

ASSET-LIABILITY MODELLING FOR PENSION SCHEMES

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The principle of selecting assets for a financial institution that are appropriate to the liabilities of that institution is a very old one. Modern asset-liability modelling simply formalises this ancient principle using mathematical methods.

It is commonly stated that the assets should take account of the *nature* and *term* of the liabilities. One aspect of the nature is the currency of the obligations, and banks (which generally have assets and liabilities expressed in nominal currency terms) are generally careful to match the currencies of their assets and liabilities. Many life assurance contracts are also expressed in nominal currency terms and some currency matching is appropriate for them too.

However, the liabilities of a pension scheme are frequently designed to be in what we call "real terms" (i.e. in units of some index of consumer prices), either because they are related to the salary of the employee at or near retirement (or possibly related to the value of an earnings index in a revalued average salary scheme), or because the pensions in payment are indexed (formally or informally) to a consumer price index. For such liabilities the appropriate matching assets may not exist. No salary- or earnings-related assets are available anywhere (and even if bonds indexed to an index of national earnings were issued, this would not necessarily match the earnings of the employees of any particular company). Some governments (those of the United Kingdom, Australia, Canada, the USA, Sweden, Greece and most recently France) have issued bonds indexed to appropriate consumer price indices, and these provide matching assets for index linked pensions.

Generally self-administered pension funds invest in ordinary shares, since these are considered by many to be a suitable match to salary- and price-related liabilities. All these are related to some extent to price inflation, all being expressed in current currency units. Further, if the economy prospers, company profits and personal earnings may well rise together. However, company profits in aggregate have proved more stable than share prices, which generally fluctuate around some multiple of company earnings or dividends rather than remaining as a fixed multiple of either of these factors.

It should be noted that the currency unit in which the nominal value of a share is expressed is of very little relevance in this context. It is the profits of the company that matter, not the currency in which the shares were issued. An obvious example of this is provided by the two companies Unilever NV and Unilever PLC, one a Dutch company one British; the shares of one are denominated in guilders, the shares of the other in pounds. Yet they jointly own the whole Unilever empire, which does business in a huge number of countries and earns profits in very many currencies, and the companies are of equal economic value. The shares trade at equivalent prices.

Royal Dutch and Shell provide another example. It would be futile to pretend that Unilever NV shares were suitable for nominal guilder liabilities, Unilever PLC shares for nominal pound liabilities, and not vice versa.

So much for the *nature* of the assets.

Matching by *term* means paying attention to the dates that the liabilities are due to be paid.

Banks generally have short term liabilities (withdrawable on demand or with short notice) and so generally lend on the same terms. Life insurance companies generally have much longer term liabilities and invest in longer term bonds. Banking supervision worldwide is now based on the principles of date matching with extra reserves being required to the extent that the assets and liabilities do not match.

One approach to term matching is what is known as "immunisation"; this means arranging the assets and liabilities so that small changes in interest rates affect the values of them in the same way. This has been elaborated by those that trade in derivative securities, who arrange their "books" so that small changes in any of the factors have small effects on the net position. (Mathematically this means considering the partial derivatives with respect to each of the relevant factors.) "Hedging" is another word for immunisation.

Modern asset-liability modelling stems from the original work of Harry Markowitz in the 1950s. There has always been a desire to maximise the return on investments and minimise the risk. Markowitz showed how this could be done mathematically, if return were represented by the expected (or mean) return, risk by the variance (or standard deviation or volatility) of that return. This is mathematically tractable, but with modern computers it is possible to use other definitions of return (such as median), and other definitions of risk (such as semivariance or shortfall).

One cannot at the same time maximise return and minimise risk; one must minimise the risk measure for each given choice of the return measure. The possible combinations of minimum risk for given return, plotted in a risk-return diagram, give a graph of the "efficient frontier" of optimal positions. It is possible to choose sub-optimal points by arranging the assets badly; it is not possible, within the chosen assumptions, to improve on a point on the efficient frontier. It is also possible to find the efficient frontier by maximising "expected utility" with various assumed utility functions.

Let me demonstrate this by a diagram. Here (Figure 1) is a risk-return diagram, with return (measured by the mean or average return) on the horizontal axis, and risk (measured by standard deviation of return, or the likely variability about the average) on the vertical axis. We wish a high return, so we wish to be as far to the right as possible, and we wish a low risk, so we wish to be as low as possible. Thus the desirable direction is to the "south-east" corner of the diagram.

Many books show the diagram with the axes reversed, so that return is on the vertical axis, and risk on the horizontal one. In this case we wish to be as far to the "north-west" as possible. However, Markowitz himself puts the diagram in the way that I do, and there are some advantages in this.

For the examples that I shall show you I have used the mean-variance principle, because it is mathematically tractable, so that I can draw the complete diagram. Real asset-liability modelling studies are much more complicated, deal with real sets of liabilities that emerge over many years, and produce only some of the answers that can be produced if we simplify the model.

I shall assume that we work in units of an index of salaries or earnings. This covers the liabilities of a company that promises defined benefit pensions based on the final salary of the

employees. Such pensions are often indexed thereafter in relation to prices, but I shall ignore that in the example. I simplify by assuming a single time horizon of 20 years, and I assume that we have now 100 units to invest which we shall see are probably enough to meet a liability of about 120 units in 20 years time.

The assets that we have available are shown in this diagram (Figure 2). Each is shown in its position in terms of mean return and standard deviation of return, measured over 20 years, and per 100 units invested now. We see that index-linked bonds are the least risky investment, and would provide a mean return of 130 salary units, with a standard deviation of 20 units. They are not risk-free in salary terms, because they are indexed to a retail price index, not a salary index.

Shares (also called "common stocks" or "equities") give the best mean return, of 260 units, but with a standard deviation of 130 units, so are more risky than index-linked bonds. Conventional government bonds (or government stocks) give the same mean return as index-linked (130) but are more risky, with a standard deviation of 100. Remember that we are measuring in units of a salary index, so bonds expressed in any monetary currency are by no means risk-free.

Commercial bonds are assumed to give a slightly higher mean return than government bonds (160), with a much higher standard deviation (150). The standard deviation is rather high, but I want to show that even a rather risky investment may have its place in a portfolio, even if it is not very attractive by itself. Cash, or short term monetary investments, are assumed to give a poorer mean return than bonds (only 100), with a lower standard deviation (60).

All these numbers are realistic ones taken from the stochastic investment model that I have developed, the details of which are published in a paper of mine in *British Actuarial Journal* (Wilkie, "More on a stochastic asset model for actuarial use", *B.A.J.*, 1, 777-964). I also allow for some correlations between returns, in accordance with the results in that paper. However, my final possible investment is an unrealistic one. I call it just "bad investments". It gives a mean return of only 65, with a high standard deviation of 150, so it has low return, high risk, and is not attractive. However, it helps to spread out my diagrams and makes them easier to see. We also see that bad investments are unattractive to those who do good asset-liability modelling.

If we assume that the distributions of returns for each of these investments is lognormal, then we get the probability density functions shown here (Figure 3). Don't worry if you are not familiar with these. However, you can see that index-linked, which have a low standard deviation, are likely to give a return close to the mean of 130, whereas all the other investments are much more uncertain.

I shall assume in this talk (but see the Appendix) that index-linked bonds are not available, which has been true in most EU countries up to now. Investors could buy UK or US index-linked stocks, but there are then two risks: one that the retail price index in the UK or the US diverges from the consumer price index in the investor's country, the other that the exchange rate fluctuates. These in principle should cancel out, but they may not do so. I also ignore the introduction of the Euro, and the consequences that this will have.

The first portfolio of investments that I consider has no restrictions on it. Any of the investments that I have described (other than index-linked) may be bought or sold in any quantity.

Negative holdings of investments are permitted. This gives us a large area of possible or "feasible" portfolios. Each resulting portfolio can be represented as a point in the risk-return diagram according to its mean return and standard deviation of return, which can be calculated from the means and standard deviations of the investments and the proportions of the portfolio invested in each type of asset. The results are shown here (Figure 4). We find that any portfolio above a certain boundary line is possible, even if not desirable, but that any portfolio below the line is impossible to reach. We cannot get perfect investment portfolios, with both a high return and a low risk.

Desirable portfolios lie along the boundary line. Each portfolio on the boundary has lower risk than the portfolios above it with the same return. However, even along the boundary line, not all portfolios are desirable. Only those to the right-hand side of the minimum point are "efficient".

Portfolios on the left-hand section of the boundary line have lower return for the same risk than the portfolios to their immediate right, and *a fortiori* to those on the right-hand section of the boundary line.

I should say here that I am assuming at present that we know what the means and standard deviations of all the possible assets are, in relation to the liabilities. But in fact many elements are unknown, and we have to do the best we can to make estimates. This means that the true picture is rather fuzzy as compared with the neat diagrams I have drawn, and efficient portfolios may lie in the neighbourhood of the boundary line, rather than exactly along it. So if the actual investments of the relevant institution were near the boundary line we would probably say that they were satisfactory. However, asset-liability modelling does pick out sets of assets that are very unsuitable for the liabilities, so that we can avoid such portfolios.

In this first example, which I call portfolio 0, negative holdings of assets are permitted. This is unrealistic in some respects, but not in other ways. It is quite possible for an investor to borrow cash. It is now possible through the purchase or sale of suitable derivatives to "go short" of shares or government bonds. But since this can lead to portfolios with as high risk as you like (or rather would not like), because any point above the boundary line is feasible, it is sensible for institutions to restrict themselves generally to non-negative holdings of all assets.

This leads to a much smaller possible area, as shown in the next figure (Figure 5). The area outlined in thicker lines is the boundary of all possible portfolios with non-negative holdings of assets. Again, it is the south-eastern boundary of this area that is desirable, and we can see that the boundary (in this particular case) is the same as the unconstrained boundary for part of its length. The spikes at the top of the area show portfolios invested 100% in certain assets. The top left-hand corner is the portfolio consisting all of bad investments (a very bad one to choose). The next spike is the portfolio with all commercial bonds (also an undesirable one), and the top right is the portfolio consisting 100% in shares, which is potentially acceptable to the investor who wishes the best expected return, and is willing to accept the accompanying risk.

We can see what investments each of the portfolios on the lower boundary of the region consist of in this diagram (Figure 6), which shows the fractions (adding up to unity) of each of the possible investments in the portfolios that give the smallest standard deviation for each value of the mean return. Remember that only those portfolios with an expected return bigger than about 120 are desirable, so the left-hand section that includes my bad investments is not sensible. Note,

however how the proportion of shares rises as one goes to the right; how the rest of the portfolio is mostly taken up by cash (i.e. short term investments), and note how commercial bonds and government bonds never take a large part of any portfolio, though commercial bonds are used to a small extent in almost all portfolios from left to right.

Now what happens if we impose a constraint that the investor may not invest more than 20% of his assets in shares. This has been a restriction in some countries for some institutions. The region that is now feasible is shown here (Figure 7) outlined in thicker lines. We see how we are now restricted to the left-hand section of the previous region. The area with the lowest possible risk (where holdings of shares were in any case less than 20%) are not affected, but all the area to the right-hand side is cut off. Yet this rule does not stop a foolish investment manager from choosing a point near the top of the possible region, with high risk and poor return.

Let us now consider instead a different rule, that the investor must invest at least 40% of his assets in government bonds. This again is a not untypical requirement, though the figure may well be 20% or 30% rather than 40%. We see the results here (Figure 8), in what I call portfolio 3, with the now possible region outlined in thicker lines. This pulls down the top limit of the region, so that foolish managers can't do so badly, but it also pushes up the bottom boundary, so that those managers who do follow good asset-liability modelling systems are forced to carry more risk than they need, or would like.

The point here is that an fixed money bonds, even those issued by governments, are not a good match to liabilities expressed in salary terms, or in consumer price terms. This is quite different from the situation of life insurance companies that offer policies with sums assured expressed in local currency terms, for whom bonds would turn out to be a fairly safe investment. Therefore the requirement to invest a fraction in government bonds is not too severe a requirement for them (even though such a requirement for insurance companies has now disappeared because of the Third Life Directive).

What happens if we impose both requirements? Clearly we are restricted to the area which is covered by both the previous possible regions, and this is outlined in thicker lines here (Figure 9). As compared with the original position with no restrictions, we are prevented from obtaining a high return, even if we are prepared to take the risk. The foolish manger is prevented from going too far to the top or to the left of the original region, but the sensible manager is obliged to adopt a more risky portfolio than he needs to without the restrictions. He could, for example, look at the minimum standard deviation position in the outlined area, at about 130 expected and 62 standard deviation, and move to the right to the position on the thinner line with the same standard deviation, and a return of 165; the same risk, but a better expected return.

Once the efficient frontier has been found, within whatever constraints are imposed, there is still the question of choosing an appropriate portfolio of assets from among the many choices on the efficient frontier. At one extreme is the minimum risk portfolio (in some cases this may involve choosing assets that exactly match the liabilities so that risk is reduced to zero, though this does not happen in my examples), but usually this minimum risk portfolio provides a low return. At the other extreme is the maximum return portfolio, at the extreme top right-hand corner, which may be uncomfortably risky for some investors.

The choice of asset portfolio from among the efficient set of portfolios has to be determined according to some criterion that reflects the attitude to risk of the investor. This in turn may involve consideration of the present surplus of assets over liabilities (a higher surplus would usually allow a more risky position to be taken) and also on what might happen if there were any shortfall of assets as compared with the liabilities. In this respect a self-administered company pension fund is in a special position as compared with "free-standing" financial institutions (such as commercial banks and insurance companies or "open pension funds", that is open to all employers or employees who wish to join), since a shortfall can be made up by the sponsoring employer of such a self-administered scheme (unless the company itself is in financial difficulties).

The structure of what may be called "pension funds" is quite different in different countries. This is something that Mr van Rees, Chairman of the EFRP, may like to explain.

The efficient frontier also depends on what assets are available. I omitted index-linked bonds from the analysis. If we allow them to be included, we get different results, but I shall not go into that now. (See Appendix)

Pension schemes within the EU are arranged in so many different ways that no single format for asset-liability supervision would be suitable for all arrangements. At one extreme are personal pension arrangements, insured by a life insurance company and providing the beneficiary exactly with the benefits of the insurance policy; in this case it is the insurer that is supervised. At the other extreme are schemes, such as those for government employees in many countries, where the employer promises pensions but pays them out of current revenues, no funds are accumulated and there are no assets. In some countries (especially in Germany) the pension liabilities may be treated as liabilities of the company ("book reserves") and the assets are the assets of the company (buildings, machinery, stocks, debtors, etc) usually together with an insolvency insurance arrangement.

Large self-administered funds in those countries where they exist (typically in the UK, Netherlands and Ireland) have generally carried out asset-liability modelling exercises, and use them to establish "benchmarks", which give limits within which the investment managers operate. If the benchmark is stated as, for example, 60:20:20 in certain assets, the investment manager may be allowed to work within limits of 50-70: 10-25: 10-25.

In several countries there is a requirement for any shortfall in the assets of such a fund to be made up within a reasonably short time.

As I said earlier, real asset-liability modelling studies are very much more complicated than what I have shown you. But they all have the same principles: there is some criterion for return (the higher the better), and some criterion for risk (the lower the better); there is a choice of efficient portfolios, from the lowest risk one (which also has the lowest return of any efficient portfolio) to the highest return one (which also has the highest risk of any efficient portfolio). Where one chooses to go along that range depends on circumstances, like the available surplus, like the nature of the liabilities (for example whether pension increases are discretionary so that they can be made to depend on whether there have been good investment returns or not) and also like what happens if things turn out badly (whether the sponsoring employer just has to put in a higher contribution, and whether he can afford to). The result of any such exercise is then a set of

proportions in the asset categories that have been considered (which may be many more than I have talked about, including shares in many countries and bonds in many currencies), which then can form a benchmark for the investment manager.

Restrictions on what investments can be chosen, as I have shown for specific cases, always move the responsible investor away from his choice of possible efficient portfolios (unless they are quite ineffective). They may also prevent the foolish or irresponsible investor from making too big a fool of himself, and I suppose that that has been their justification in the past. But I suggest that with modern asset-liability modelling techniques they are no longer necessary even for that purpose.

There is another purpose of a requirement that certain institutions should invest a certain proportion in government bonds: that is to make it easier for the government to borrow, and thereby perhaps reducing the interest rate that the government would otherwise need to pay. This is clearly not in the best interest of the investors. Further, with the introduction of the Euro, it will hardly even be effective. Interest rates will be determined by the whole Euro market, not by the actions of any one government; and it is hardly in the spirit of the EU for a requirement that investors should invest in the bonds of one particular government rather than in the bonds in the same currency, the Euro, issued by the governments of any other member state. So such requirements are likely in future to be ineffective either in helping governments, or in providing security to the beneficiaries of institutional investors.

Appendix: paragraphs and Figures drafted but not used

If we allow index-linked bonds to be included, what results do we get? Here (Figure 10) is the result. This time I have excluded the bad investments category, so now the worst return we can get is by putting all the assets into cash. The risk that we *must* take is very much reduced as compared with the previous examples. The restrictions do much the same as before in terms of the shape of the resulting areas.

Here (Figure 11) are the proportions invested in various categories of asset for each of the lowest risk portfolios for all the possible mean returns, from the low of 100 to the maximum of 260. Note that any return below 128 is for an inefficient portfolio. We see that in practice the only two significant investments are shares and index-linked bonds, and how much one chooses of each depends on one's attitude to risk. The pension fund of a large, stable and prosperous company can readily afford to be invested heavily in shares. The pension fund of a company that has reduced in size compared with its pensioner population (and there are several of these), or which no longer has a strong company behind it (and there are these too), should invest substantially in index-linked, if it provides index-linked pensions. For some companies this is precisely what has happened, as a result of asset-liability modelling studies.

Figure 1

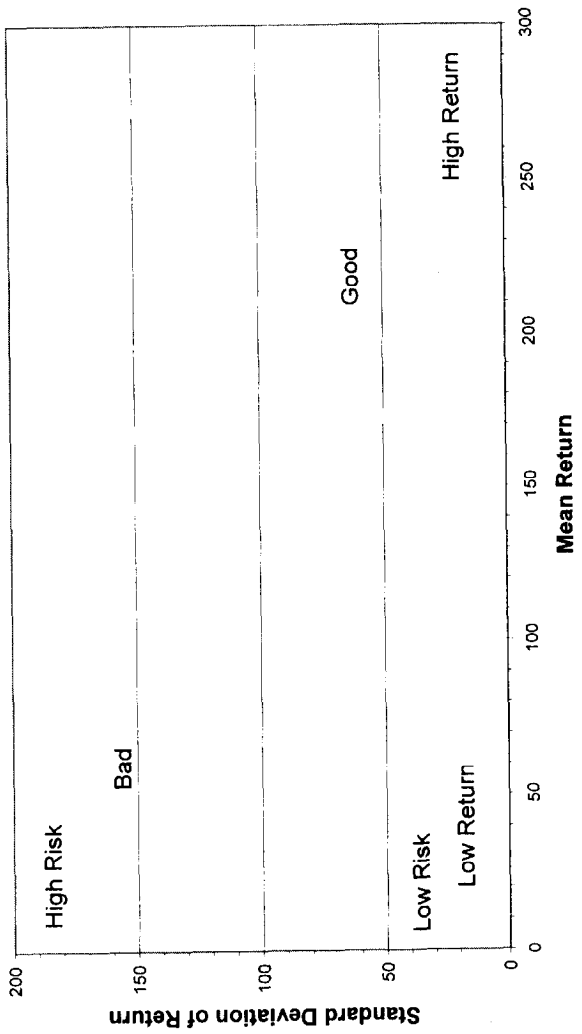


Figure 2

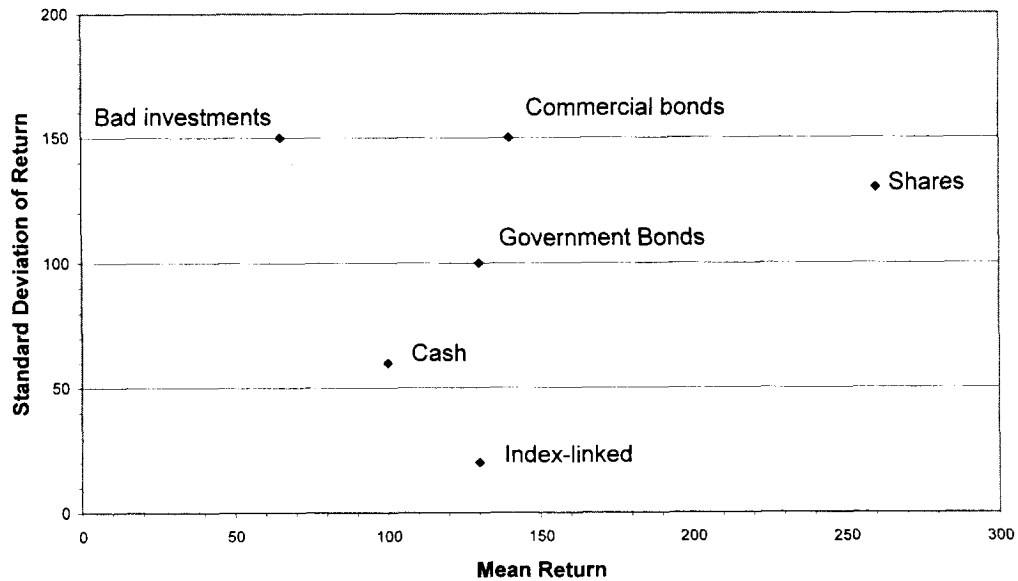


Figure 3

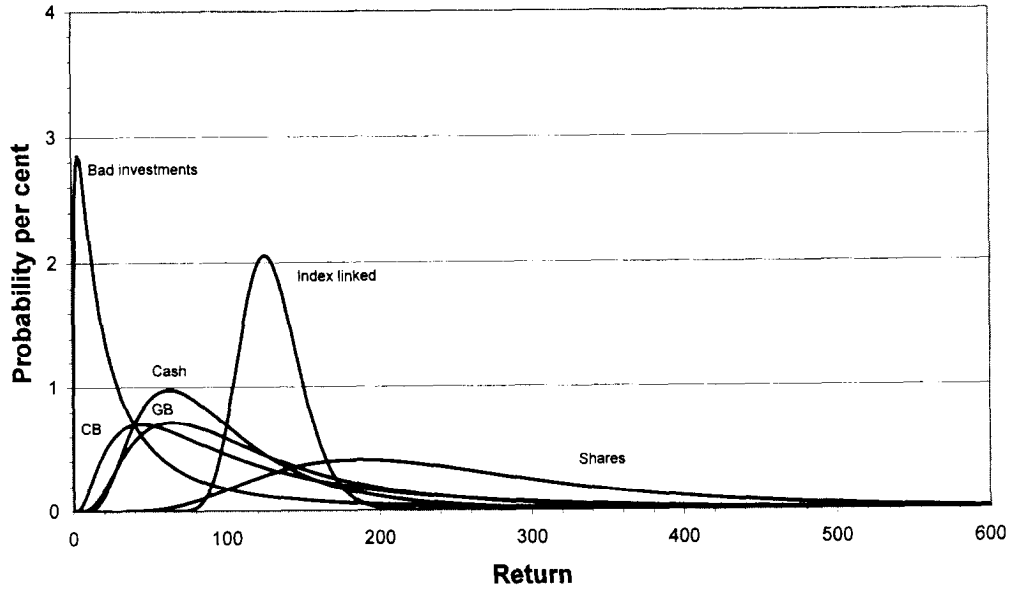


Figure 4: portfolio 0: unrestricted

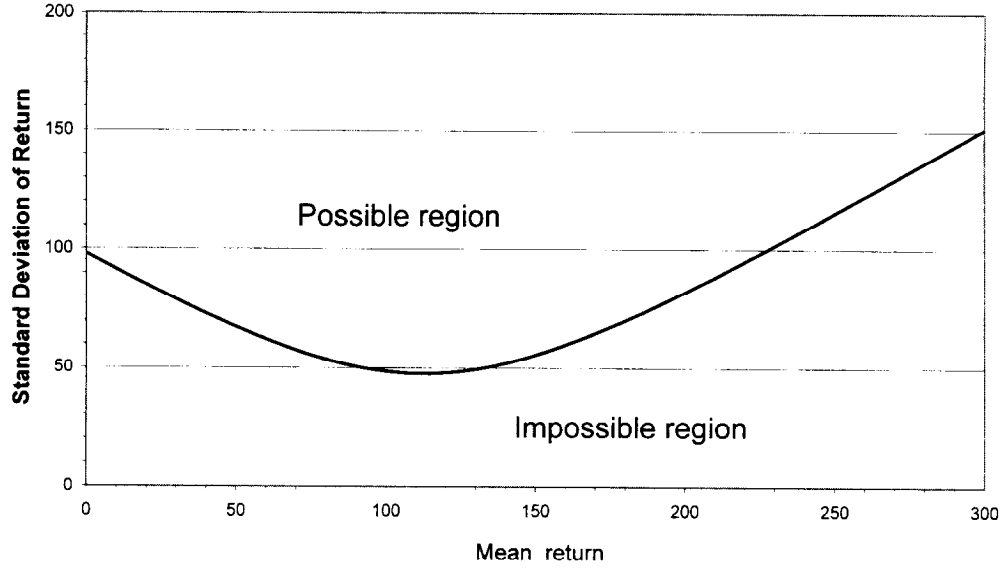
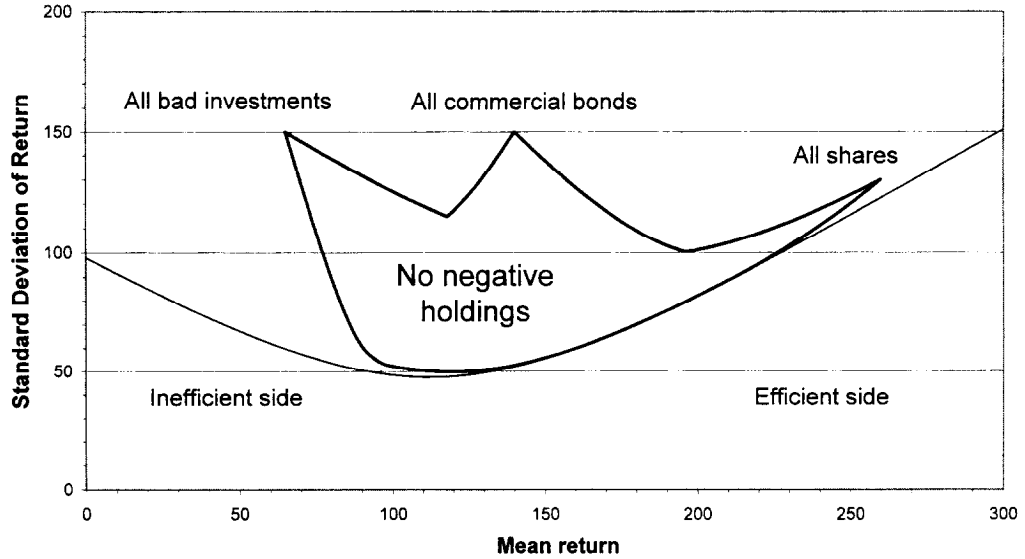


Figure 5: portfolio 1: no negative holdings



**Figure 6: fractions invested for minimum variance portfolios:
non-negative holdings and no other restrictions (portfolio 1)**

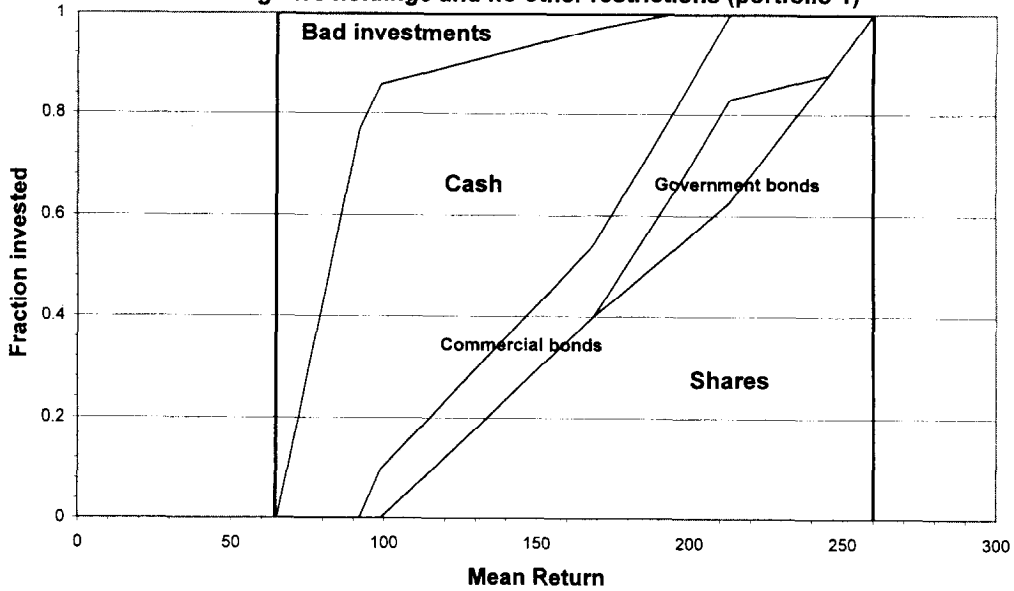


Figure 7: portfolio 2: shares not more than 20%

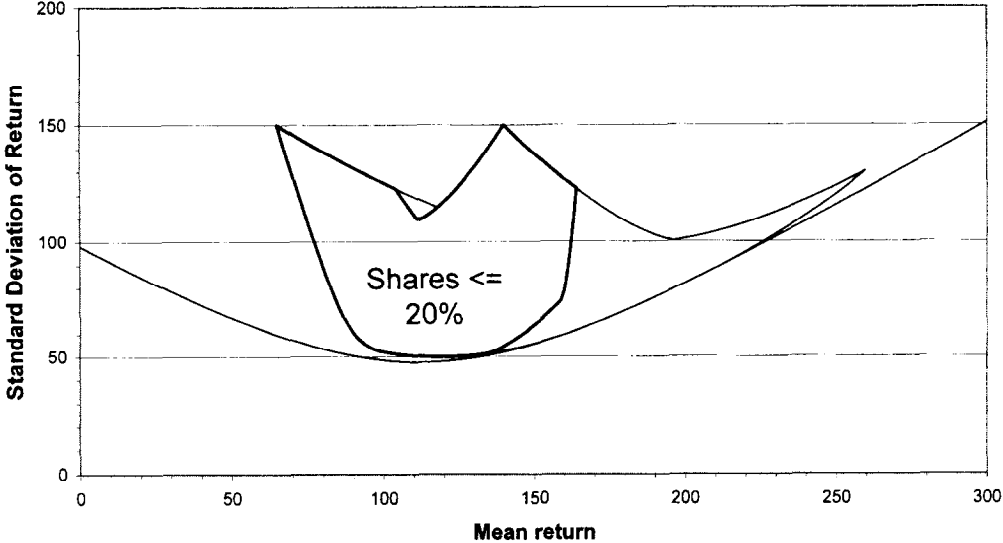


Figure 8: portfolio 3: government bonds at least 40%

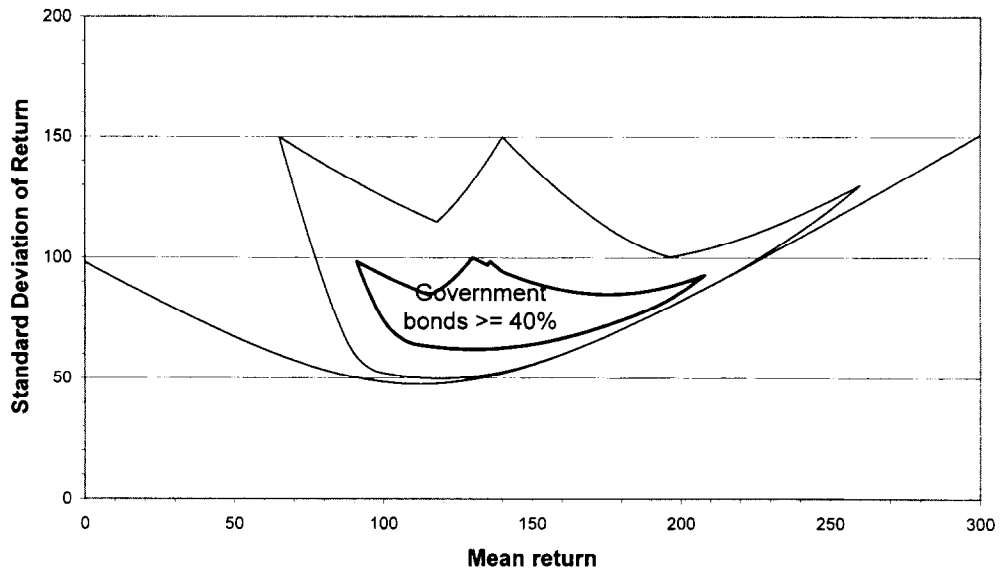


Figure 9: portfolio 4: both restrictions

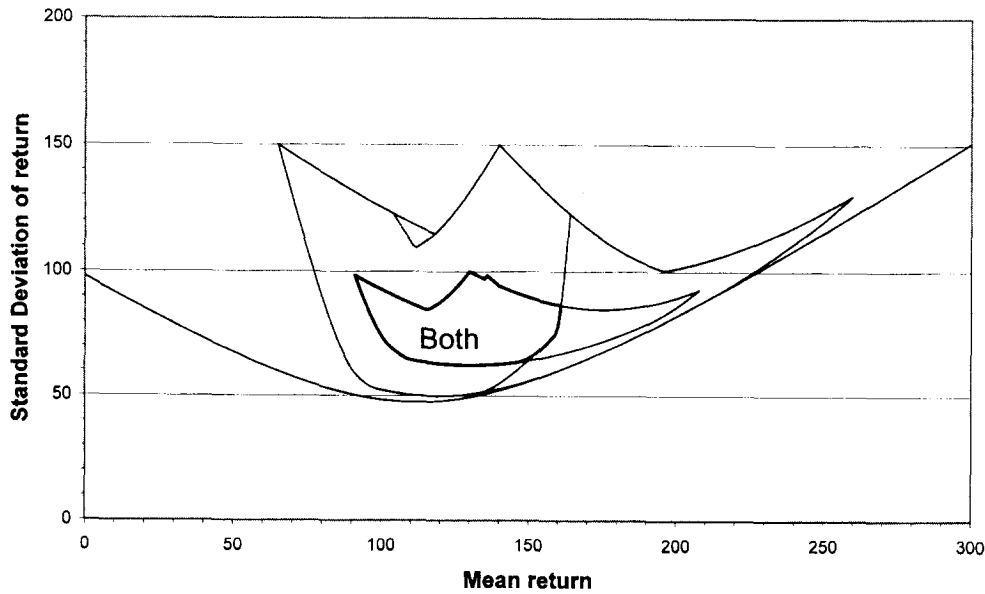


Figure 10: portfolio 5: with index linked, non-negative holdings

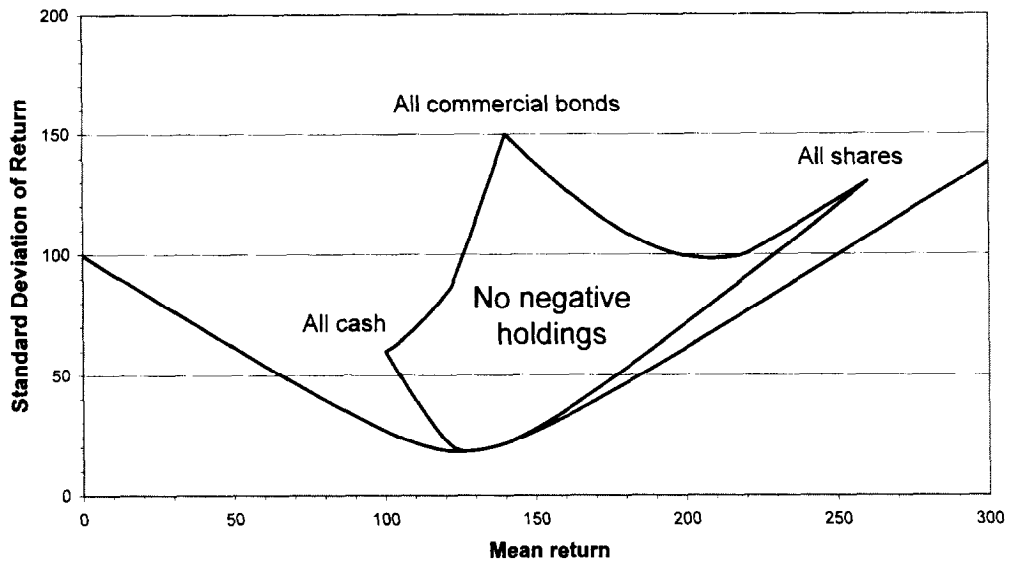


Figure 11: fractions invested for minimum variance portfolios: non-negative holdings (portfolio 5), with index-linked bonds

