Risk measures and efficient use of capital

Ph. Artzner, ULP Strasbourg, F. Delbaen ETH Zürich, P. Koch-Medina, Swiss Re

- Intuition on FX risk
- Simple example of capital choice
- Generalisation to currency invariance
- Difficulties with TailVaR

Currency risk

- Business in euros and pounds: (buffer-) capital better be in euros and pounds
- Same cost, same value, of extra basket of capital, both in Francfort and in London ?

Example of P/L - FX correlation

- Date 0   FX rate 1/1
- Date 1 random FX rate : # of **foreign** to get one *domestic* :

(1/2 , 1 , 3)

Date 1 net worth domestic *(-2, 6, -1)*

required date 0 capital *domestic* : + *2* (worst-case risk measure)

required date 0 capital **foreign** : +**3**

Efficient choice of capital

Deal with *(-2, 6, -1)* and rate ***domestic*** to **foreign** of (1/2, 1 , 3)

solve : min (d + $\mathbf{f}$) such that (zero int. rates)

$$d + 2\,\mathbf{f} - 2 > 0$$

$$d + 1/3\,\mathbf{f} - 1 > 0$$

and get d = 4/5, $\mathbf{f}$ = 3 / 5 , at a date 0 cost of 7/5 (domestic or foreign) !

Generalisation

Exchange rates e_0 and e_1 , P/L X *domestic*

$$\min\{\, m;\ d + \mathbf{f} / e_1 + X \in \mathcal{A},\ d + \mathbf{f} / e_0 = m \}$$

and with numéraires *r* and **s** :

$$\min\{\, m;\ rd + \mathbf{sf} / e_1 + rX \in \mathcal{A},\ d + \mathbf{f} / e_0 = m \}$$

Results

With a coherent acceptance set A in future domestic currency the optimisation above leads to an extended acceptance set B such that the coherent risk measures ρ and σ defined by B and by $e_1\mathsf{B}$ satisfy for each X

$$\sigma(e_1 r/\mathbf{s}\ X) = e_0\ \rho(X)$$

London Francfort

About distribution invariance

Back to FX rate (1/2, 1, 3) , and a permuted P/L:

(-1, 6, -2) instead of *(-2, 6, -1)*

solve for min (d + **f**) such that:

$$d + 2\mathbf{f} - 1 > 0$$

$$d + 1/3\mathbf{f} - 2 > 0$$

and get d = 2 , **f** = 0 at a cost of 2 : the derived measure is not distribution invariant !

TailVaR in Francfort and London do not fit via e_0 !