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A Multi-Period View on Actuarial and Financial Pricing for Guaranteed Minimum Death Benefits in Unit-Linked Life Insurance

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Introduction

- Consider the following product:
  - 1000 insured aged 50 invest S_0 in a risky asset $(S_t)_{t\geq 0}$.
  - Guaranteed Minimum Death Benefit of K.
    If insured dies in month $(t, t+1]$, GMDB leads to payment of
    $$(K - S_{t+1})_+ = \max\{0; K - S_{t+1}\}$$
    at time $t+1$
  - No surrender
  - No guarantee at retirement (age 65).
- Possibility to invest in a risk-less bond process $B_t = B_0 e^{rt}$

Introduction

- Financial Reserving:
  - Under Black-Scholes market model and assuming mortality risk can be completely eliminated, a risk-less hedging strategy exists
  - Unhedged liability and additional costs due to:
    - Transaction costs
    - Discrete hedging intervals
    - Log-returns are not normally distributed
    - Mortality risk
- Actuarial Reserving: no hedging but capital allocation
- Multi-period setting $\Rightarrow$ Future information about:
  - Underlying Asset
  - Mortality

Introduction

- How to come to a price in the different reserving strategies?
- Potential price differences?
- Impact of future information:
  - On capital and technical provisions?
  - In pricing?
  - In function of reserving strategy?

Financial Reserving

- Black-Scholes Price of GMDB at $t \in \{0, \ldots, T-1\}$:

$$BSP_t = N_t \sum_{s=t}^{T-1} {}_{s-t|}q_{x,t} P(t, s+1),$$

where $P(t, s+1)$ denotes the Black-Scholes price of a put with strike K on underlying asset $(S_i)_{i \in [t, s+1]}$ and N_t denotes the number of survivors at t

- Value of hedging portfolio from period $[t-1, t)$ at t:

$$V_t^- = N_{t-1}(\xi_{t-1} S_t + \beta_{t-1} B_t)$$

- Hedging error at $t \in \{1, \ldots, T\}$:

$$HE_t = BSP_t + (N_{t-1} - N_t)(K - S_t)_+ - V_t^-, \text{ with } BSP_T = 0$$

- Transaction costs at $t \in \{0, \ldots, T\}$: $TC_t = \tau S_t |\xi_t - \xi_{t-1}|, \xi_{-1} = 0 = \xi_T$

Actuarial versus Financial Reserving: Discounted Future Costs

- Financial Reserving:

$$D_0^{(F)} = BSP_0 + \sum_{s=1}^{T} HE_s e^{-rs} + \sum_{s=0}^{T} TC_s e^{-rs},$$

$$D_t^{(F)} = \sum_{s=t+1}^{T} HE_s e^{-r(s-t)} + \sum_{s=t+1}^{T} TC_s e^{-r(s-t)}, \text{ for } t \in \{1, \ldots, T-1\}$$

- Actuarial Reserving:

$$D_t^{(A)} = \sum_{s=t+1}^{T} (N_{s-1} - N_s)(K - S_s)_+ e^{-r(s-t)}, \text{ for } t \in \{0, \ldots, T-1\}$$

Multi-Period Capital Allocation

- Notations:
  - $\mathbb{F} = (\mathcal{F}_t)_{t\in[0,T]}$: filtration describing the information
  - D_t: discounted future costs at t
  - P_t = provision, K_t = capital and $TSL_t = P_t + K_t$ = total solvency level at t
- We compare 2 approaches:

Not using future information

$$P_t^{(N)} = E[D_t|\mathcal{F}_0]$$
$$K_t^{(N)} = TVaR_{.99}[D_t|\mathcal{F}_0] - P_t^{(N)}$$

- Asset ↓↓ ⇒ Solvency ↓
  Asset ↑↑ ⇒ Unnecessary capital costs
- Easy to calculate

Using future information

$$P_t^{(U)} = E[D_t|\mathcal{F}_t]$$
$$K_t^{(U)} = TVaR_{.99}[D_t|\mathcal{F}_t] - P_t^{(U)}$$

- More rational
- Future reserves and capitals are random variables as seen from 0

Cash-flow Model

- We look at product on a stand-alone basis
- Model average in-and outflows for shareholders:
- Inflows:
  - Premium income
  - Net mean return on reserves
  - Net mean return on capital
- Outflows:
  - Net mean claim payments
  - Net mean change in reserves
  - Net mean change in capital
- Discount average cash-flows at cost of capital

Cash-flow Model

Technico-Financial Premium (TFP)

- TFP : premium which makes the sum of the discounted inflows equal to the sum of the discounted outflows

$$TFP = P_0 + \frac{\sum_{t=0}^{T} e^{-tCOC}[\Delta k_t - R_t(k)]}{1-\gamma}$$

where:

- P_0 is the reserve taken at time 0
- γ is the tax rate
- Δk_t is the mean change in the capital
- $R_t(k)$ is the net mean return on the capital (after taxation)

Modelling Methods and Assumptions

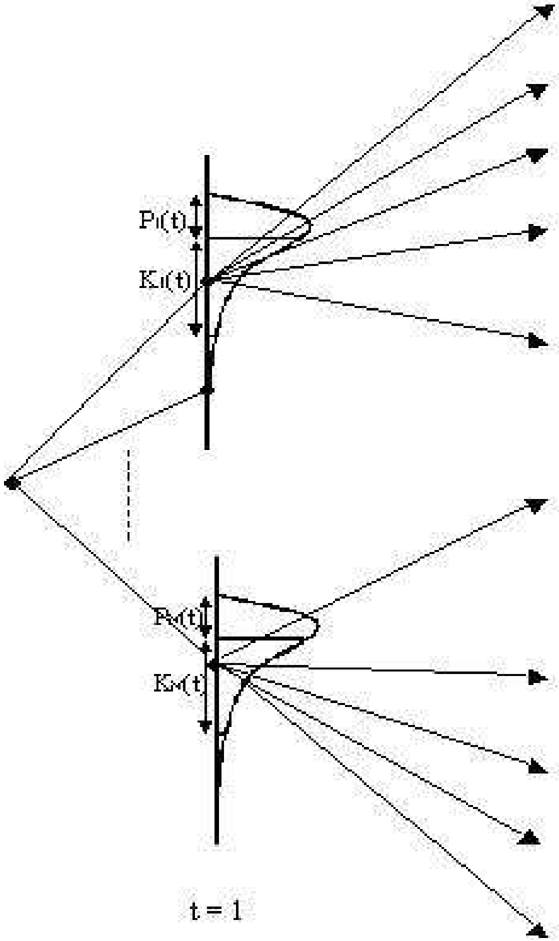
- Monthly basis for simulation of asset and mortality
- Underlying risky asset:
  - Regime-Switching Log-Normal model
  - Parameters based upon maximum likelihood estimates for $S\&P500$ from 1960 to 2003
  - Volatility for financial reserving: estimated on $S\&P500$ from 1960 to 2003
- Mortality
  - Gompertz-Makeham approach
  - Parameters based upon table 197 Assuralia (Belgian Union of Insurance Companies)

Future Information: Number of Survivors

Year	Mean	StDev	Skewness	Kurtosis
1	996	1.91	-0.47	3.14
2	992	2.79	-0.32	3.05
...	...	...	...	...
5	976	4.73	-0.17	2.92
...	...	...	...	...
14	890	9.81	-0.10	3.00

- Volatility increases with time and is very small at initiation
- Distribution more and more symmetric with time
- Impact on provisions and TSL's is linear
- Considering future information for pricing is not necessary

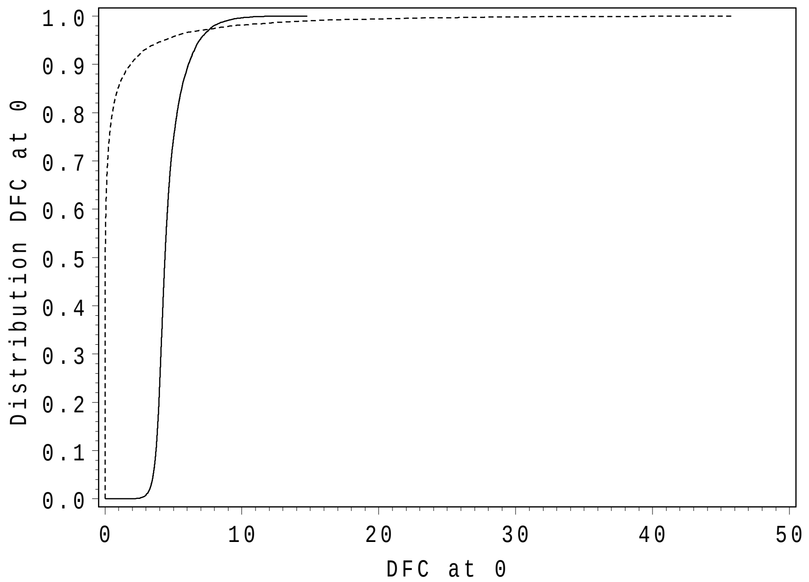
Future Information: Underlying Asset



- Actuarial approach:
  - Limited number of future values
  - Avoid resimulating
- Financial approach:
  - Same techniques
  - Many hedging errors are needed $\Rightarrow$ Black-Scholes prices needed
  - Solution: reference values for which Black-Scholes prices and hedge are calculated beforehand
- Cfr. paper for more details

Distribution of DFC's at time 0

$K = S_0 = 1$



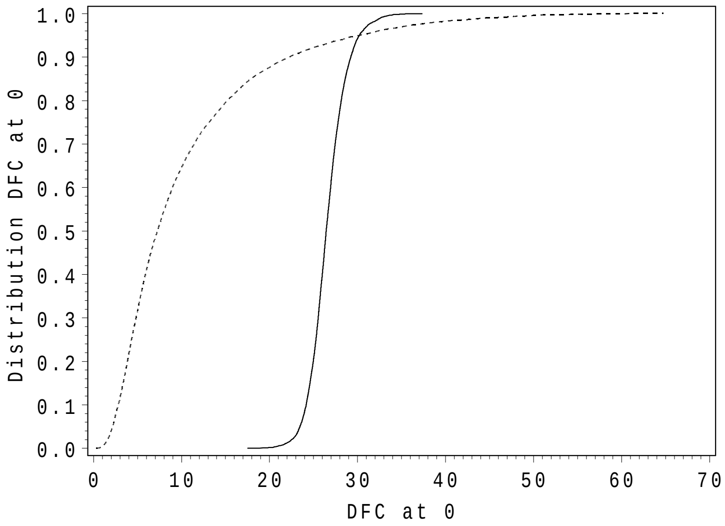
- Dashed line: actuarial approach
  Full line: financial approach
- With a probability of 97,3%, actuarial reserving leads to less costs than financial reserving
- Volatility and tail are a lot more important in actuarial approach

Approach	BSP_0	Mean	StDev	Skewness	TVaR$_{0.95}$	TVaR$_{0.99}$
$D_0^{(F)}$	4.24	4.68	1.11	1.98	8.07	9.88
$D_0^{(A)}$	-	0.83	2.91	7.07	10.79	23.30

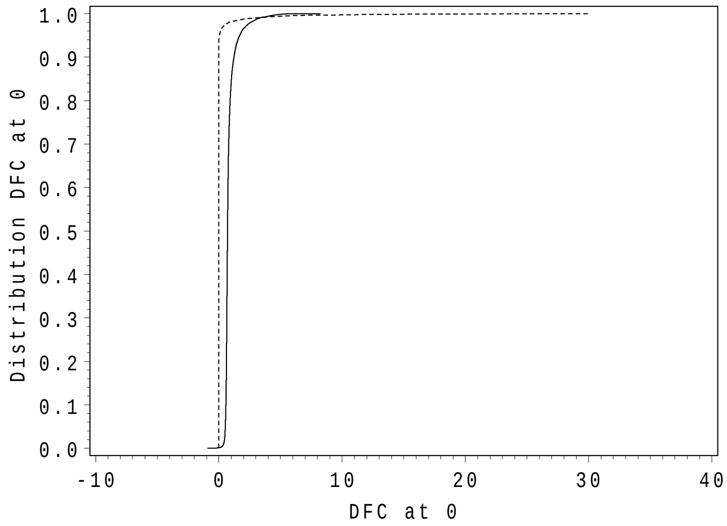
Distribution of DFC's at time 0

Impact of Ratio S_0/K

S_0 = 0.5 and K = 1



S_0 = 1.5 and K = 1

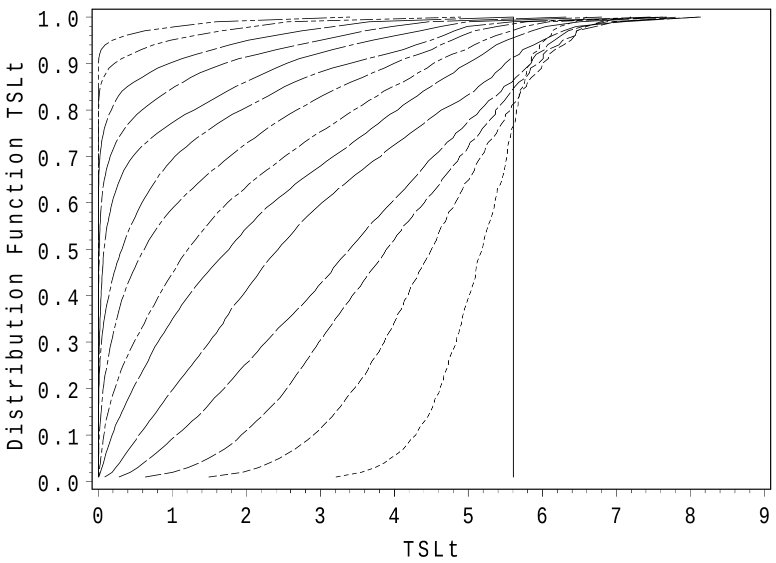


Approach	BSP_0	Mean	StDev	Skewness	TVaR$_{0.95}$	TVaR$_{0.99}$
$D_0^{(F)}$ + $S_0 = 0.5$	26.14	26.58	2.09	0.41	31.64	33.63
$D_0^{(A)}$ + $S_0 = 0.5$	-	10.33	9.06	2.00	38.70	50.29
$D_0^{(F)}$ + $S_0 = 1.5$	0.68	0.84	0.51	4.56	2.66	4.28
$D_0^{(A)}$ + $S_0 = 1.5$	-	0.11	1.09	17.00	2.22	8.62

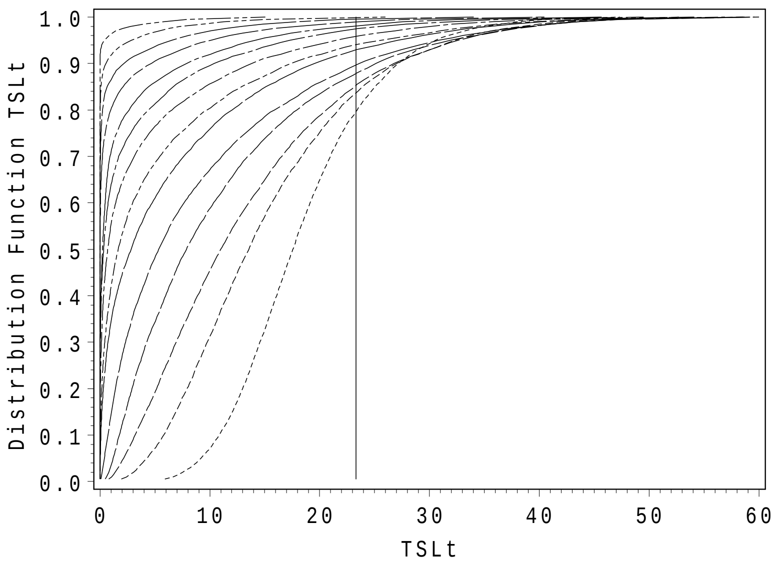
Distribution of Future TSL's

$K = S_0 = 1$

$TSL_t^{(F)} = TVaR_{0.99}[D_t^{(F)}|\mathcal{F}_t]$



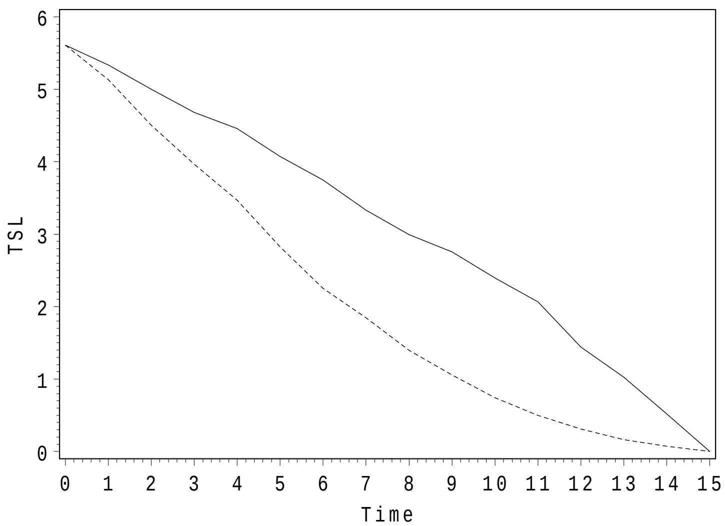
$TSL_t^{(A)} = TVaR_{0.99}[D_t^{(A)}|\mathcal{F}_t]$



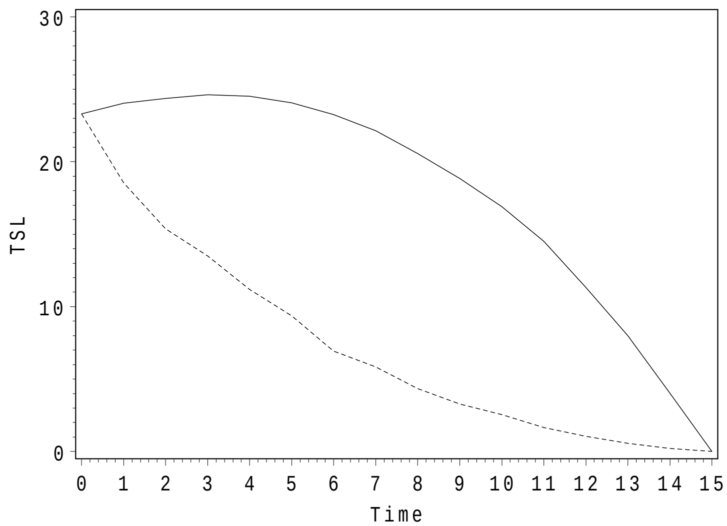
- Vertical line: initial TSL
- For other lines: the higher at the left, the further in time
- Both relative and absolute potential increases are smaller in financial approach
- Larger skewness in actuarial approach

Impact of Future Information in Pricing for $K = S_0 = 1$

$\text{TVaR}_{0.99}[D_t^{(F)}|S_0]$ (full line) vs.
$\text{E}[\text{TVaR}_{0.99}[D_t^{(F)}|S_t]]$ (dashed line)



$\text{TVaR}_{0.99}[D_t^{(A)}|S_0]$ (full line) vs.
$\text{E}[\text{TVaR}_{0.99}[D_t^{(A)}|S_t]]$ (dashed line)



- Relative and absolute differences are importantly larger in actuarial approach
- Average capital is substantially smaller in financial approach
- For lower security levels of the TVaR, the relative differences are smaller

Pricing

$K = S_0 = 1$

Risk Measure + Approach	$\text{TVaR}_{0.99}$		$\text{TVaR}_{0.95}$	
	Act	Fin	Act	Fin
BSP	-	4.24	-	4.24
Average DFC at 0	0.83	4.68	0.83	4.68
TFP without future info	14.74	6.83	5.86	5.85
TFP with future info	7.01	6.42	3.48	5.71

- Cost of capital is assumed to be 10% both in actuarial and financial approach
- **Financial approach:**
  - Price is mainly determined by the Black-Scholes price of the option
  - Transaction costs + Hedging errors: on average about 10% of BSP
  - Rest: capital costs
  - Impact of future information on price is not very important

Pricing

$K = S_0 = 1$

Risk Measure + Approach	$\text{TVaR}_{0.99}$		$\text{TVaR}_{0.95}$	
	Act	Fin	Act	Fin
BSP	-	4.24	-	4.24
Average DFC at 0	0.83	4.68	0.83	4.68
TFP without future info	14.74	6.83	5.86	5.85
TFP with future info	7.01	6.42	3.48	5.71

- **Actuarial approach:**
  - No fixed cost of Black-Scholes price and no transaction costs
  - Impact of capital costs and corresponding security level is very important
  - Impact of using future information is very important

Influence of Ratio K and S_0

Approach	BSP	E	TVaR$_{0.95}$	TVaR$_{0.99}$	TFP$^{(U)}_{0.95}$	TFP$^{(U)}_{0.99}$
(F) with $S_0 = 0.5$	26.14	26.58	31.64	33.63	28.80	29.89
(A) with $S_0 = 0.5$	-	10.33	38.70	50.29	20.57	25.48
(F) with $S_0 = 1$	4.24	4.68	8.07	9.88	5.71	6.42
(A) with $S_0 = 1$	-	0.83	10.79	23.30	3.48	7.01
(F) with $S_0 = 1.5$	0.68	0.84	2.66	4.28	1.28	1.75
(A) with $S_0 = 1.5$	-	0.11	2.22	8.62	0.73	2.24

- $S_0 = 0.5$:
  - Actuarial approach leads to lowest price under wider range of conditions
  - Prices are less sensitive to level of security (distribution DFC's less skewed)
- $S_0 = 1.5$:
  - Financial approach leads to lowest price under a wider range of conditions
  - Prices are more sensitive to level of security

Conclusion

- Actuarial reserving: future information has important impact
  - On price
  - On technical provisions and required solvency level
- Financial reserving: future information has
  - Smaller influence on price
  - Important impact on technical provisions and required solvency level
- Choice between actuarial or financial reserving depends on
  - Required security level
  - The portfolio where the GMDB is part of
- Important sensitivity to ratio of guarantee and initial investment

A Used Parameters

Simulations		
Number of simulations	N_S	10000
Number of classes for using future information for the underlying asset	N_A	200 for (A), 100 for (F)
Contractual Parameters		
Portfolio composition	$\{N_0, x, C\}$	$\{1000, 50, 1\}$
Initial value underlying asset	S_0	1
Guarantee at death	K	1
Age of retirement	x_R	65
Mortality Parameters		
Gompertz-Makeham parameters	α	0.000591
	β_{0-65}	0.00000738
	γ_{0-65}	0.118
	β_{65-99}	0.000619
	γ_{65-99}	0.0532
Financial Parameters		
Risk free rate	r	0.0035 (= 0.0425 yearly)
Volatility underlying asset	σ	0.0432 (= 0.150 yearly)
Tax rate	γ	0.4
Average return on invested capital	δ	0.00458 (= 0.055 yearly)
Cost of capital	COC	0.0083 (= 0.10 yearly)
Transaction costs	τ	0.2%
Parameters RSLN model (monthly)		
Average log-return in regime 1	μ_1	0.0135
Average log-return in regime 2	μ_2	-0.0109
Volatility in regime 1	σ_1	0.0344
Volatility in regime 2	σ_2	0.0645
Probability to move from regime 1 to 1	p_{11}	0.0483
Probability to move from regime 2 to 1	p_{21}	0.1985