The Influence of FX Risk on Credit Spreads

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Agenda

A. The Problem

B. Mathematical Model

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A. The Problem

Q: *What is the price of Credit Event Protection in different currencies?*

Some Notation

- X_t is the US$-JPY FX rate at time t.
- τ is the default time of an obligor.
- q_τ is the loss given default (LGD) rate of this obligor's bonds.
- $S_\$$ and S_Y are T-year US$ and JPY Credit Default Swap (CDS) rates at time 0.
- $b_\$(t)$ is a US$- and $b_Y(t)$ a JPY-bank account at time t.

CDS Cash Flows

(Viewpoint of protection buyer)

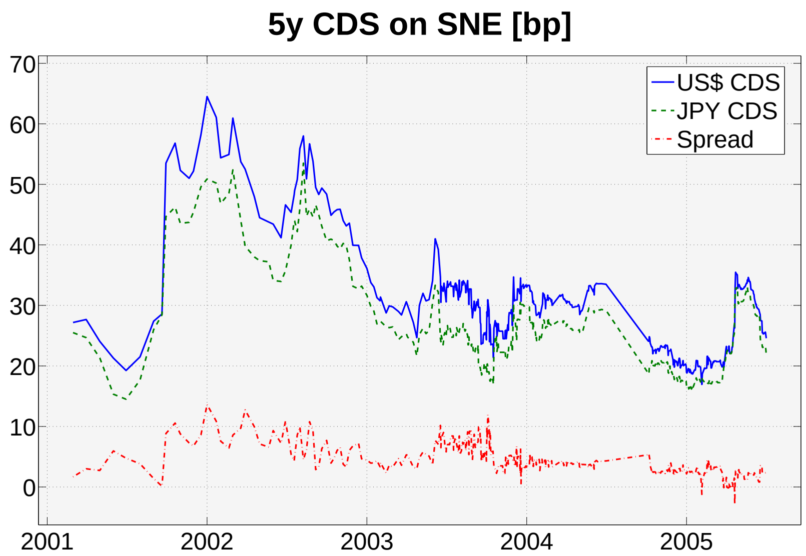
at	US$ CDS	JPY CDS	Payment
$t \leq \min(\tau, T)$	$-S_{\$} dt$ US$	$-S_Y dt$ JPY $= -X_t S_Y dt$ US$	Fee for $[t, t+dt)$
τ iff $\tau \leq T$	q_τ US$	q_τ JPY $=$ $X_\tau q_\tau$ US$	Protection

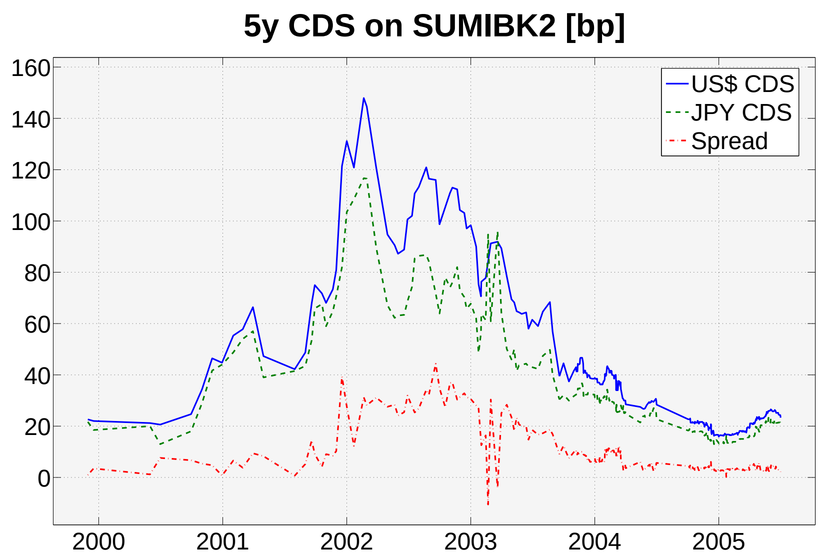
Intuition. Dependence of X and (τ, q_τ) is essential for relationship of $S_{\$}$ and S_Y.

Benchmark case. X independent of (τ, q_τ) and $b_{\$}(t) = b_Y(t) = 1$, then

$$S_{\$} = S_Y.$$

Evidence from the CDS Markets





Economic Reasons

- US$-JPY FX impacts on profits of Japanese Export/Import companies and on liability side of Japanese companies that issued debt in US$ (*direct* dependence).

- Credit quality of major Japanese obligors (in particular banks) may influence investors' confidence in JPY (*indirect* dependence).

B. Mathematical Model

Preliminaries. Probability space $(\Omega, \mathcal{F}_T, \mathbf{Q})$, finite time horizon T, filtration $(\mathcal{F}_t)_{t \leq T}$, $\mathbf{Q}$ the US$ spot martingale measure ($SMM) chosen by the market.

Proposition.

$$\left.\frac{\mathrm{d}\widetilde{\mathbf{Q}}}{\mathrm{d}\mathbf{Q}}\right|_{\mathcal{F}_t} := L_t := \frac{X_t b_Y(t)}{X_0 b_\$(t)}$$

defines a probability $\widetilde{\mathbf{Q}}$ on $(\Omega, \mathcal{F}_T)$. $\widetilde{\mathbf{Q}}$ is a JPY spot martingale measure (YSMM).

Change of Measure Method. We have

$$S_\$ = \frac{\mathbf{E}^{\mathbf{Q}}\left[\frac{1}{b_\$(\tau)}\mathbf{1}_{\{\tau \leq T\}} q_\tau\right]}{\int_0^T \mathbf{E}^{\mathbf{Q}}\left[\frac{1}{b_\$(t)}\mathbf{1}_{\{t \leq \tau\}}\right] dt} \qquad \text{and} \qquad S_Y = \frac{\mathbf{E}^{\widetilde{\mathbf{Q}}}\left[\frac{1}{b_Y(\tau)}\mathbf{1}_{\{\tau \leq T\}} q_\tau\right]}{\int_0^T \mathbf{E}^{\widetilde{\mathbf{Q}}}\left[\frac{1}{b_Y(t)}\mathbf{1}_{\{t \leq \tau\}}\right] dt}.$$

Default Risk under Q

- $(\mathcal{F}_t)_{t\le T}$ carries a Brownian Motion (BM) W and a jump measure $\mu(dz,dt)$ on $\mathcal{Z}\times[0,T]$ with predictable $\mathbf{Q}$-compensator of the form

$$\nu^{\mathbf{Q}}(dz,dt) = dF_t(z)\lambda(t)dt,$$

where $F_t(\cdot)$ is a distribution function on $\mathcal{Z}$ and $d\lambda(t) = \gamma_\lambda(t)dt + \sigma'_\lambda(t)dW_t$.

- $\tau = \inf\{t > 0;\ N_t > 0\}$, where $N_t := \int_0^t\int_{\mathcal{Z}} \mu(dz,ds)$.

- $q_\tau = \int_0^\tau\int_{\mathcal{Z}} q(z,t)\mu(dz,dt)$ for some predictable function $q(z,t)$.

Limitting Property.

$$\lim_{T\downarrow 0} S_\$(T) = \overline{q}(0)\lambda(0) \qquad \mathbf{Q}\text{-a.s.},$$

where $\overline{q}(t) := \int_{\mathcal{Z}} q(z,t)dF_t(z)$.

FX Rate Modelling

Key Assumption. L satisfies the stochastic differential equation (SDE)

$$\frac{dL_t}{L_{t-}} = dM_t := \sigma'_X(t)\,dW_t - \int_{\mathcal{Z}} \delta(z,t)\big[\mu(dz,dt) - \nu^{\mathbf{Q}}(dz,dt)\big]$$

where $\delta(z,t) < 1$ is a predictable function.

That means there can be

- Correlation *before default*: $d[X,\lambda]_t = \sigma'_X(t)\sigma_\lambda(t)X_{t-}dt$,
- Devaluation *at default*: $X_\tau = \big(1-\delta_\tau\big)X_{\tau-}$, where $\delta_\tau := \int_0^\tau\int_{\mathcal{Z}} \delta(z,t)\mu(dz,dt)$.

Default Risk under $\widetilde{\mathrm{Q}}$ (Girsanov)

- Correlation $\Longrightarrow$ Drift changes in underlying diffusions (e.g. λ)

- Devaluation $\Longrightarrow$
  - ⋆ Different default intensity $\widetilde{\lambda}(t) := \left[1 - \overline{\delta}(t)\right]\lambda(t)$,
  - ⋆ Different LGD distribution $d\widetilde{F}_t(z)$

  where $\overline{\delta}(t) := \int_{\mathcal{Z}} \delta(z,t) dF_t(z)$.

Remember:

$$\lim_{T \downarrow 0} S_Y(T) \quad = \quad \widetilde{\overline{q}}(0)\widetilde{\lambda}(0) = \widetilde{\overline{q}}(0)\left[1 - \overline{\delta}(0)\right]\lambda(0) \qquad \mathbf{Q}\text{-a.s.},$$

where $\widetilde{\overline{q}}(t) := \int_{\mathcal{Z}} q(z,t) d\widetilde{F}_t(z)$.

C. Empirical Results

Word on the Data. ValuSpread CDS data base by *Lombard Risk Systems Ltd.*

Devaluation Fraction. We use the estimator $\widehat{\delta}_T := \text{mean}\left\{1 - \frac{S_Y(T)}{S_\$(T)}\right\}$.

	1 year CDS		5 year CDS	
Ticker *(Company)*	$\widehat{\delta}_1$ [%]	p-Value [%]	$\widehat{\delta}_5$ [%]	p-Value [%]
EJRAIL *(East Japan Railway Company)*	20.2	0.00	18.4	0.00
HONDA *(Honda Motor Co., Ltd.)*	12.5	0.00	19.2	0.00
MATSEL *(Matsushita Electric Industrial Co., Ltd.)*	24.6	0.00	17.0	0.00
MITCO1 *(Mitsubishi Corp)*	18.4	0.00	10.7	0.00
MITSCO *(Mitsui & Co., Ltd.)*	14.7	0.00	11.8	0.00
NIPSTL *(Nippon Steel Corporation)*	16.2	0.00	11.8	0.00
NTT *(Nippon Telegraph & Telephone Corporation)*	27.0	0.00	19.0	0.00
SHARP *(Sharp Corporation)*	31.4	0.00	21.1	0.00
SNE *(Sony Corporation)*	27.0	0.00	16.5	0.00
SUMIBK2 *(Sumitomo Mitsui Banking Corporation)*	21.7	0.00	20.6	0.00
SUMT *(Sumitomo Corporation)*	13.5	0.00	9.6	0.00
TOKELP *(Tokyo Electric Power Co., Inc.)*	31.0	0.00	18.9	0.00
TOKIO *(Tokio Marine and Fire Insurance Company Limited)*	29.0	0.00	22.8	0.00
TOYOTA1 *(Toyota Motor Corporation)*	30.6	0.00	25.6	0.00
YAMAHA *(Yamaha Motor Co., Ltd.)*	16.9	0.00	16.6	0.00

Affine Diffusion Correlation Model

$$
\begin{aligned}
d\lambda(t) &= \kappa\big[\theta - \lambda(t)\big]dt + \sigma\sqrt{\lambda(t)}dW_t^1 \\
dM(t) &= \xi_1\sqrt{\lambda(t)}dW_t^1 + \xi_2 dW_t^2 - \delta\big[dN_t - \lambda(t)dt\big]
\end{aligned}
$$

Ticker *(Company)*	$\widehat{(\frac{\xi_1}{q\sigma})}$	95%-CI	Implied $\widehat{\delta}_5$ [%]
EJRAIL *(East Japan Railway Co.)*	-2.97	[-10.22, 4.29]	1.0
HONDA *(Honda Motor Co. Ltd.)*	-2.38	[-7.28, 2.52]	0.8
MATSEL *(Matsushita Electric Industrial Co. Ltd.)*	-1.05	[-4.03, 1.93]	0.9
MITCO1 *(Mitsubishi Corp.)*	-3.97	[-8.18, 0.23]	1.7
MITSCO *(Mitsui & Co. Ltd.)*	-4.20	[-8.29, -0.12]*	1.2
NIPSTL *(Nippon Steel Corp.)*	-5.61	[-9.19, -2.03]*	-0.8
NTT *(Nippon Telegraph & Telephone Corp.)*	-2.73	[-10.58, 5.12]	2.3
SHARP *(Sharp Corp.)*	-3.96	[-7.28, -0.65]*	0.6
SNE *(Sony Corp.)*	-1.97	[-6.12, 2.17]	-3.0
SUMIBK2 *(Sumitomo Mitsui Banking Corp.)*	-1.94	[-5.69, 1.81]	-4.6
SUMT *(Sumitomo Corp.)*	-3.80	[-7.39, -0.21]*	-1.2
TOKELP *(Tokyo Electric Power Co. Inc.)*	-3.10	[-7.892, 1.70]	-2.0
TOKIO *(Tokyo Marine & Fire Insurance Co. Ltd.)*	-1.51	[-4.39, 1.37]	0.9
TOYOTA1 *(Toyota Motor Corp.)*	-1.68	[-11.02, 7.66]	1.5
YAMAHA *(Yamaha Motor Co., Ltd.)*	0.26	[-1.76, 2.28]	3.4

D. Application in Cat Insurance

- τ is time of next earthquake in Tokio of magnitude > 8.5
- X_t^* is replacement value [US$] of a skyscraper in Tokio ($X_0^* = 1\mathrm{bn}$ US$)

Example: 3 Property Insurance Contracts

Contract	Premium [US$]	Sum Insured	
A	$S_\$$	1 bn	US$
B	S_Y	111 bn	JPY
C	S_{repl}	X_τ^*	US$

Analogy. If FX rate or replacement value may be devaluated at τ, then possibly

$$S_\$ \neq S_Y \qquad \text{or} \qquad S_{repl} \neq S_\$.$$

E. Conclusion

- US$ and JPY CDSs on the same Japanese reference entity are not equal.
- FX risk and default risk of these obligors are not independent.
- Our model explains this spread by
  - ⋆ Correlation of US$-JPY FX and CDS rates before default,
  - ⋆ Devaluation of JPY at default.
- Correlation does not sufficiently explain our CDS data.
- The market expects JPY devaluation at default of major Japanese obligors.
- Applications in Cat Insurance when τ is "Catastrophe Time".