A Feasibility Study of the Optimal Asset Mix for Japanese Life Insurer's General Account

Naoki Matsuyama
Planning and Research Dept., Meiji Life Ins.Co.
1-1,Marunouchi 2chome, Chiyodaku,Tokyo,100-0005Japan
tel: (03)3283-8959, e-mail: t14605@meiji-life.co.jp
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Abstract

Japanese life insurers are often requested by their corporate pension customers to disclose their baseline portfolios for general accounts. In current low yield and negative spread situation for life insurers, baseline portfolio strategies would be, if possible, different from immunized portfolio and result in more risky asset mix.

As a feasibility study of the optimal asset mix for general account, we refer to implications of basic financial theory, Merton's problem and option theory, numerical techniques for financial modeling and its fitness to algorithm for dynamic optimization.

Keywords : ALM, Baseline portfolio, Dynamic programming, Option theory, Utility theory

1. Requested baseline portfolio

It is popular for Japanese life insurers to set the static baseline portfolio in their investment strategy for their separate pension accounts and even disclose to their customers what their baseline portfolio is. Recently, corporate pension customers often request life insures to disclose their general account's baseline portfolio, too. Although the risk-return profile of general account is difficult to estimate only from its asset allocation because it has guaranteed interest rate, it is plausible for the customers to try to analyze general account products based on the same framework used for analysis of other asset management products, such as separate account.

Besides the customers' requests, it seems an attractive idea to portfolio managers of general account to allocate their asset to form the static baseline portfolio because the amount of their assets are comparatively large and it is difficult to change their asset allocation quickly.

It might be needless to say from the viewpoint of ALM, basic investment strategy for fixed interest liability should be immunization by fixed income assets and investment to risky assets must be limited within the amount of company's capital, but the actual asset allocations of Japanese major life insurers are far from such an ideal immunized portfolio, as shown below.

Proportions of risky assets and surplus to general account asset

(7 major life insurers in Japan at 97 fiscal year-end)

Risky assets(stocks and foreign securities)	Surplus + Reserves for asset value fluctuation
average 25.4%	average 1.8%
(20.7% to 29.7%)	(1.3% to 2.3%)

*figures on accounting basis

Cutting down of guaranteed interest rates lagged behind the current market interest rates drop, immunization strategy has remained of limited use. Resulting negative spread becomes a severe problem after all. Then, the requested static baseline portfolio for general account under current situation would not be an immunized portfolio, but, if any, might be a portfolio that allow insurers to invest in more risky asset. It should be derived as a solution to the problem of multi-period optimal asset mix which could earn enough for guaranteed interest mainly by its capital gain. We will examine feasibility of such a baseline portfolio for general account.

2. Implications of simplified Merton's problem

Generally, to solve optimal asset mix problems, we have to specify return distributions of assets and a utility function. And then we can get the solution by maximizing the expected utility under expected utility theory. The mean-variance (MV) models which have been well-known in asset management business, are simple tools consistent with the theory of expected utility if the assumed utility function is quadratic or the forecasted distribution of asset return conforms to (multivariate) normal distribution.

Merton (1971) has got an closed-form solution for the problem of optimal asset mix and consumption that maximizes the utility of consumption during a period and the wealth (W) at period-end under certain assumptions. Here we concentrate on the simple asset allocation problem assuming that there are no income and no consumption and overview implications of Merton's problem for considering baseline portfolio.

We simplify the problem as allocation between two assets, risk free asset and risky asset, suppose that interest rate (r) for risk free asset is constant and price of risky asset (P) follows Geometric Brownian Motion :

$dP = \mu P dt + \sigma P dz$. ($\mu, \sigma$: constant, z: standard brownian motion)

And we assume the most popular HARA (Hyperbolic Absolute Risk Aversion) family as utility function (U(W)).

Then the problem is to find the solution of asset allocation which maximizes period-end expected utility $E[U(W_T)]$.

Technically, to solve this problem, it is necessary to use dynamic programming technique and get a solution of Bellman equation .(see Appendix)

2.1. An optimal asset allocation without Floor

We assume CRRA (Constant Relative Risk Aversion) utility function among HARA family , which is often used for portfolio analysis.

$U(W) = W^{1-\gamma}/(1-\gamma)$ if $\gamma > 0, \gamma \neq 1$

$\log W$ if $\gamma = 1$.

Then, the optimal allocation of risky asset at time(t) is decided as

$\{(\mu - r)/(\gamma \sigma^2)\}Wt$

In this case, the optimal proportion invested in risky asset becomes constant, then we can think this result support the static baseline portfolio strategy. But this utility function has no floor such as general account, then we should regard this case as a supporting evidence for using static baseline portfolio strategy in asset management business such as separate account.

2.2. An optimal asset allocation with Floor

In general account investment, different from general arguments in asset management, we have to add conditions of liability constraint into utility functions. As the most simple model for liability constraint, we assume constant floor($\underline{W}$) in utility function. We assume the following utility function as the most simple utility function with floor($\underline{W}$) among HARA family.

$U(W) = (W - \underline{W})^{1-\gamma}/(1-\gamma)$ if $\gamma > 0, \gamma \neq 1$

$\log(W - \underline{W})$ if $\gamma = 1$.

Then, the optimal allocation of risky asset at time(t) is decided as

$\{(\mu - r)/(\gamma\sigma^2)\}(W_t - \underline{W})$

Even in this simplified case, the optimal allocation depends on cushion (i.e. asset value(W) - floor($\underline{W}$)), then it becomes dynamic and is conceptually different from static baseline portfolio strategy.

Even if we allow dynamic asset allocation strategy , the variability of allocation should be realistic for life insurers' general account. If we would assume such an optimal asset allocation strategy that verify actual current asset allocation of major Japanese life insurers who have considerable amount of risky assets with their current limited capital, according to the above model, derived tactics might become considerably dynamic with high sensitivity to levels of cushion.

As an implication of Merton's problem, the optimal asset allocation strategy for general account should be solved in dynamic framework. And moreover, to solve actual asset allocation problems, we should consider actual constraint condition such as allowance for variability of asset allocation and levels of risk buffer or etc.

3. Implications of an option model for general account

It is practically difficult to specify the utility function which has above characteristics. We will examine a simple option model for general account in which we do not have to specify a utility function.

To guarantee a return on the portfolio including risky asset, life insurers need a put option, But they do not charge its premium explicitly on their customer. In this case, we can think, this premium is charged off implicitly by retaining a part of each year profit and reserving it. To discuss this interest rate guarantee mechanism, we can suppose a model for general account using a one year European option according to accounting period.

Here we assume that, as well as section 2, the portfolio consists of two assets, risk free asset and risky asset. Risk free interest rate (r) and volatility of risky asset (σ) are constant. And the proportion invested in risky asset (m) is variable. Then the volatility of portfolio is expressed by ($m\sigma$).

The value of interest guarantee can be represented as a put option : $P(i, m\sigma)$, where its strike price equals guaranteed interest rate(i), and the value of retaining each year profit can be evaluated by a call option : $C(i, m\sigma)$.

Here, below relation should exist between a put option which covers 100% of assets and a call option which covers R (0 to 100%) of assets where R is retaining proportion to profit:

$P(i, m\sigma) \leqq R \cdot C(i, m\sigma) \leqq C(i, m\sigma)$.

On the other hand, there stands so called put call parity:

$C(i, m\sigma) = P(i, m\sigma) + 1 - e^{(i-r)(T-t)}$.

Then, $e^{(i-r)(T-t)} \leqq 1$,

i.e. (guaranteed interest rate(i) $\leqq$ risk free interest rate(r))

This implies that even if we can assume constant volatility, depending on relation between guaranteed rate and risk free rate, this option model can not work, that is, no solution of asset allocation can be found.

Actually in Japan, guaranteed interest rates have been higher than one year risk free rate(12m LIBOR) for 5 years or more, and this option model have not provided any solutions for asset allocation problems.

FY	LIBOR 12m (average)	guaranteed interest rate for new business (individual par ins.)
92	3.89%	5.5%
93	2.67%	4.75%
94	2.59%	3.75%
95	0.87%	3.75%
96	0.79%	2.75%
97	0.72%	2.75%

Implication of this option model is that the current investment problem of general account should not be solved only by targeting guaranteed interest rate, because such investment solutions are too risky to sustain. To examine asset allocation, we should consider not only risk buffer but also future profit margin occurring from liability side, mortality or expense margin which decreases actual level of guaranteed interest rate. Then, to solve the problem, it is necessary to use liability models to simulate future profit margin and cash flows, i.e. full-scale ALM models.

4. Optimization of asset mix on ALM models

As we have seen above, optimal asset allocation problems of general account should be solved in dynamic framework on full-scale ALM models. We will consider of numerical characteristics of life insurer's ALM models and algorithms of optimization for asset allocation.

4.1. Risk measures in life insurer's ALM models

General requirements for ALM models are following:

1) cash flow simulation of asset and liability based on long term stochastic scenarios

2) probability distributions of values of assets, liabilities and their difference (=surplus)

Among various measures in ALM models, the most remarkable is surplus. The most natural approach for solving optimal asset allocation problems is to maximize the expected utility of surplus.

Then, even if we could assume normality of probability distribution of asset value, probability distribution of surplus would become asymmetric, because the value of liability includes embedded option of cash value guarantee. So, we should be careful about risk measures of ALM models.

4.1.1. MV approach

Under such conditions that we can not assume normality of probability distribution, the sufficient condition for MV model to match expected utility theory is that utility function should be quadratic.

A utility function related to standard deviation : $U(W) = W - aW^2 / 2$

Although quadratic utility functions have preferable characteristics locally, it might have following global difficulties:

- utility decreases if W increases over certain number $(1 / a)$
- absolute risk aversion $(a / (1 - aW))$ becomes increasing function of W

Here, the latter might not be general characteristics of utility function because absolute risk aversion of normal utility function is supposed to be decreasing function of W.

4.1.2.Other 2 parameters approach

As substitutional methods for MV model, we can think following without assuming normality of distribution:

- Directly maximizing of the specified expected utility function
- Stochastic Dominance Rule
- 3 parameters model with skewness as third parameter.
- 2 parameters model with downside risk measures

Here, from the viewpoint of practical simplicity as MV models have, we examine 2 parameters models which have expected return rate and downside risk as measures.

In ALM models, the most important risk measure should be the loss probability or expected loss of surplus, we consider methods of selecting portfolio based on these downside risk measures and expected return rate (E[R]) of surplus.

1) Portfolio selection based on loss probability and expected return

In this case, expected utility is expressed as

$E[U(R)] = F(E[R], Prob(0>R))$.

$Prob(0>R) = E[g(R)]$ where $g(R) = 0$, if $R > 0$

$= 1$, if $R \leqq 0$.

Here, U(R) should be linear expression of R and g(R), because of linearity of expectation.

Then, utility function including loss probability is expressed as

$U(R) = a \cdot R - b \cdot g(R)$ where $g(R) = 0$, if $R > 0$

$= 1$, if $R \leqq 0$.

This function is invalid as utility function because it consists of parallel two lines.

Generally, line segment utility functions do not have proper characteristics in portfolio selection, as Markowitz(1959) pointed out the possibility that these functions could lead to speculative investment strategy.

2) Portfolio selection based on expected loss and expected return

In this case, expected utility is expressed as

$E[U(R)] = F(E[R], E[\min(R, 0)])$.

Here, U(R) should be linear expression of R and min(R, 0), because of linearity of expectation.

Then, utility function including loss probability is expressed as

$U(R) = a \cdot R - b \cdot \min(R, 0) + c$

Although this utility function consists of line segments as well as 1), it also has the shape of concave function and can be a candidate for risk measure.

3) Portfolio selection based on LPM and expected return

Markowitz, on the other hand, offered Lower Semi-Variance as the most theoretically robust risk measure. Therefore, LPM (Lower Partial Moment), extended concept of Lower Semi-Variance, can be also a strong candidate for risk measure.

$LPM(K) = \int_{-\infty}^{\Theta} (\Theta - x)^k f(x)dx$.

Here we can express above 1) as LPM(0), and 2) as LPM(1).

Generally LPM results in problems of non-linear programming and at most LPM(2), which is attributed to a quadratic programming problem, can be treated with so much computational effort as in usual MV models. But most practical use of LPM models are one period model and practical use of dynamic multi-period models of LPM is future problem to be solved.

4.2. ALM models and algorithms of optimization

Next, to treat more realistic models of asset price variation and complicated cash flow caused by insurance liability, we will examine numerical techniques for the case that we can not reach closed-form solutions.

Major numerical techniques for financial models are :

1) Monte Carlo simulation

2) Lattice method

3) Finite difference method

Monte Carlo simulation is the method which generates time series sample paths by using random number series. Although this method can be applicable to models with complicated cash flow, it is not appropriate for backward induction which is the algorithm of dynamic programming used for dynamic optimal asset allocation.

On the contrary, lattice method and finite difference method are consistent with algorithms of backward induction in dynamic programming, but it is difficult to apply them to the models which have complicated cash flow because we have to either approximate probability distributions by binomial distribution or express models as differential equation to use them.

Actuaries usually want to use Monte Carlo simulation in their ALM models, because they want to express various asset return and complicated cash flow from liability. Generally speaking, these models, often called DFA (Dynamic Financial Analysis), are used to analyze "What If Test" and they can only tell which static asset mix strategy is better than the other. Static optimization can be easily done on Monte Carlo simulation model of ALM, if we don't mind taking computational time, but not dynamic.

To solve problems of optimization of dynamic asset allocation , we should select lattice method or finite differences method. But it is necessary for using them to simplify models enough for completing computation and it is doubtful that these simplified models have enough validity for life insurer's ALM.

5. A prospect

Even if we could abandon the illusion of static baseline portfolio and overcome technical problems of dynamic strategy for asset allocation, the most difficult problem to decide long term strategy of asset allocation for general account, is to forecast expected return. To forecast expected return, we have to assume long term stability of risk premium as in building block methods. Since 1998, Japanese market has been extraordinary situation that short term interest rate has been led down to 0%. It is difficult to think that currently observed return structure of each asset would continue to be stable for the future.

Especially, from now on, we have to pay attention to cash outflows which would occur when interest rate rises. It is obvious that under current situation we can not operate general account by baseline portfolio like as auto-pilot apparatus.

5.1. A future image of asset allocation of general account

Apart from tactics under present situation, it may be of some use to imagine, a future image of asset allocation under such market yield condition that, for example, the option model mentioned in above section 3 could work normally.

If we should simply regard forward rates derived from yield curve at March31,1999, as market participant's forecast of interest rate, then, six years after, short term interest rate would go up over the level of guaranteed interest rate for 97 fiscal year new business, and go up to 3.4% within 10 years.

On the other hand, Japanese mutual life companies are required to distribute as dividends more than 80% of their each year profit by article 29 of Insurance Business Law Enforcement Regulation. Then, if we suppose the option model of above section 3 and market value accounting, the upper limit of proportion invested in risky asset (m) would be decided by portfolio volatility ($m\sigma$) which is consistent with the equation :

$$P(2.75\%, m\sigma) = 0.2 \cdot C(2.75\%, m\sigma).$$

Here we assume Black-Scholes formula and risky asset volatility as 20%, then we can derive such result that the proportion invested in the risky asset should be no more than 5.5% within 10 years ahead.

. Upper limit of the proportion invested in risky asset

years later	forward rate (1year)	proportion invested in risky asset(m)
6	2.885%	1.06%
7	3.180%	3.38%
8	3.215%	3.66%
9	3.453%	5.53%
10	3.407%	5.16%

Even if we would not select immunization strategy, we can see that the proportion invested in the risky asset should be fairly limited in general account.

Appendix

[A brief explanation of solution of 2.2.]

We assume the utility function as

$$U(W) = (W - \underline{W})^{1-\gamma}/(1 - \gamma)$$

The process which generates W can be written as

$$dW = (u\mu + v\, r)\, W\, dt + u\, \sigma W\, dz, \text{ where } u + v = 1$$

Here, we define the programming value J and Φ as follows

$$J(t,W) = \max_u (E_t[U(W_T)])$$

$$\Phi(u\, ;\, t,W) = \Delta[J] = Jt + (u\mu + v\, r)\, W\, Jw + (1/2)u^2\sigma^2 W^2\, Jww \quad (1)$$

where the notation for partial derivatives is $Jt=\partial J/\partial t$, $Jw=\partial J/\partial w$, $Jww=\partial^2 J/\partial w^2$

From the principle of the optimization in dynamic programming,

$$0 = \Phi(u^*; t,W) \geqq \Phi(u\, ;\, t,W)$$

$$\therefore \quad 0 = \max \{\Phi(u\, ;\, t,W)\}$$

where u^* is the optimal proportion of wealth invested in risky asset at time (t).

Then,

$$0 = \partial(\Phi + \lambda(1-u-v))/\partial u = -\lambda + \mu\, W\, Jw + u^*\sigma^2 W^2\, Jww \quad (2)$$

$$0 = \partial(\Phi + \lambda(1-u-v))/\partial v = -\lambda + r\, W\, Jw \quad (3)$$

From (2)(3),

$$u^* = -(\mu - r)\, Jw / (W\, Jww\, \sigma^2) \quad (4)$$

From (1)(4),

$$0 = \Phi = Jt + r\, W\, Jw - (\mu - r)^2\, Jw^2 / (2\sigma^2 Jww) \quad (5)$$

Assume trial solution of the form(5) as

$$J(t,W) = y(t)\, U(W), \text{ with the period end condition : } y(T)=1.$$

Finally, we can find the solution of the form (5) and $y(t) = \exp(K(T-t))$

$$Jt = (W - \underline{W})^{1-\gamma} y'/(1 - \gamma)\ ,\ Jw = (W - \underline{W})^{-\gamma} y\ ,\ Jww = (-\gamma)(W - \underline{W})^{-\gamma-1}\, y \quad (6)$$

From (4)(6), u^* the optimal proportion invested in risky asset at time (t) can be rewritten as

$$u^*(t) = \{(\mu - r)/(\gamma\, \sigma^2)\}(Wt - \underline{W})/Wt$$

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