Risk Neutral Valuation of With-Profits Life Insurance

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Executive Summary

- **Develop Framework in which the following embedded options in participating life-insurance contracts can be valuated separately**
  - **Interest Rate Guarantee:** Consider surplus distribution mechanism (management rules)
  - **Surrender Option:** Develop valuation model in order to price complex, path-dependent options
- **Practical Implementation:**
  - Prerequisites and Downsides of the risk-neutral approach
- **Key Results:**
  - Guarantees are currently offered below their risk neutral value
  - Financial strength of the insurance company considerably affects value of a contract

Introduction

- **Guarantees in Participating Life Insurance Contracts:**
  - **Year-to-Year Cliquet Style (Ratcheting) Guarantee:** Liabilities (including already credited surplus) earn a guaranteed rate of interest p.a. plus some surplus (if possible)

    → Value of the guarantee depends on **regulatory prerequisites** (guaranteed rate of interest, participation levels) and on the company's **surplus distribution mechanism** (management decision)
  - **Deferred annuities: Guaranteed annuity factors** (not considered here)
- **Current Problems:**
  - Low interest rates
  - plunging stock markets
  - Insurers are stuck with old, non-hedged guarantees

Introduction (2)

- **Literature:**
  - Mostly focusing on unit-linked products
  - Mostly valuation of contract as a whole – no consideration of the embedded options separately (Implementation?)
  - Most models are not adequate for some markets (e.g. the German market)
  - Mostly, the financial strength of the issuer is not taken into account (e.g. via reserve quota)
- **Target of this paper:** Fill the gap –

  Valuate and Analyze offered guarantees / options of a participating contract in a framework taking into account the prevailing surplus distribution rules (in e.g. the German market) and the financial strength of the issuer

Model

- **Insurer‘s financial situation**

Market value of assets ← A_t

Assets	Liabilities
A_t	L_t
	R_t
A_t	A_t

L_t → Book value of Policyholder’s account

R_t → Reserves (Valuation Reserves, equity, etc.)

- **Development of the assets and liabilities (general):**

“ - ”: before dividend payment

“ + ”: after dividend payment

Annual return of the “reference portfolio” *A*

$$A_t^- = A_{t-1}^+ (1 + r_t)$$

$$A_t^+ = A_t^- - D_t$$

Reserve Quota R_{t-1}/L_{t-1}

$$L_t = L_t\left(A_t^-, A_{t-1}^+, x_{t-1}, L_{t-1}, \ldots\right)$$

Dividend Payments

$$D_t = D_t\left(A_t^-, A_{t-1}^+, x_{t-1}, L_{t-1}, \ldots\right)$$

Model

- **Considered distribution schemes:**
  1. **MUST-case:**
     - Only obligatory payments considered
     - Year-to-year cliquet style guarantee on the liabilities (minimum rate *g)*
     - Proportion δ of earnings on book value have to be credited to policyholder's accounts
     - "Rest" of the earnings on book values paid out as dividend
     - "Rest" of the earnings on market values remains in the company (Reserves)
     - e.g.

BV(·) denotes the function transforming earnings on market values to earnings on book values (depends on regulatory framework and management decisions) – for details see paper

$$L_{t+1} = (1+g)L_t + [\delta\, BV(A_{t+1}^{-} - A_t^{+}) - gL_t]^{+}$$

→ **Obligatory payments should be considered in any meaningful valuation – in particular interesting for regulatory organs**

Model(2)

- **Considered distribution schemes:**
  2. **IS-case:**
     - In addition to obligatory payments, typical surplus distribution schemes of German Insurers are considered (model by Kling/Richter/Russ)
     - Target interest rate z is credited to the policyholders' account as long as the reserve quota stays within a given range $[a,b]$
     - If crediting target rate leads to a quota below a or above b, the company credits exactly the rate that leads to $x_t=a$ or $x_t=b$, respectively (however, the guarantees must still be fulfilled!)
     - Dividends amount to a portion α of any surplus credited to the policy reserves

→ **Such corporate political issues might be of interest for companies' actuaries as well as for customers, who are interested in the value of their product**

Risk Neutral Valuation

- **General Approach:**
  - **As usual:** There exists a risk neutral measure *Q* and a numéraire process $(B_t)_{t \in [0,T]}$
  - **General pricing formula:** $P^* = E_Q[B_T^{-1} L_T]$
- **Problems:**
  1. **Underlying security not traded:** Company's portfolio is subject to management and investment decisions
  2. **We want to price the embedded option rather than the whole contract:** A possible hedging strategy could not be implemented within the company, since then the underlying would change
- **Solution:**
  1. **Approximate reference portfolio by traded benchmark portfolio**
  2. **Alternative Approach: Price the cash flows**

Risk Neutral Valuation(2) – Cash Flow Approach

- **Relevant Cash Flows:**
  - **Dividends** are paid out and reduce the value of the reference portfolio (but not the asset allocation)

    Value: $$Z_0 = E_Q\left[\sum_{t=1}^{T} B_t^{-1} D_t\right]$$

  - If the return of the reference portfolio is so poor, that granting the **minimum interest guarantee** would result in negative reserves, capital is needed in order to fulfill obligations (capital shots C_t) – they increase the value of the reference portfolio, asset allocation stays the same

    Value: $$F_0 = E_Q\left[\sum_{t=1}^{T} B_t^{-1} C_t\right]$$

  - **Similarly:** Value WAO_0 of the **Surrender Option** as the supremum of the values of all possible surrenders

Risk Neutral Valuation(3) – Cash Flow Approach

→ **Equilibrium Condition for a fair contract:**

$$F_0 + WAO_0 + R_0 \overset{!}{=} Z_0 + E_Q\left[B_T^{-1} R_T\right]$$

$$\Leftrightarrow P^* = E_Q\left[B_T^{-1} L_T\right] \overset{!}{=} L_0$$

since

$$P^* = E_Q\left[B_T^{-1} L_T\right] = L_0 + R_0 + F_0 - Z_0 - E_Q\left[B_T^{-1} R_T\right] + WAO_0$$

Numerical Analysis

- **Model market** by a geometric Brownian motion (*A*) with constant volatility and the numéraire (*B*) by riskless asset with constant interest rate *r*

- **Simulation methods:**

  1. **Monte Carlo Simulation:**

     → No consideration of the Surrender Option, but allows for pricing value of Dividends, Interest Guarantee and Reserve-Delta separately

  2. **Discretization Approach (Extension of Lukkarinen and Tanskanen's (2004) Approach):** Use Black-Scholes PDE in order to solve valuation problem within one period and use arbitrage arguments at the transitions

     → Consideration of the Surrender Option possible

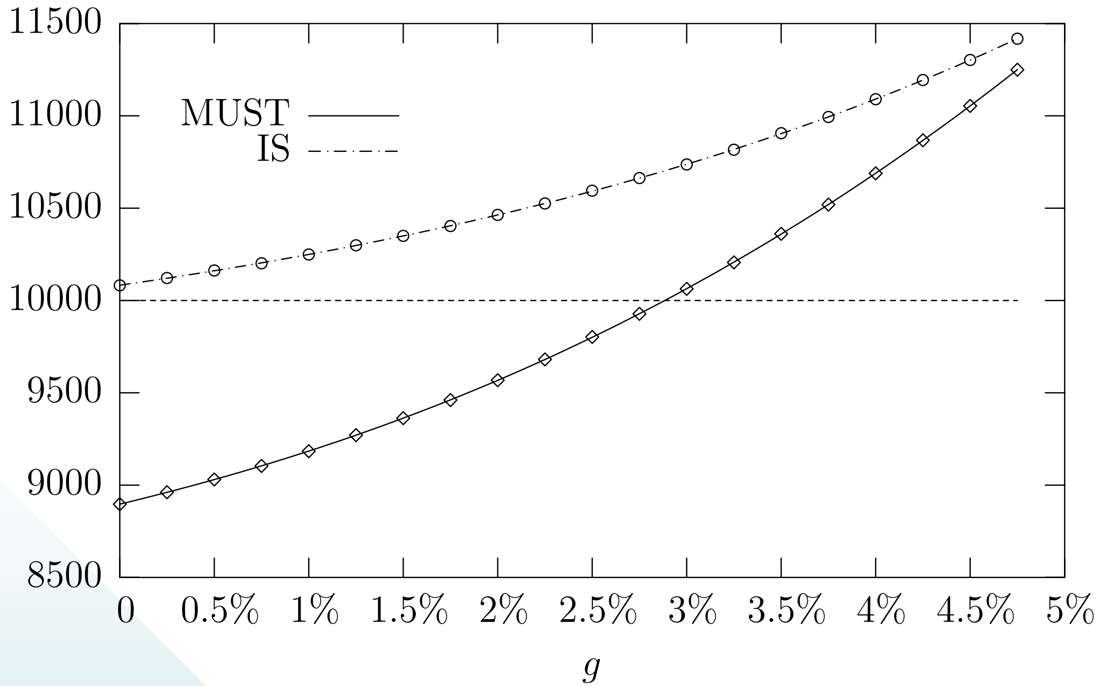
Selected Results – Parameters for the Simulations

Guaranteed rate of interest g	3.5%
Minimum participation rate δ	90%
Insurer's initial reserve quota x_0	10%
Target interest rate z	5%
Reserve corridor $[a,b]$	[5%,30%]
Portion provided to share holders α	5%
Asset volatility σ	7.5%
Risk free rate of interest r	4%
Time horizon T	10 years
Initial investment L_0	10,000

Selected Results(2) – Values

	MUST-Case	IS-Case
Premium	10,000.00	10,000.00
+ Interest Guarantee	+868.42	+998.99
- Dividends	-238.16	-74.36
- Reserve-Delta	-275.76	-20.30
Value of a “European Contract”	10,354.50	10,904.33
+ Surrender Options	0	0
Value of an “American Contract”	10,354.50	10,904.33

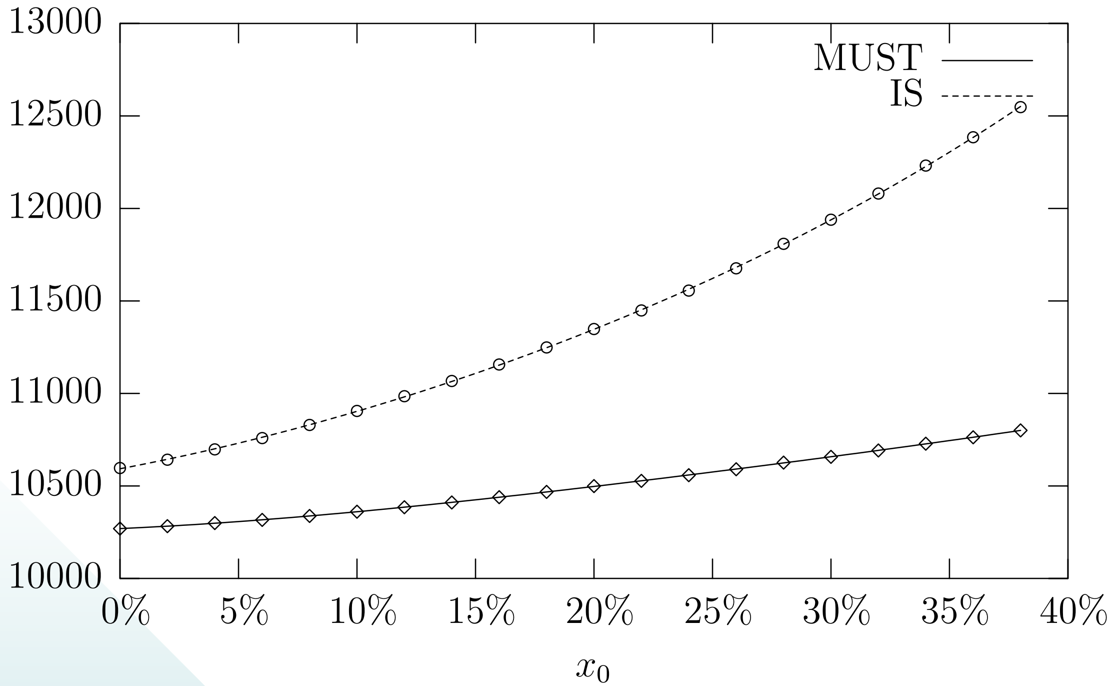
Selected results(3) – Influence of the guaranteed rate *g*



-Obviously, the value of a contract **increases with increasing guaranteed rate.**

-However, the level is very high: In the Must-Case, the value of a contract **exceeds the insured's investment at a level of 2.75%** (current level) and in the **Is-Case even with a 0% guarantee.**

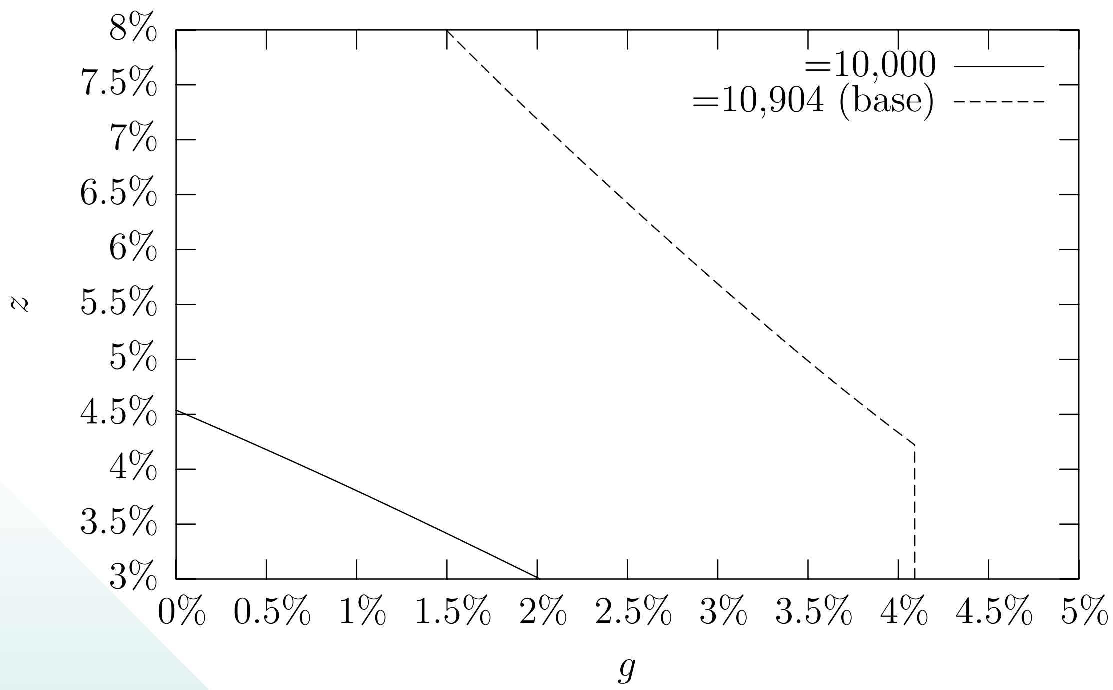
Selected results(4) – Influence of the initial reserve quota x_0



-**Is-Case:** The value of a contract increases with increasing reserve quota, since more is paid out if the reserves are high.

-**Must-Case:** Value of a contract **also** increasing in x_0.

Selected Results(4) – Interaction of guaranteed rate *g* and target rate *z*



-In order to treat contracts alike (same risk-neutral value), **contracts with a lower guaranteed rate should be granted a higher target rate**, if the reserve situation is good!

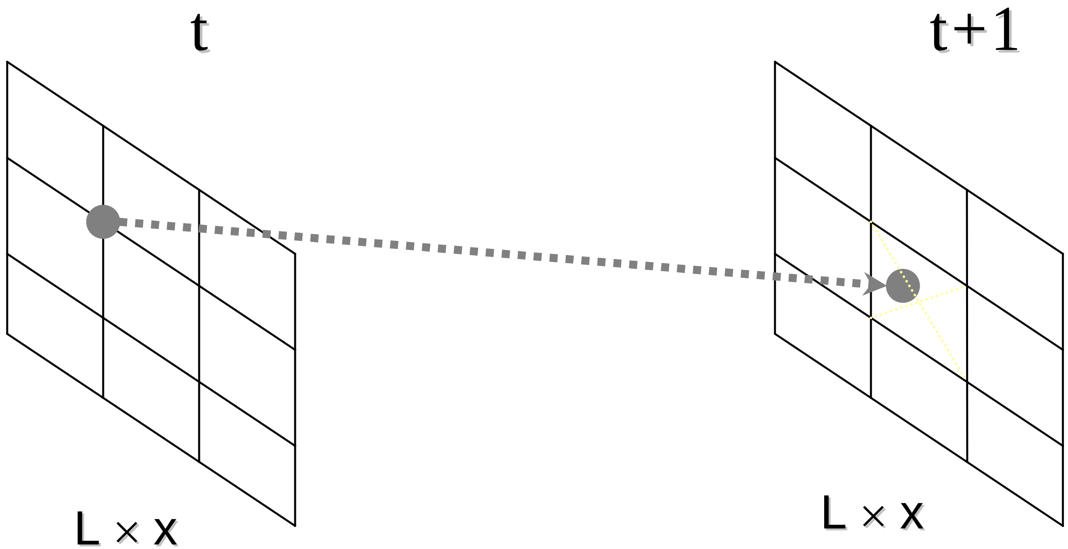
-**However:** Relationship depends on other parameters, too: In company with higher reserves, the "slope" is smaller.

Outlook

- **Extensions of our model**:
  - **Stochastic interest rate model**
  - **More advanced model for the underlying assets**
  - **Empirical Research:** Is it possible to find a adequate traded benchmark portfolio?
  - **Implementation:** How to hedge the options/guarantees?

THANK YOU!

Backup (1) – Possible Question to Tanskanen/Lukkarinen‘s (2004) approach



- Within one period: Black-Scholes Problem → Heat Equation
- t=1,2,3,...,T – No Arbitrage condition:

$$V(\nu-, A_{\nu-1}, A_{\nu}, L_{\nu-1}, x_{\nu-1}) =$$
$$V(\nu, A_{\nu}, A_{\nu}, L(A_{\nu-1}, A_{\nu}, L_{\nu-1}, x_{\nu-1}), X(A_{\nu-1}, A_{\nu}, L_{\nu-1}, x_{\nu-1}))$$

- Numerical Implementation: Mesh... know values at t+1 und solve for values at t