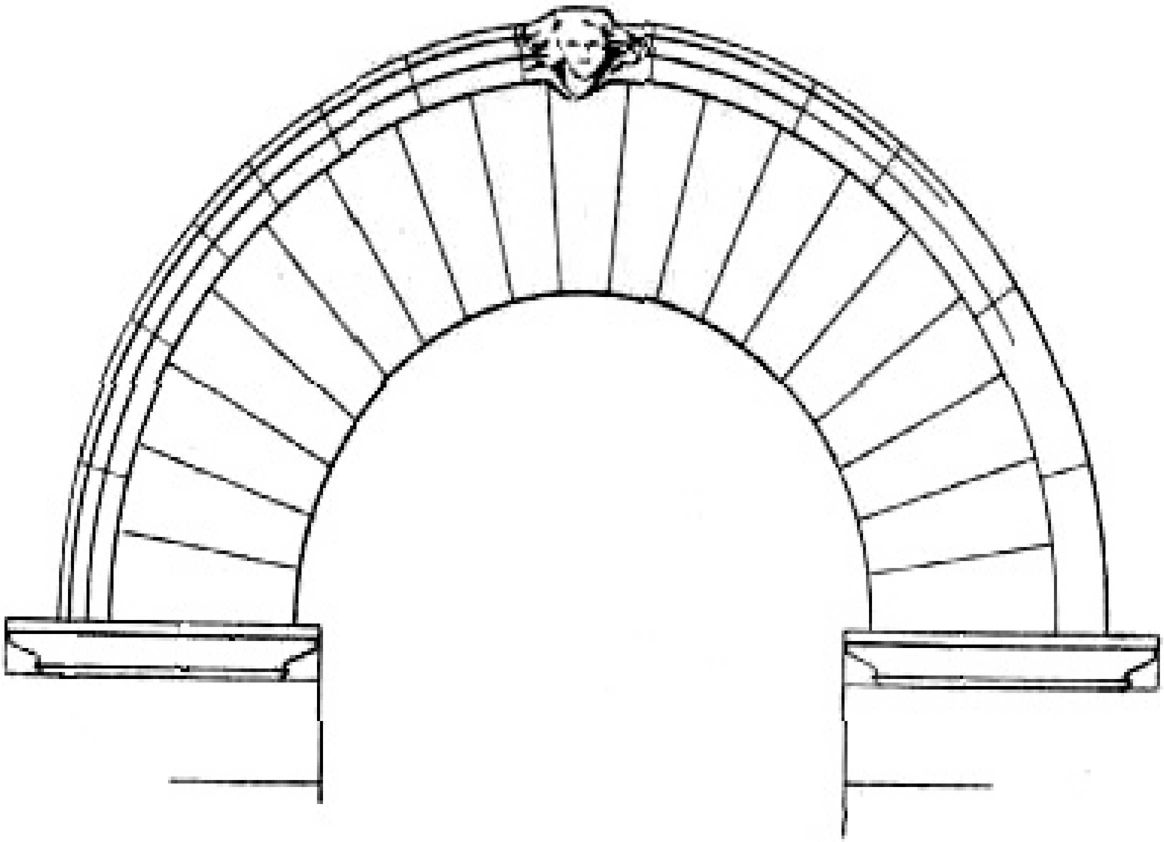
- EVALUATE ANY MEASURE BY REPEATEDLY SIMULATING FROM THE ONE PERIOD CONDITIONAL DISTRIBUTION:
- METHOD: $$f_t\left(r_{t+1}\right)$$
  - Draw r_{t+1}
  - Update density and draw observation t+2
  - Continue until T returns are computed.
  - Compute measure of downside risk

A MODEL

- MEAN ZERO, TIME VARYING VOLATILITY
- $$r_t = \sqrt{h_t}\varepsilon_t,\ \ \varepsilon_t \sim i.i.d.$$
  $$E_{t-1}\left(r_t\right) = 0, h_t = V_{t-1}\left(r_t\right)$$
- ASYMMETRY
  - FOLLOWS FROM ASYMMETRY IN SHOCKS
  - HOWEVER FOR MULTI-PERIOD RETURNS, THERE IS ANOTHER SOURCE – *ASYMMETRIC VOLATILITY.*

The ARCH Model

- The ARCH model of Engle(1982) is a family of specifications for the conditional variance.
- The q^{th} order ARCH or ARCH(q) model is a weighted average of squared returns.
- $$h_t = \omega + \sum_{j=1}^{q} \alpha_j r_{t-j}{}^2$$
- Notice that it is a simple generalization of both constant variance and rolling variance estimates.



GARCH

- The Generalized ARCH model of Bollerslev(1986) is an ARMA version of this model.
- The GARCH(1,1) is the workhorse

$$h_t = \omega + \alpha r_{t-1}{}^2 + \beta h_{t-1}$$

GARCH

Asymmetric Volatility

- Often negative shocks have a bigger effect on volatility than positive shocks
- Nelson(1987) introduced the EGARCH model to incorporate this effect.
- I will use a Threshold GARCH or TARCH which is like a GARCH but where negative returns get an extra boost.

$$h_t = \omega + \alpha r_{t-1}^2 + \gamma r_{t-1}^2 I_{(r_{t-1}<0)} + \beta h_{t-1}$$

WHERE DOES ASYMMETRIC VOLATILITY COME FROM?

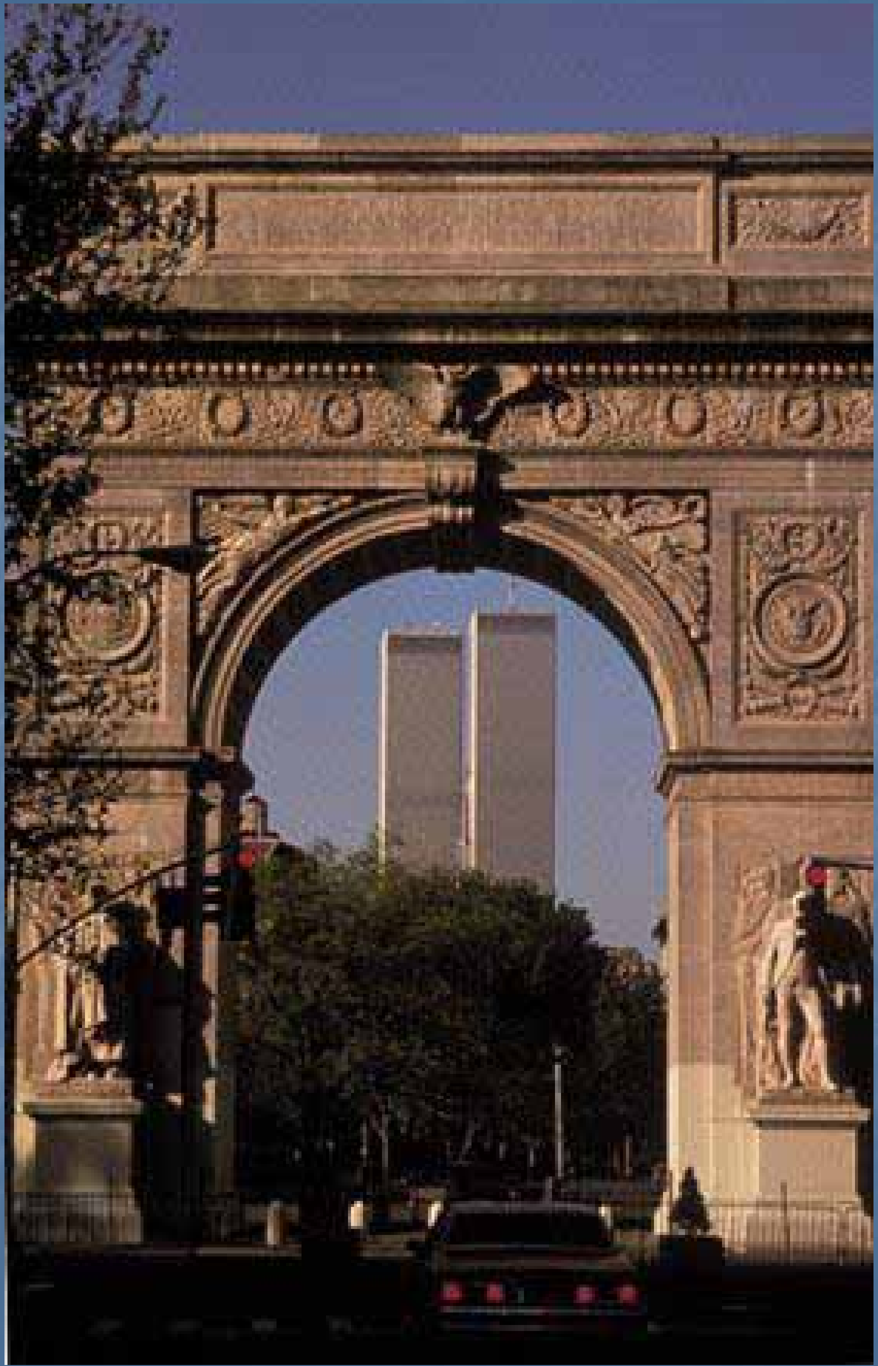
- LEVERAGE - As equity prices fall the leverage of a firm increases so that the next shock has a greater effect on stock prices.
- This effect is usually too small to explain what we see.

WHERE DOES ASYMMETRIC VOLATILITY COME FROM?

- RISK AVERSION– News of a future volatility event will lead to stock sales and price declines now. Subsequently, the volatility event occurs. Since events are clustered, any news event will predict higher volatility in the future.
- This effect is especially relevant for broad market indices since these have systematic risk.

NEW ARCH MODELS

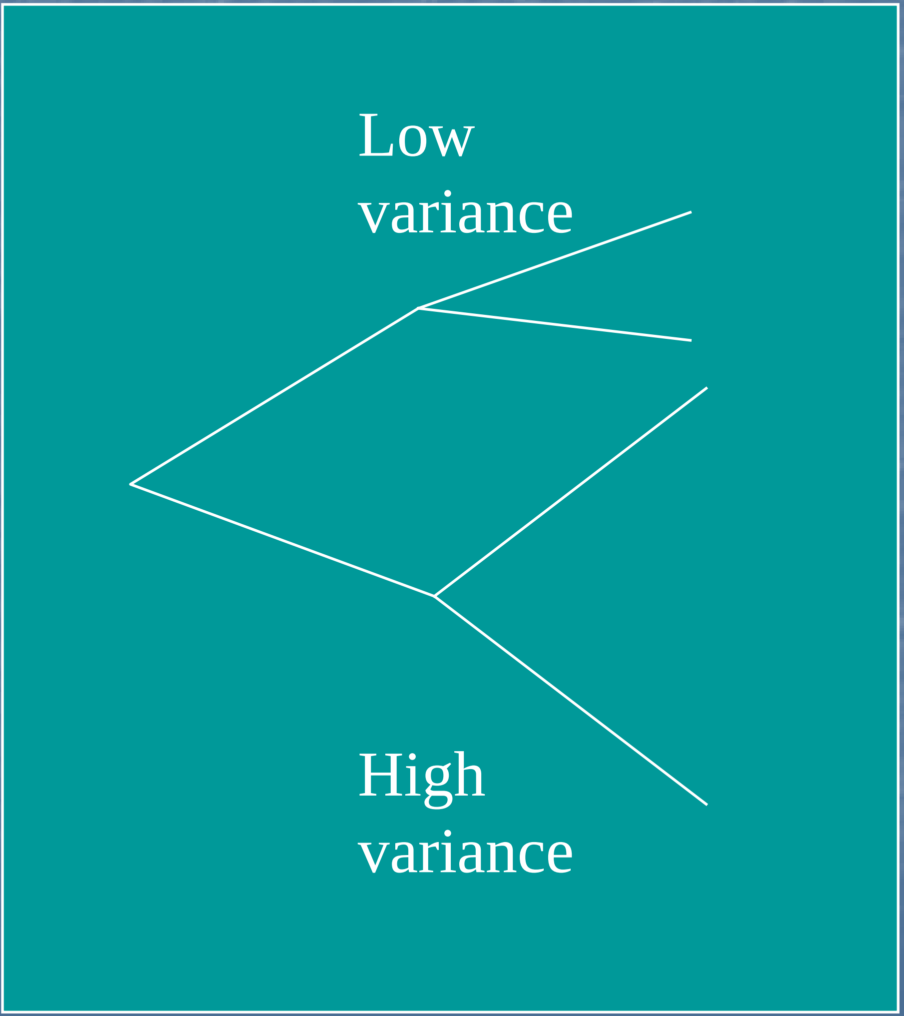
- GJR-GARCH
- TARCH
- STARCH
- AARCH
- NARCH
- MARCH
- SWARCH
- SNPARCH
- APARCH
- TAYLOR-SCHWERT
- FIGARCH
- FIEGARCH
- Component
- Asymmetric Component
- SQGARCH
- CESGARCH
- Student t
- GED
- SPARCH
- Autoregressive Conditional Density
- Autoregressive Conditional Skewness




Château d'Arche
GRAND CRU CLASSÉ
Sauternes
1986
MISE EN BOUTEILLES AU CHATEAU

TWO PERIOD RETURNS

- Two period return is the sum of two one period continuously compounded returns
- Look at binomial tree version
- Asymmetric Volatility gives negative skewness



ANALYTICALLY: TARCH
WITH SYMMETRIC INNOVATIONS

$$E\left(r_t + r_{t+1}\right)^3 = E\left(r_t^3 + 3r_t^2 r_{t+1} + 3r_t r_{t+1}^2 + r_{t+1}^3\right)$$

$$= 0 + 0 + 3E\left(r_t h_{t+1}\right) + 0$$

$$= 3E\left[r_t\left(\omega + \alpha r_t^2 + \gamma r_t^2 I_{(r_t<0)} + \beta h_t\right)\right]$$

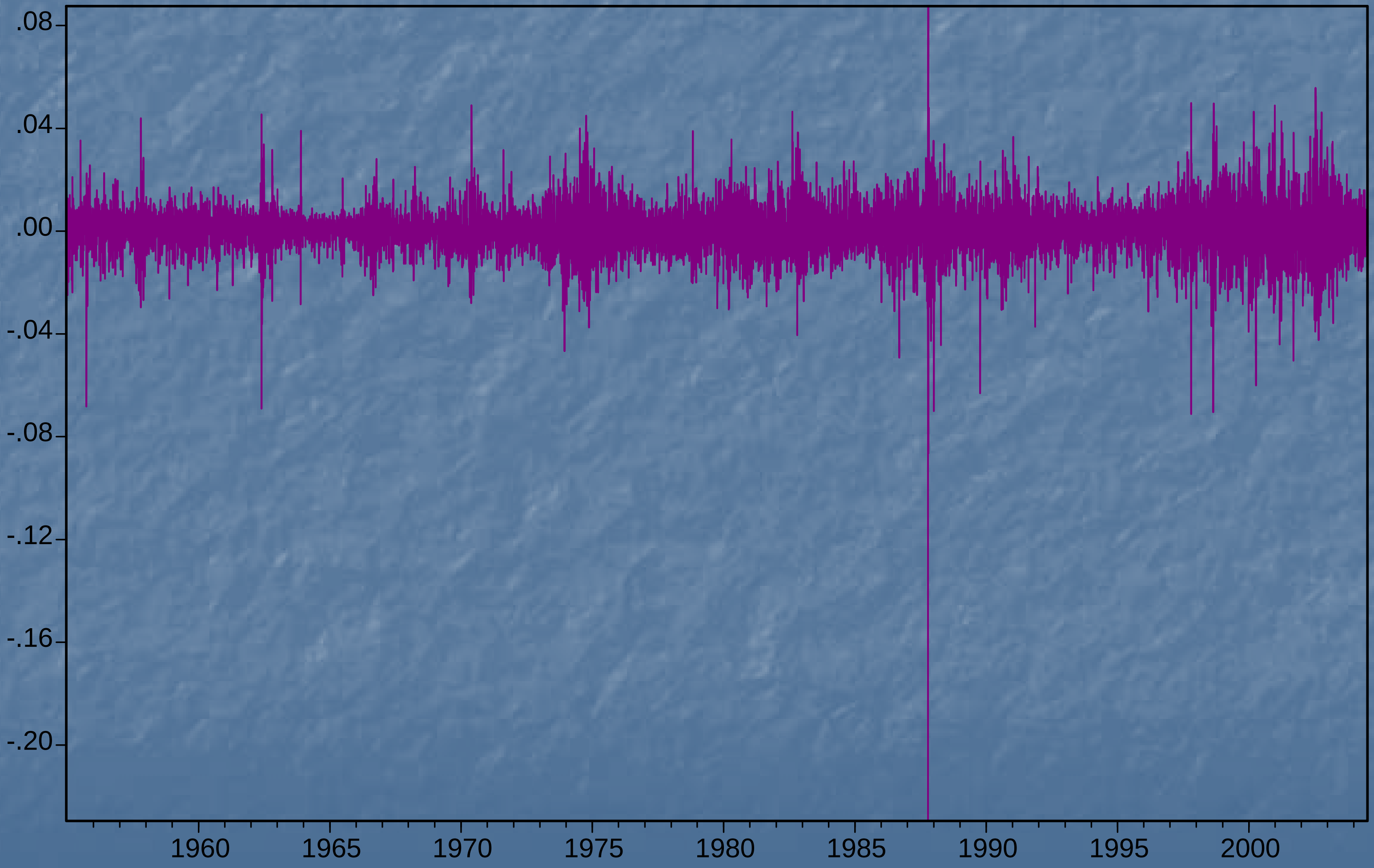
$$= 3\gamma E\left(r_t^3 I_{(r_t<0)}\right) < 0$$

and the conditional third moment is

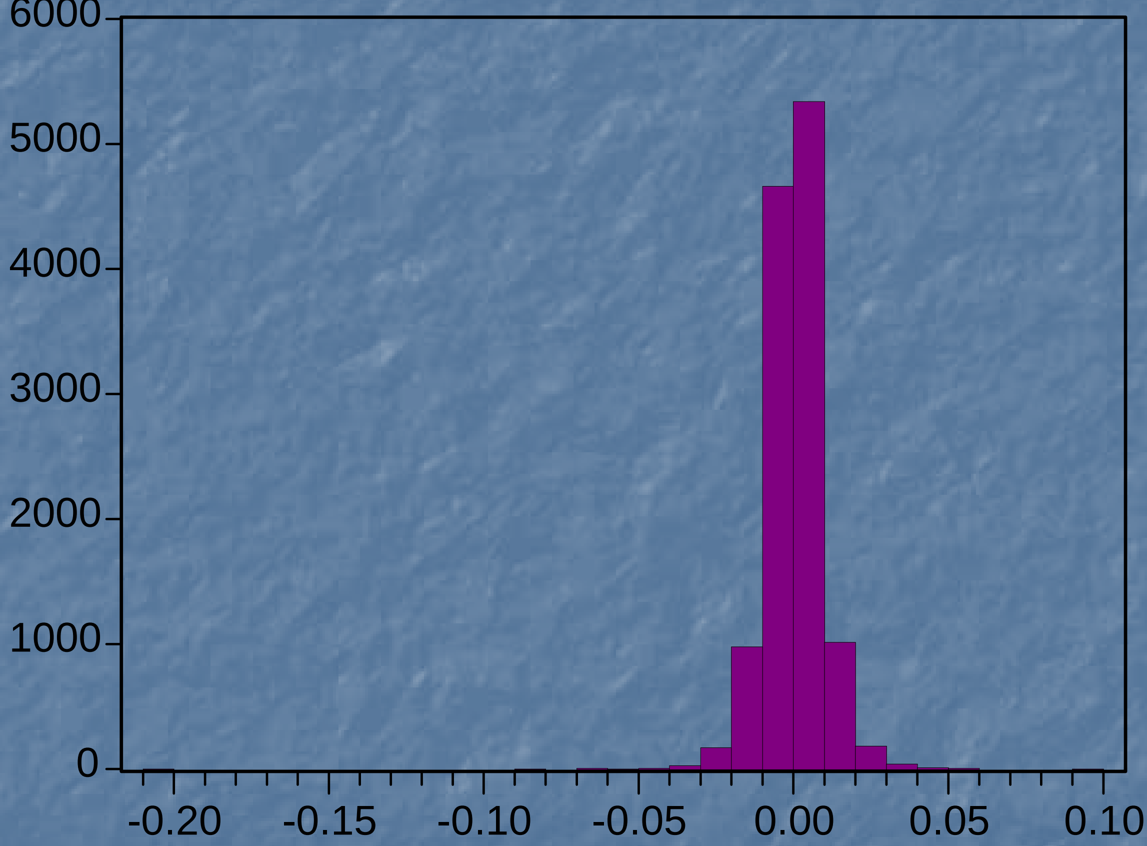
$$E_{t-1}\left(r_t + r_{t+1}\right)^3 = 3\gamma E_{t-1}\left(r_t^3 I_{(r_t<0)}\right) < 0$$

STYLIZED FACTS

S&P 500 DAILY RETURNS



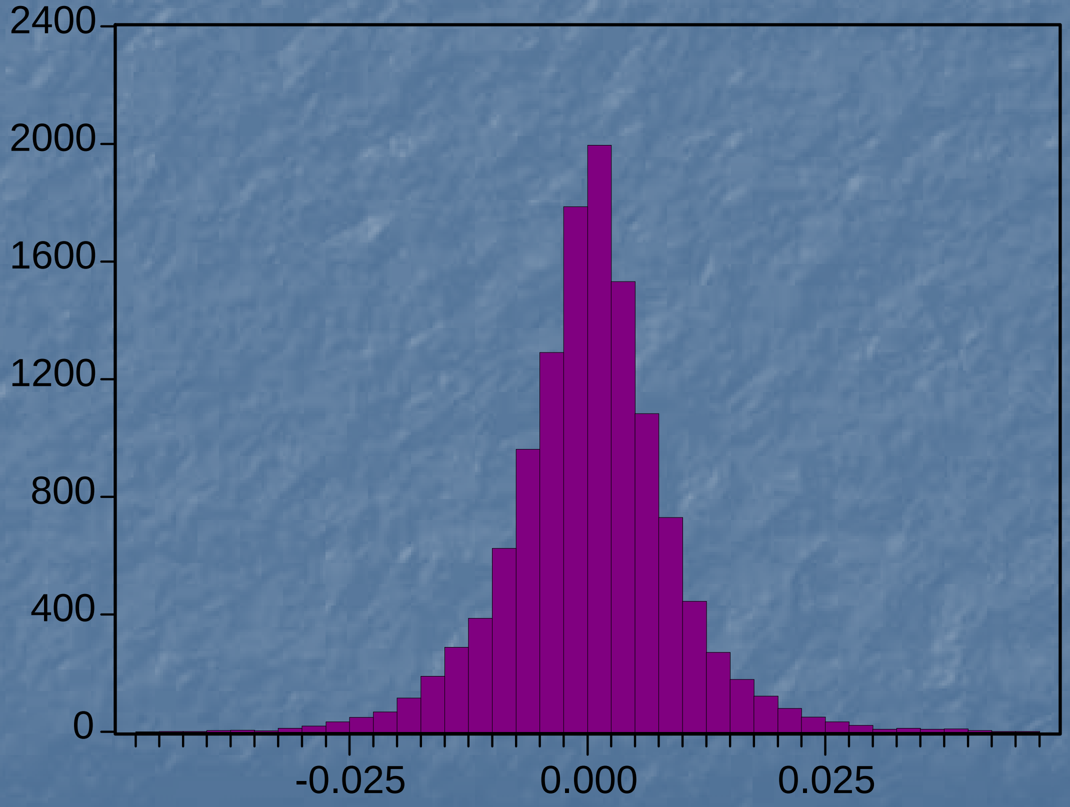
HISTOGRAM OF S&P500 DAILY RETURNS



Series: RETUSSP
Sample 1/4/1955 TO 6/25/2004
Observations 12455

Mean	0.000318
Median	0.000375
Maximum	0.090994
Minimum	-0.204669
Std. Dev.	0.009179
Skewness	-0.926286
Kurtosis	28.00273
Jarque-Bera	326200.8
Probability	0.000000

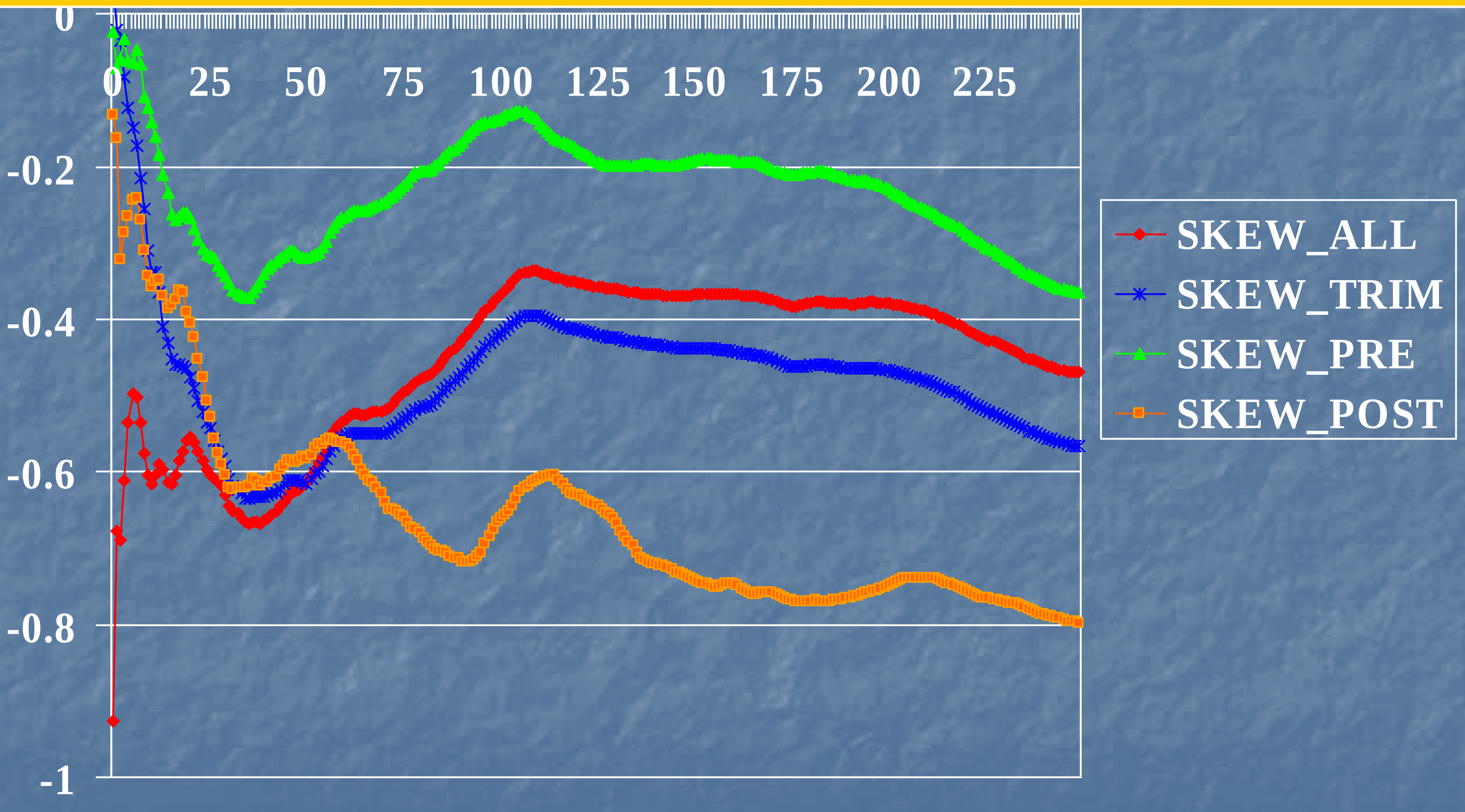
TRIMMING .001 IN EACH TAIL
(8 DAYS)



Series: RETUSSP
Sample 1 12455 IF Y>LOWTRIM AND Y<HIGHTRIM
Observations 12431

Mean	0.000338
Median	0.000375
Maximum	0.046486
Minimum	-0.045594
Std. Dev.	0.008633
Skewness	0.049617
Kurtosis	5.380668
Jarque-Bera	2940.670
Probability	0.000000

SKEWNESS OF MULTIPERIOD RETURNS



STANDARD ERRORS

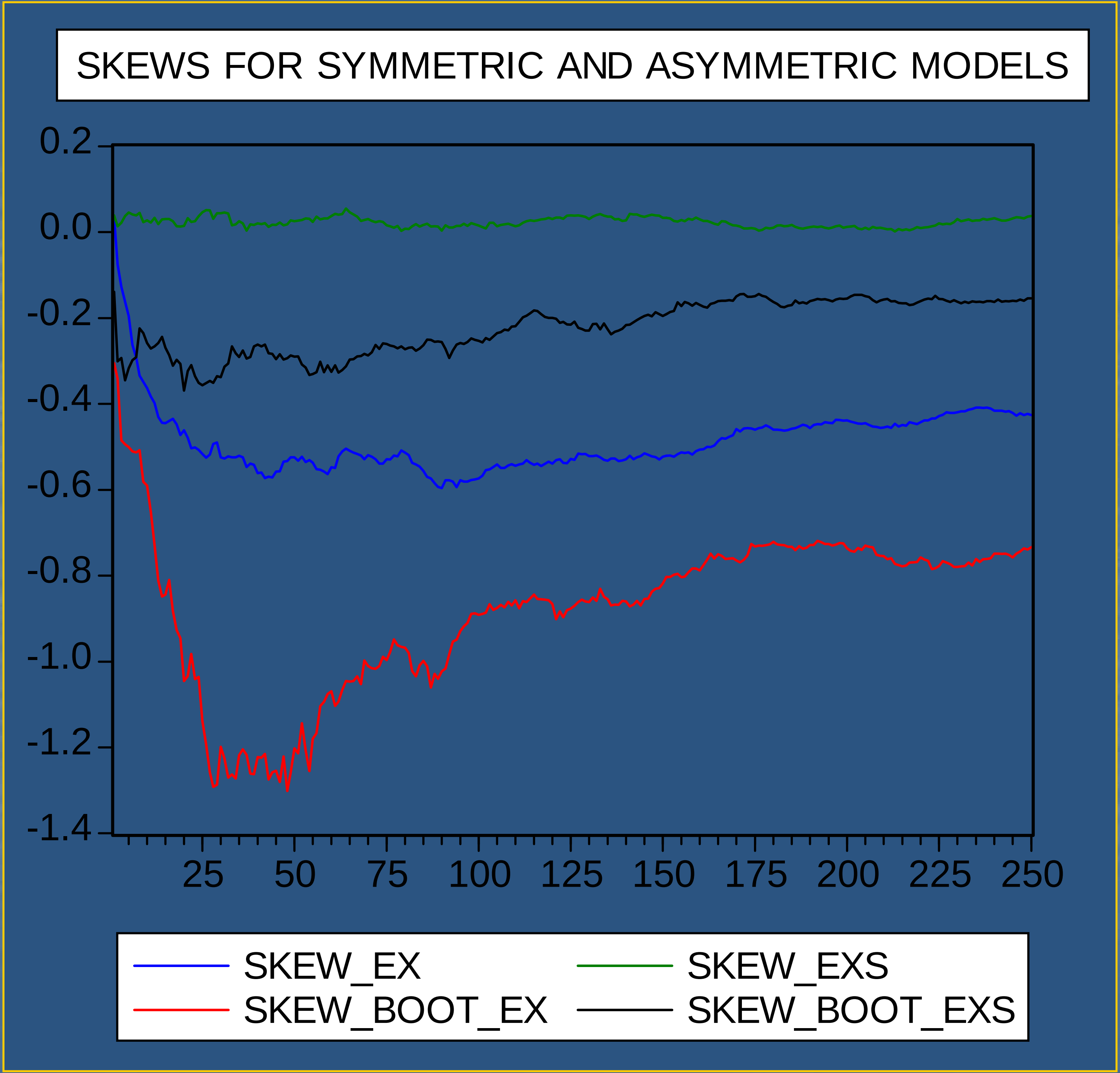
- ARE THESE DIFFERENCES SIGNIFICANT?
- THE INFERENCE IS COMPLICATED BY THE OVERLAPPING OBSERVATIONS AND BY THE DEPENDENCE DUE TO ESTIMATING THE MEAN.
- FROM SIMPLE ROBUST TESTS, SIZE CORRECTED BY MONTE CARLO, THESE ARE SIGNIFICANT.

EVIDENCE FROM DERIVATIVES

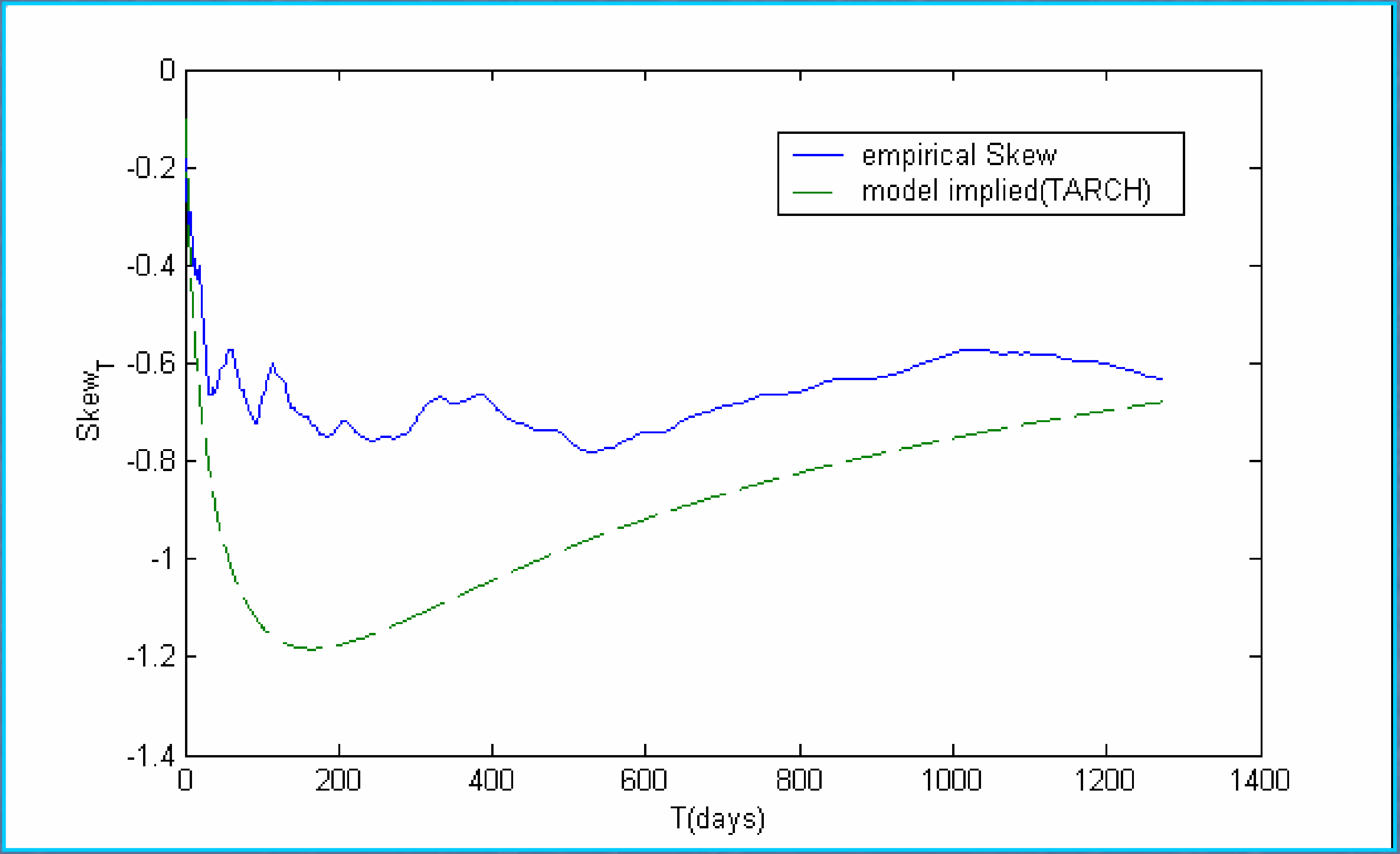
- THE HIGH PRICE OF OUT-OF-THE-MONEY EQUITY PUT OPTIONS IS WELL DOCUMENTED
- THIS IMPLIES SKEWNESS IN THE RISK NEUTRAL DISTRIBUTION
- MUCH OF THIS IS PROBABLY DUE TO SKEWNESS IN THE EMPIRICAL DISTRIBUTION OF RETURNS.
- DATA MATCHES EVIDENCE THAT THE OPTION SKEW IS ONLY POST 1987.

MATCHING THE STYLIZED FACTS

- ESTIMATE DAILY MODEL
- SIMULATE 250 CUMULATIVE RETURNS 10,000 TIMES WITH SEVERAL DATA GENERATING PROCESSES
- CALCULATE SKEWNESS AT EACH HORIZON
- ANALYTICAL CALCULATION


SKEWS FOR SYMMETRIC AND ASYMMETRIC MODELS
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
-1.4
25
50
75
100
125
150
175
200
225
250
SKEW_EX
SKEW_EXS
SKEW_BOOT_EX
SKEW_BOOT_EXS

Time Aggregation of TARCH



IMPLICATIONS

- Multi-period empirical returns are more skewed than one period returns (omitting 1987 crash)
- Asymmetric volatility is needed to explain this.
- Skewness has increased since 1987, particularly for longer horizons.
- These findings match options markets.

MULTIVARIATE MODELS

DOWNSIDE RISK RESULTS FROM TIME AGGREGATION WITH:

- ASYMMETRIC CORRELATIONS
  - CORRELATIONS RISE PARTICULARLY AFTER TWO ASSETS BOTH DECLINE. (Asymmetric DCC (Cappiello, Engle, Sheppard(2004))
- VOLATILITY SHOCKS ARE CORRELATED
  - PURE VARIANCE COMMON FEATURES(Engle, Marcucci(2005))
  - FACTOR MODELS (Engle Ng and Rothschild(1992))
  - CREDIT RISK MODEL(Engle, Berd, Voronov(2005))

DOWNSIDE RISK IN THE CAPM

- The return on a stock can be decomposed into systematic and idiosyncratic returns using the beta of the stock
- $$r_{i,t} = \beta_i r_{m,t} + \varepsilon_{i,t}$$
- If the market declines substantially, many stocks will decline. There will be skewness in each stock and downside risk in the portfolio.

SKEWNESS

- Under the standard assumptions, the skewness of return *i* is related to the skewness of the market by $s_i = s_m \rho^3$ where ρ is the correlation between stock and market.
- Notice that all stocks will then have skewness but that it will be less than for the market.

TAIL DEPENDENCE

- The probability that two stocks will both underperform some threshold can be calculated conditional on the market return.
- When the market return is a fat-tailed distribution, tail dependence rises.

SUMMARY

- ASYMMETRIC VOLATILITY IN THE MARKET FACTOR IMPLIES
  - SKEWNESS IN MULTIPERIOD MARKET RETURNS
  - SKEWNESS IN MULTIPERIOD EQUITY RETURNS
  - LOWER TAIL DEPENDENCE IN EQUITY RETURNS

IMPLICATIONS FOR FINANCIAL MANAGEMENT

Don’t ask….

IMPLICATIONS FOR RISK MANAGEMENT

- MULTI-PERIOD RISKS MAY BE SUBSTANTIALLY DIFFERENT FROM ONE PERIOD RISKS.
- THE MULTI-PERIOD RISK CHANGES OVER TIME AND CAN BE FORECAST.
- BIG MARKET DECLINES ARE MORE LIKELY WHEN VOLATILITY IS HIGH

IMPLICATIONS FOR DERIVATIVE HEDGING

- AS EACH NEW PERIOD RETURN IS OBSERVED, THE DERIVATIVE CAN BE REPRICED AND THE HEDGE UPDATED.
- GREEKS CAN BE CALCULATED FROM SIMULATION PRICING TO SIMPLIFY THE UPDATING

IMPLICATIONS FOR PORTFOLIO SELECTION

- MEAN VARIANCE PORTFOLIO OPTIMIZATION WILL MISS THESE ASYMMETRIES.
- HIGH FREQUENCY REBALANCING WILL GIVE *EARLY WARNING* OF DOWNSIDE RISK.

HOW TO DO THIS?

- SUBOPTIMAL METHOD 1
  - MYOPIC ASSET ALLOCATION ON A HIGH FREQUENCY BASIS.
  - AS VOLATILITIES RISE, YOU NATURALLY SHIFT OUT OF RISKY ASSETS.
- SUBOPTIMAL METHOD 2
  - MULTI-PERIOD FORECAST OF RISK GIVES AN EX-ANTE OPTIMAL PLAN.
  - OVERINVEST WHEN VOLATILITY IS LOW AND UNDERINVEST WHEN IT IS HIGH

OPTIMAL METHOD

- DYNAMIC PROGRAMMING:
  - WHEN VOLATILITY IS LOW, UNDERINVEST, RECOGNIZING THAT THIS PLAN MAY CHANGE WHEN THE SUBSEQUENT VOLATILITY IS OBSERVED
  - SEE COLACITO AND ENGLE(2004)

EXPECTED RETURNS

- EACH OF THESE METHODS REQUIRES EXPECTED RETURNS-*COORDINATION OF RISK MANAGEMENT AND ALPHA ESTIMATION*
- THE LISTED IMPLICATIONS ARE BASED ON THE ASSUMPTION THAT EXPECTED RETURNS ARE UNCHANGED.
- IS THIS REASONABLE?

BUT IF EVERYBODY DID THIS?

- IF ALL AGENTS FOLLOW THIS STRATEGY, THEN EXPECTED RETURNS WOULD NECESSARILY ADJUST. RETURNS WOULD INSTANTANEOUSLY MOVE ENOUGH TO RESTORE EQUILIBRIUM. CAMPBELL AND HENTSCHEL(1992)
- IN A REPRESENTATIVE AGENT WORLD, THERE WOULD NO LONGER BE A MOTIVE FOR ADJUSTING TO CHANGES IN RISK.

IN GENERAL EQUILIBRIUM

- CHANGES IN RISK WOULD INSTANTLY LEAD TO CAPITAL GAINS OR LOSSES.
- INVESTORS WOULD TAKE SMALLER POSITIONS BECAUSE OF THE MULTI-PERIOD RISKS OR WOULD REQUIRE HIGHER RETURNS.
- WE SAY IN THIS CASE, “DOWNSIDE RISK IS PRICED”.

HOWEVER EVEN IN EQUILIBRIUM

- THERE IS NO REASON TO BELIEVE DOWNSIDE RISK WOULD DISAPPEAR OR COLLAPSE IN TIME.
- WITH HETEROGENEITY, THERE WOULD STILL BE REASONS TO REBALANCE.
- DERIVATIVE REPLICATION STRATEGIES CONTINUE TO BE USEFUL.
- DERIVATIVE PRICING FOR NON-LINEAR PAYOFFS SUCH AS OPTIONS AND CREDIT DERIVATIVES WILL NEED TO BE MODIFIED.

CONCLUSIONS

- ASYMMETRIC VOLATILITY AND CORRELATION MODELS ARE POWERFUL TOOLS FOR ANALYZING DOWNSIDE RISK
- ONE PERIOD MODELS HAVE BIG IMPLICATIONS ABOUT THE LONG HORIZON RETURNS
- THE UPDATING OF VOLATILITY AND RISK MEASURES HAS A NATURAL APPLICATION TO DERIVATIVE HEDGING, PRICING, AND POSSIBLY PORTFOLIO REBALANCING.


THE ARCH
Cafe
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