VaR Control, as a Source of Profit

A Paper for AFIR 1999 in Tokyo

Miwaka Yamashita

Portfolio Manager, The Tokio Marine and Fire Insurance Co., Ltd.
Tel: 03-5223-3630 Fax: 03-5223-3539 E-mail: miwaka.yamashita@tokiomarine.co.jp

April 30, 1999

[Summary]

Basically, VaR is a risk control tool and itself not a source of profit. Needless to say, investment results will be decided by the accuracy of the return prediction of the investing assets. In this paper, tactics of the investment decision making process, how to use VaR control smart, will be discussed other than how to forecast asset returns appropriately and precisely.

Usually, pension funds or insurance companies perform ALM analysis before investing and can fix the surplus or the loss of their assets versus their liabilities by using interest rate instruments. The pure risk-taking, absolute return mandate investment will be conducted aiming to get an absolute amount of profit in order to increase the surplus, or compensate the loss. VaR controlled investment is the most popular strategy for that purpose.

In the section II, mainly, VaR controlled investment is defined and its superiority is discussed. In section III, the method of risk allocation, not cash allocation, is developed. Others such as the effect of the short strategy and advanced VaR controlled investment is also discussed.

I. Introduction

Basically, VaR is a risk control tool and itself not a source of profit. Needless to say, investment results will be decided by the accuracy of the return prediction of the investing assets. In this paper, tactics of the investment decision making process, how to use VaR control smart, will be discussed other than how to forecast asset returns appropriately and precisely.

Usually, pension funds or insurance companies perform ALM analysis before investing and can fix the surplus or the loss of their assets versus their liabilities by using interest rate instruments. The pure risk-taking, absolute return mandate investment will be conducted aiming to get an absolute amount of profit in order to increase the surplus, or compensate the loss. VaR controlled investment is the most popular strategy for that purpose.

In the section II, mainly, VaR controlled investment is defined and its superiority is discussed. In section III, the method of risk allocation, not cash allocation, is developed. Others such as the effect of the short strategy and advanced VaR controlled investment is also discussed.

II. Superiority of VaR controlled Investment

1. The Best Strategy

Under the condition that allowed maximum VaR amount (V_0) is fixed (VaR is defined as 2.33*volatility*amount of asset mix if the limit probability of the draw down is set 1%.), the investment amount is decided by the size of volatility of the investment asset mix. In order to maximize the total amount of profit from the investment, the asset mix whose return per unit volatility is maximum should be chosen as the optimal one. This investment strategy is defined as VaR controlled investment.

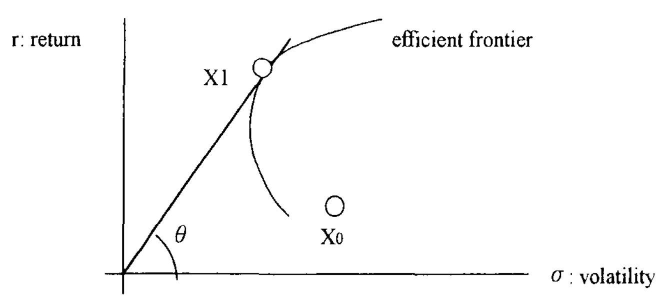


fig.1 Optimal point of VaR controlled investment

The initial investment asset mix is denoted X_0 in the volatility-return plane as shown above. The efficient frontier at the next time period is drawn as a curved line and X_1 represents the next time period's optimal asset mix. X_1 will be decided so that the Sharp Ratio becomes maximum where Sharp Ratio is defined as return dividend by volatility. (The $\tan\theta$ becomes maximum in the fig.1.)

Four strategies are compared and they are as follows:

a) VaR controlled investment

In this strategy, total VaR amount of asset mix will be maintained constant. The next time period's asset mix will be decided as shown above and the total asset amount will be calculated as follows:

$$Pa \times 2.33\,\sigma a = \text{Constant} = V0$$

V0: maximum limit of total VaR amount of asset mix

Pa: total asset amount at the beginning of the next time period

σa: volatility of the asset mix at the beginning of the next time period

The amount of the profit Ra will be as follows denoting the return as Ra:

$$Ra = Pa \cdot ra = \frac{V0}{2.33} \cdot \frac{ra}{\sigma a}$$

b) Constant volatility strategy

In this strategy, volatility of the investment asset will be maintained constant when asset mix is determined. The next time period's asset mix will be the point on the efficient frontier as σb = Constant = $\sigma 0$ ($\sigma 0$ is initial asset mix volatility) with a higher (expected) return rb (versus ra). The total amount of the asset mix Pb and the profit Rb will be decided as follows under making the amount of VaR is V0:

$$Pb \times 2.33\,\sigma b = V0$$

$$Rb = Pb \cdot rb = \frac{V0}{2.33} \cdot \frac{rb}{\sigma b} = \frac{V0}{2.33} \cdot \frac{rb}{\sigma 0}$$

c) Constant asset mix amount (cash amount) strategy

In this strategy, initially, total asset amount is decided and will be maintained constant. The profit Rc will be decided as follows (The total amount of the asset mix is Pc (= P0 (initial asset mix amount)), possible volatility size is σc, and the profit is Rc:

$$Rc = Pc \cdot rc = P0 \cdot ra = \frac{V0}{2.33} \cdot \frac{ra}{\sigma 0}$$

(VaR amount is independent from maximum amount V0 and return and volatility are rc = ra and σc

= $\sigma 0$)

The result of the comparison of the total amount of profits Ra, Rb, Rc and Ro (profit from initial asset mix with the return denoted r0) depends on the size order of the Sharp Ratio of each. For example, under the assumption of the efficient frontier as shown below, the order is as just they are (Ra-Rb-Rc-Ro).

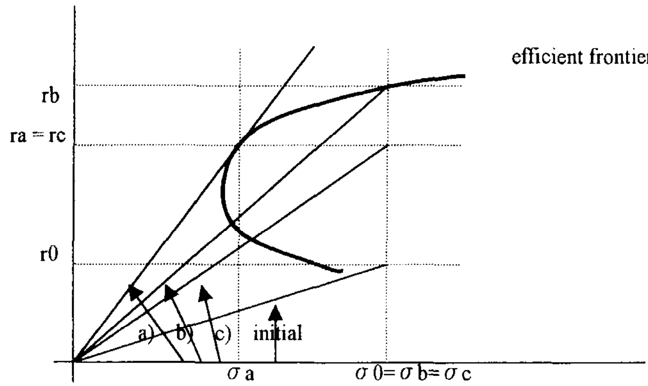


fig. 2 Comparing strategies

2. Broader Opportunity to Get Optimal

In case that at least the part of the efficient frontier of the next time period is in the upper side of the line O-X0 (X0: initial asset mix) shown in fig. 3, it is clear that there is the optimal asset mix X1, whose Sharp Ratio is larger than that of the initial asset mix X0.

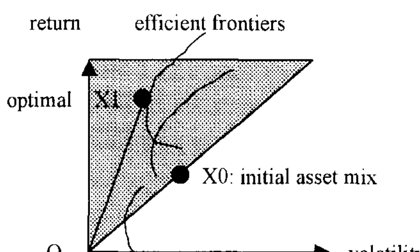


fig. 3 Efficient frontiers which have optimal points

In case the asset mix consists of two kinds of assets, in what circumstances, it is worth while investing into both two kinds of asset, other than only to invest one asset.

Suppose the case that initial asset mix is one asset and another asset will be added to the initial asset mix. The condition it that part of the efficient frontier is above the line O-X0 is described:

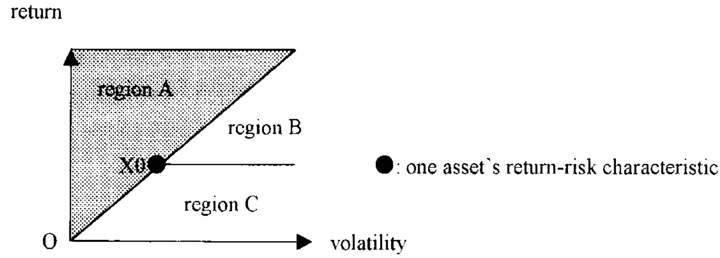


fig. 4 Areas for conditions

region A:

$$\frac{r1}{\sigma 1} > \frac{r0}{\sigma 0}$$

region B:

$$\frac{r1}{\sigma 1} < \frac{r0}{\sigma 0}, r1 > r0, \frac{\sigma 0}{\sigma 1} > \rho$$

region C:

$$\frac{r1}{\sigma 1} < \frac{r0}{\sigma 0}, r1 < r0, \frac{r1}{\sigma 1} / \frac{r0}{\sigma 0} > \rho$$

ρ : correlation coefficient between two assets

$\sigma 0$: volatility for initial asset mix

$\sigma 1$: volatility for the other asset

r0: forecasted return for initial asset mix in the next time period

r1: forecasted return for the other asset in the next time period

3. Short Strategy as a New Frontier of Investment

Suppose all the negative return will be predicted, short strategy will give additional profit opportunity. The following shows how the return prediction distribution change will affect on the value of Sharp Ratio under perfect negative return prediction circumstance.

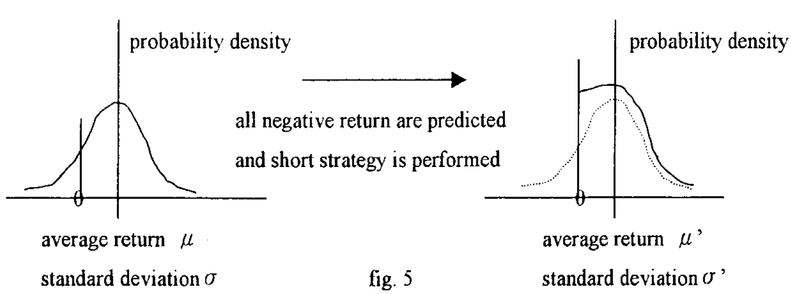


fig. 5

$$\mu' = \mu[\sqrt{\frac{2}{\pi}}\exp\{-\frac{1}{2}(\frac{\mu}{\sigma})^2\} + G\pm]$$

$$G\pm = \int_{-\frac{\mu}{\sigma}}^{+\frac{\mu}{\sigma}} \mathrm{Z(x)dx}$$

$$\mathrm{Z(x)} \sim \mathrm{N(0,1)}$$

Here Sharp Ratio is defined as the return divided by the second moment (σ ') and it is as follows:

$$\sigma'^2 = \sigma^2[1 + \frac{\mu}{\sigma}\sqrt{\frac{2}{\pi}}\exp\{-\frac{1}{2}(\frac{\mu}{\sigma})^2\} - (\frac{\mu}{\sigma})^2\{\frac{\sigma}{\mu}\sqrt{\frac{2}{\pi}}\exp\{-\frac{1}{2}(\frac{\mu}{\sigma})^2\} + G\pm\}]$$

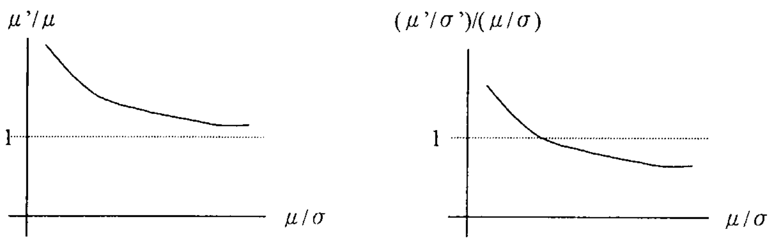


fig. 6 μ amd μ', σ and σ'

Needless to say, average return surly is improved ($\mu' > \mu$).

However, Sharp Ratio is not always improved.

In a more general case, an investment strategy which includes both long and short position is introduced and the returns are depicted as X~•Y~, where;

X~ ∈ N(μ, σ); function : Z(x; μ, σ)

Y~ ∈ {+1, -1}   +1: long strategy, -1: short strategy.

As an investment result, for example, a set of long strategy (Y~) and positive return (X~) is appropriate. Here the probability of the pair of X~ and Y~ will be set as following (a) ~ (d):

	Y~ = +1	Y~ = -1	Sum.
X~ ≧ 0	(a) k(1-G)	(b) (1-k)(1-G)	1-G
X~ < 0	(c) (1-k)G	(d) kG	G
Sum.	k+(1-2k)G	1-k-(1-2k)G	1

where; 0<k<1 : parameter, probability to chose right strategy.

If k>0.5, Probability of Y~ = +1 is over 0.5.

$$G = \int_{-\infty}^{0} \mathrm{Z}(\mathrm{x}; \mu, \sigma)\mathrm{dx}$$

In terms of the probability that the return falls short of a certain return level (r0), the condition that average return from strategy X~•Y~ is smaller than that of strategy X~ is as follows:

$$k > 1 - \frac{G0}{1-(G+G0)+2GG0}$$

$$G0 = \int_{-\infty}^{r0} Z(x;\mu,\sigma)dx$$

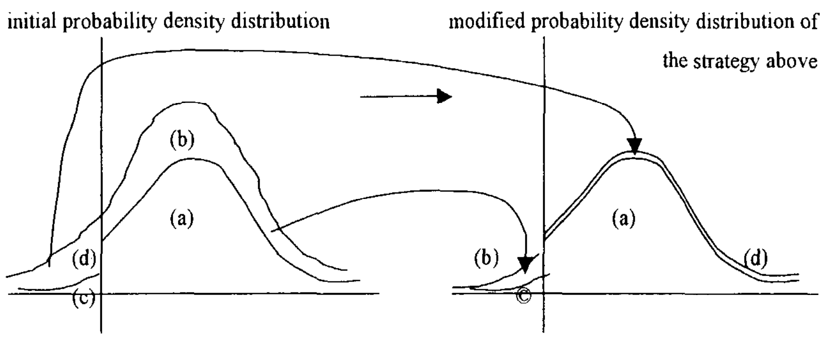


fig. 7 Change of the return probability density distribution

Usually, this condition of k is to be very near to 1. The reason why, in order to improve average return, the probability to chose right strategy (long at positive return and short at negative return) should be almost 1 is that there is a certain extent of chance to loose much by short strategy and empirically this is right, winning small and losing much.

III. Risk Allocation Method

1. RoR – Risk Plane

At return prediction, each view for each investment asset has each time horizon. It is not appropriate that predictions for asset returns are performed only on fixed time horizon base. For each asset, appropriate time horizon will be set when return prediction is performed.

Based on a random walk assumption, the size of standard deviation for returns become wider in proportion to the square of the size of the time period. (In actual, although, each asset has each volatility structure, and this specific structure will easily be implimented.)

The popular formation to find maximum Sharp Ratio point on the efficient frontier for two assets can be denoted as follows:

$$\text{Max} \quad r/\sigma$$

$$r = w1*r1+w2*r2$$

$$\sigma^2 = w1^{2}*\sigma 1^2+2\sigma 1\sigma 2\rho+\sigma 2^2*w2^2$$

ri: return prediction for asset i

wi: weight for asset i as an investment amount base

σ i: (prediction of) standard deviation for asset i as an investment base

ρ : correlation coefficient between asset 1 and asset 2

$$\text{s.t.} \quad w1+w2 = 1$$

The equations can be rewritten into as follows:

$$\text{Max} \quad S$$

$$S = (W1*S1 + W2*S2) / R$$

$$R^2 = W1^2+2*W1*W2\rho+W2^2$$

$$S = r / (2.33*\sigma)$$

$$R^2 = (2.33*\sigma)^2$$

$$Wi = 2.33*wi*\sigma i$$

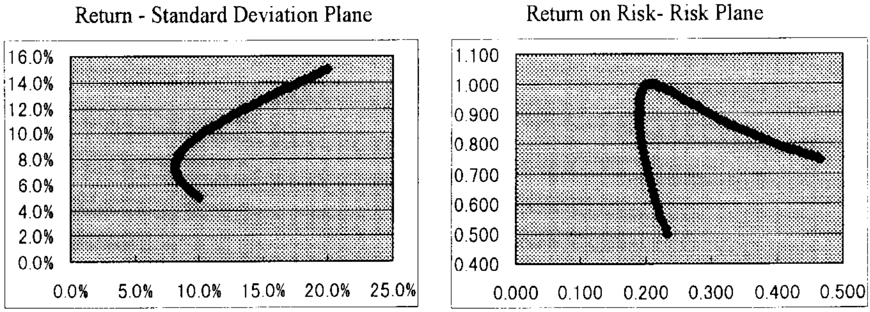
$$Si = ri/ (2.33*\sigma i)$$

$$\text{s.t.} \quad W1 / (2.33*\sigma 2) + W2 / (2.33*\sigma 2) = 1$$

W1,W2 are unit risk amount on the condition that those two asset returns are perfectly correlated. S depicts return on risk and R depicts the amount of risk.

efficent frontier on Return - Standard Deviation Plane

efficent frontier on Return on Risk- Risk Plane



Conditions: r1=5%, r2=15%, σ 1=10%, σ 2=20%, ρ=-0.20

fig. 8 Return-Standard Deviation Plane and Return on Risk-Risk Plane

2. Time – Horizon of Returns

In the VaR controlled investment here, each time horizon is taken into account as using Si defined the following definition:

$$Si = \frac{ri}{2.33 * \sigma i \sqrt{time\_horizon}}$$

In the case the return is recalculated for unit time horizon base, the result will be as follows:

$$\frac{ri}{2.33 * \sigma i \times time\_horizon}$$

This "time horizon adjusted return on risk" can be used only in the formula described the previous section and this allows appropriate consideration about time-horizon.

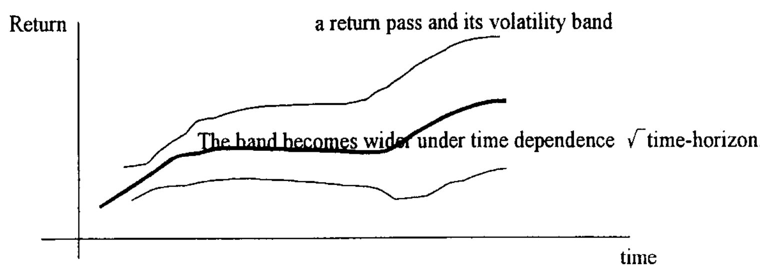


fig. 9 The volatility band

IV. Advanced VaR Control

1. Portfolio of Japanese Insurance Companies

At Japanese insurance companies, in terms of the market value weight of the asset, Japanese stock portfolio is dominant and its risk (volatility of market value) is almost all the risk, which should be taken care of.

In this section, the model is introduce as follows:

$$(Ps+P0) \times 2.33\sigma < V0 + k(Ps-Ps0)$$

Ps: market value of a risk asset

Ps0: initial market value of a risk asset

P0: market value of the other assets: constant

V0:initial allowed VaR amount

$$V0 = 2.33*(Ps0 + P0)*\sigma 0$$

σ : volatility of the whole assets

$\sigma 0$: initial volatility of the whole assets

In this model, VaR is controlled and the amount of maximum VaR increases as the risk asset's market value increases with the multiple k (k<1). The condition is re-written into the following inequation:

$$k > 2.33\sigma + 2.33\frac{\sigma - \sigma 0}{Ps - Ps0}(Ps0 + P0)$$

The parameter k could be under 1 and this inequation means that the important thing to manage VaR is to reduce the volatility. This implicates that, if the unrealized gain of the risk asset is included as a risk buffer of VaR, the important thing is to reduce the volatility of the whole asset mix, not the size of a risk asset.

2. Advanced VaR Controlled Investment

Here the maximum VaR amount for each time period increases/decreases in proportion to the profit amount of the previous time period. The following shows the example.

P1 depicts the total amount of the asset mix at the beginning of the time period 1， $\sigma 1$, the volatility of the investment asset mix at the beginning of the time period 1 ,V1, the maximum VaR (and actually the amount of the VaR) of the time period 1 and r1, the return of the investment asset mix at time period 1. P2, $\sigma 2$、V2 are set for the same notation for the time period 2 and more for the time period 3. The relation between them are as follows:

$$P1 \times 2.33\sigma 1 = V1$$

$$P2 = P1 \times (1+r1)$$

$$(P2 + \Delta P2) \times 2.33\sigma 2 = V2$$

$$V2 = V1 + k \times P1 \times r1$$

$$(P3 + \Delta P3) \times 2.33\sigma 3 = V3$$

$$V3 = V2 + k \times (P2 + \Delta P2) \times r2$$

where $\Delta P2$ and $\Delta P3$ depict additional asset amount of time periods 2 and 3, which only come from controlling VaR limit. The coefficient k represents the ratio how the previous time period's return will be added to VaR amount of the next time period.

From the equations above, the value of $\Delta P2$ and $\Delta P3$ are described as follows:

$$\frac{\Delta P2}{P1} + r1 = -\frac{1}{\frac{\sigma 2}{\sigma 1}} \cdot \frac{\sigma 2 - \sigma 1}{\sigma 1} + \frac{k * r1}{2.33\sigma 2}$$

$$\frac{\Delta P3}{P2 + \Delta P2} + r2 = -\frac{1}{\frac{\sigma 3}{\sigma 2}} \cdot \frac{\sigma 3 - \sigma 2}{\sigma 2} + \frac{k * r2}{2.33\sigma 3}$$

In each equation, there are two sources of changing total investment amount. The first term is leverage part and in case the total volatility of new asset mix becomes smaller, more leveraged investment will be allowed. The second term comes from changing VaR amount according to the return obtained in the previous period.

According to the equations above, the continuous equation can be described as follows:

$$\frac{1}{\mathrm{P}}\cdot\left(\frac{\partial \mathrm{P}}{\partial \mathrm{V}}\cdot\frac{\partial \mathrm{V}}{\partial t}+\frac{\partial \mathrm{P}}{\partial t}\right)=-\frac{1}{\sigma}\cdot\frac{\partial \sigma}{\partial t}+\frac{k}{2.33\sigma}\cdot\frac{1}{P}\frac{\partial \mathrm{P}}{\partial t}$$

P: investment amount

V: VaR amount

σ :standard deviation of the portfolio

V. Summery

VaR controlled investment has many advantages if used smart.

The point is that always to use all the maximum limit VaR. Leveraged effect will give higher performance.

In addition, risk allocation method is developed, in order to be able to take different time horizon return predictions into account.

In a Mean-Variance optimization, return and volatility are taken their place by Retrun on Risk and Risk Amount respectively.

References

Best, Philip, Implementing Value at Risk, John Wiley and Sons, Inc.,1971
Dowd, Kevin, Beyond Value at Risk, John Wiley and Sons, Inc.,1998
Theil, Henri, Principles of Economics, John Wiley and Sons, Inc.,1998