Development and Pricing of a New Participating Contract

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Presentation and Pricing of Standard Contracts

Standard Contracts (the Company)

Assets	Liabilities
A_0	$E_0 = (1-\alpha)A_0$ $L_0 = \alpha A_0$

– E_0 = initial equity value

– L_0 = initial policyholders' investment

Standard Contracts (Minimum Guarantee)

Existence of a minimum guaranteed rate r_g :

$$L_T^g = L_0 \, e^{r_g T} \quad \text{at } T$$

➠ **Solvency at time T : $\mathbf{A_T \geq L_T^g}$**

Policyholders receive L_T^g

➠ **Default at time T : $\mathbf{A_T < L_T^g}$**

Policyholders receive A_T

Standard Contracts
(Participating Bonus)

Policyholders are given a part δ of the benefits when

$$A_T > \frac{L_T^g}{\alpha} \quad \text{where} \quad \alpha < 1.$$

Policyholders now receive at T :

$$\Theta_L(T) = \begin{cases} A_T & \text{if } A_T < L_T^g \\ L_T^g & \text{if } L_T^g \le A_T \le \frac{L_T^g}{\alpha} \\ L_T^g + \delta(\alpha A_T - L_T^g) & \text{if } A_T > \frac{L_T^g}{\alpha} \end{cases}$$

Standard Contracts (Early Default)

The firm pursues its activities until T if :

$$\forall t \in [0, T[\quad , \quad A_t > L_0 e^{r_g t}$$

Let τ be the default time

$$\tau \; = \; \inf\{ \; t \in [0, T] \; / \; A_t < L_0 e^{r_g t} \; \}$$

In case of prior insolvency, policyholders receive :

$$\Theta_L(\tau) = \; L_0 e^{r_g \tau}$$

Standard Contracts
(Valuation Formula)

- With a Constant Minimum Guaranteed Rate,
- a Participation in the Assets Performance,
- and the Possibility of an Early Default, we have :

$$\mathbf{V_1} = \mathbb{E}_Q \Big(e^{-\int_0^T r_s ds} \left[L_T^g + \delta(\alpha A_T - L_T^g)^+ - (L_T^g - A_T)^+ \right] \mathbb{1}_{\tau \geq T} + e^{-\int_0^\tau r_s ds} \left[L_0 \, e^{r_g \tau} \right] \mathbb{1}_{\tau < T} \Big)$$

Reminder
Underlying Model

The dynamics under Q of the ZC bonds $P(t,T)$ are :

$$\frac{dP(t,T)}{P(t,T)} = r_t dt - \sigma_P(t,T) dZ_1^{\mathbb{Q}}(t)$$

whilst the assets dynamics under Q are :

$$\frac{dA_t}{A_t} = r_t dt + \sigma dZ^{\mathbb{Q}}(t)$$

where $Z^{\mathbb{Q}}$ and $Z_1^{\mathbb{Q}}$ are correlated Q-Brownian motions. ($dZ^{\mathbb{Q}}.dZ_1^{\mathbb{Q}} = \rho dt$).

Standard Contracts (Pricing Methodology)

To price this contract :

1. decorrelation of the assets and interest rate risks
2. introducing the forward-neutral measure.

➠ a 2D problem in (r, τ)

➠ an extension of Collin-Dufresne and Goldstein [2001] *solves the problem in terms of a recurrence equation*

➠ **Yet, No Closed-Form Formulae Can Be Obtained**

Building of New Contracts

Obtaining Closed-Form Formulae

Introducing...

... A New Contract

which is :

- ➠ only very slightly different from the previous one
- ➠ in a totally identical framework for A and r
- ➠ where only the Guaranteed Amount is modified

and Indexed on Government Zero-Coupon Bonds

The New Contract's Guaranteed Amount

The Minimum Guaranteed Rate is proportional to the yield of a Government Zero-Coupon Bond.

The Guarantee is the one of an equivalent position in $\frac{\beta \mathbf{L_0}}{\mathbf{P(0,T)}}$ Government ZC Bonds Maturing at time T. Indeed :

☞ At time 0, this Guarantee is worth βL_0,

☞ At time t, it is worth $l_t^g = \frac{\beta L_0}{P(0,T)} P(t,T)$,

☞ At time T, it is worth $l_T^g = \frac{\beta L_0}{P(0,T)}$.

The New Contract's Guaranteed Amount

What is the main Implication of Introducing a Guarantee such as the one Defined in the Previous Slide ?

Indeed the default time becomes :

$$\tau = \inf \left\{ \; s < T \; / \; A_s \; < \; l_s^g \; = \; l_T^g P(s,T) \; \right\}$$

and this allows pricing in closed-form :

$$\mathbf{V_2} \;=\; \mathbb{E}_{\mathbb{Q}} \Big[e^{-\int_0^T r_s ds} \Big(l_T^g + \delta \left(\alpha A_T - l_T^g \right)^+ - \left(l_T^g - A_T \right)^+ \Big) \mathbb{1}_{\tau \geq T} + e^{-\int_0^\tau r_s ds} \; l_\tau^g \; \mathbb{1}_{\tau < T} \Big] \text{ !!!}$$

The New Contract (Valuation)

So, how can V_2 be priced in closed-form ?

We Illustrate our Approach by Computing the Forward-Neutral Ruin Probability

$$\mathsf{Q}_T\,(\ \tau < T\ ) \quad = \quad \mathsf{Q}_T\left(\inf_{u\in[0,T[}\left(\frac{A_u}{P(u,T)}\right) < l_T^g\right)$$

The New Contract (Valuation)

The solution of our problem lies in the fact that :

$$\frac{A_u}{P(u,T)} = \frac{A_0}{P(0,T)} e^{N_u - \frac{1}{2}\xi(u)}$$

where the differential of the martingale N is defined by :

$$dN_s = (\sigma_P(s,T) + \rho\sigma)\, dZ_1^{Q_T}(s) + \sigma\sqrt{1-\rho^2}\, dZ_2^{Q_T}(s)$$

and the quadratic variation of N is :

$$\xi(u) = \langle N \rangle_u = \int_0^u [(\sigma_P(s,T) + \rho\sigma)^2 + \sigma^2(1-\rho^2)]ds$$

Conclusion (Valuation)

Thanks to Dubins-Schwarz Theorem

$$FNRP = \mathbb{Q}_T \left\{ \min_{s \in [0, \xi(T)]} \left(B_s - \frac{1}{2} s \right) < \ln(\beta\,\alpha) \right\}$$

Therefore the contract can be priced as a linear combination of Gaussian functions (and so the pricing is instantaneous).

As the Minimum Guarantee moves with the interest rates some of the Participating Contract's Characteristics become very interesting for the company...

Numerical Analysis

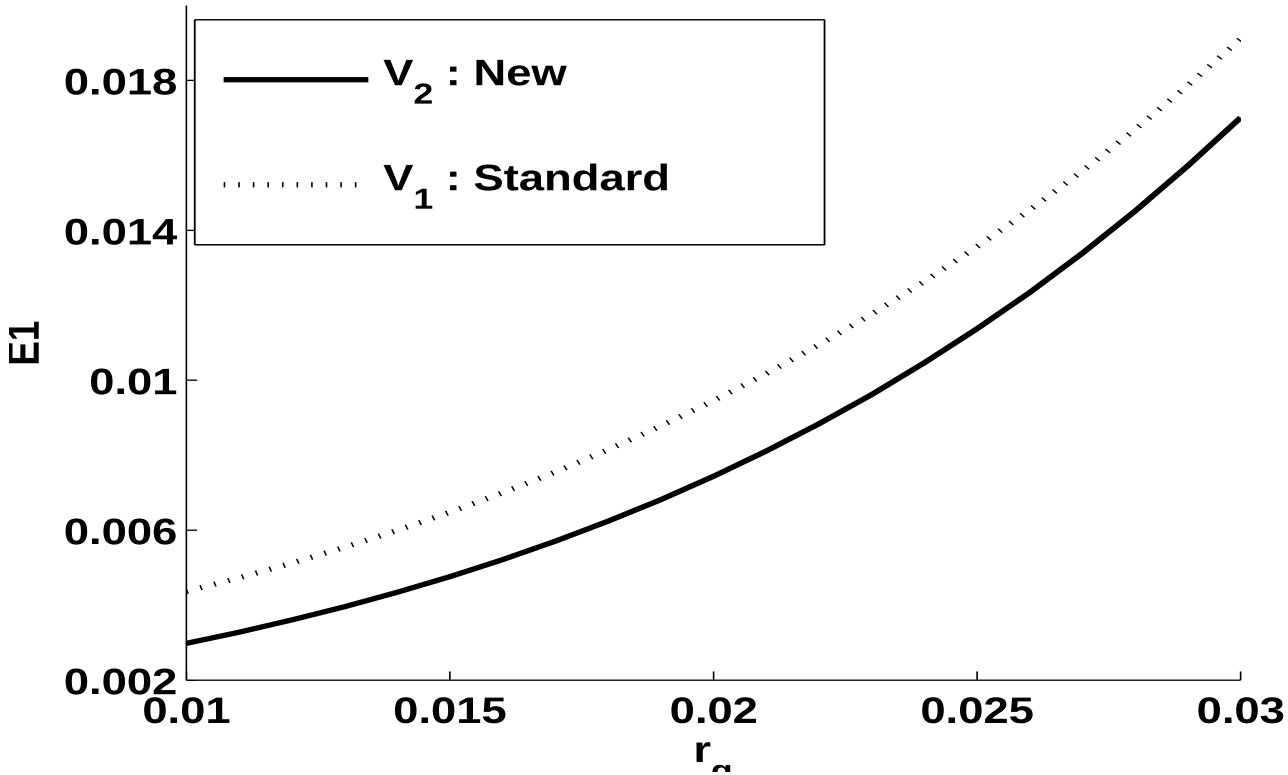
We set our parameter range according as :

A_0	α	ρ	σ	T	δ
100	0.85	0.2	10%	10	90%

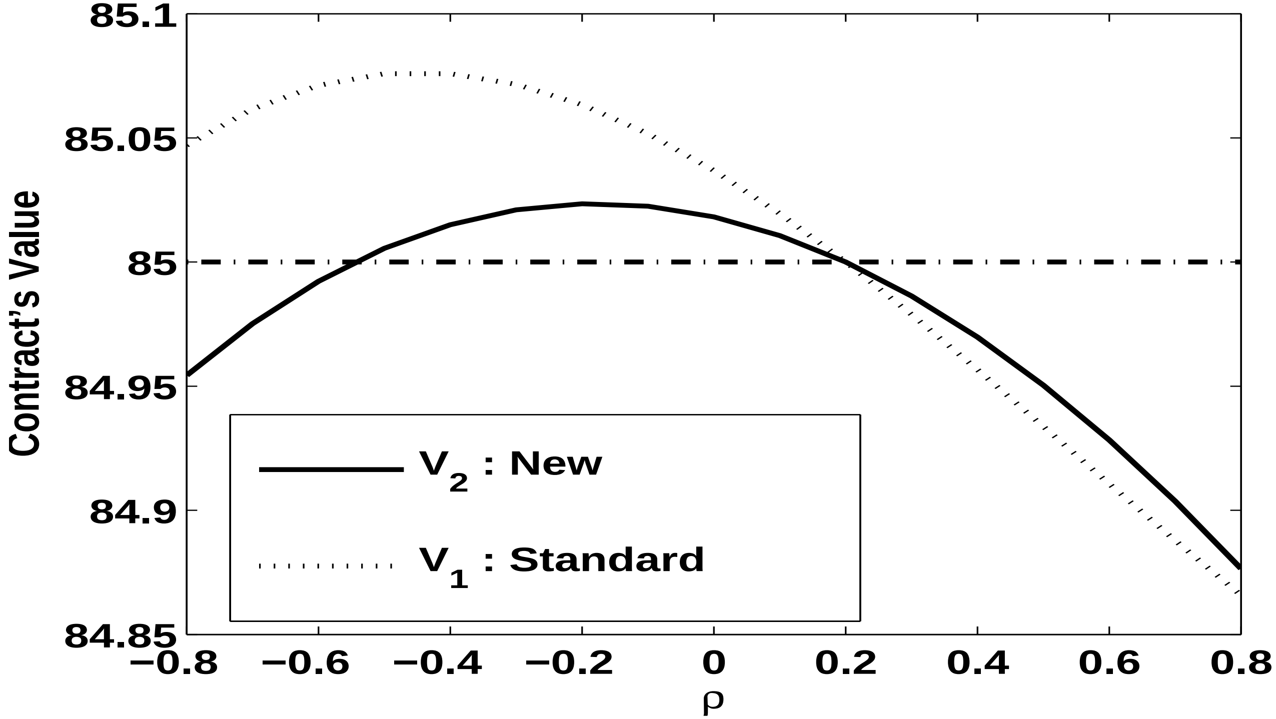
$$L_0 = \alpha A_0 = 85$$

Contract Maturity : 10 years

Ruin Probability E_1 w.r.t. r_g



Contract Value w.r.t. ρ (Correlation A / r)



General Conclusion

A Proposal for New Participating Contracts.

Instead of guaranteeing at time t a constant minimum rate, guarantee a yield proportional to the one of a ZC bond.

In such a setting, closed-form formulae can be obtained.

This method also allows to price exotic options (sharks options for instance).