Cost of Capital for equity Holders and related credit risk premium (The empirical test of the Leland model)

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Abstract

The purpose of this research is to understand the relation among the cost of capital, the cost of equity, the cost of debt, and the credit risk premium by using Leland model (1996). And empirical test is performed about how a bankruptcy risk premium should be shown in the actual Japanese market. It is assumed that there are no taxes throughout this paper. The firm value, the equity value and the debt value can be computed in consideration of the bankruptcy from the Leland model. To calculate a bankruptcy risk premium, it is necessary to formulate the relation between the cost and the value. In this research, it is also assumed that the stochastic process of firm value can be derived from the stochastic process of the cash flow, and a cash flow discount model is used to evaluate the firm value.

By this formulation, the bankruptcy risk premiums are calculated as the spreads between the costs when there is no bankruptcy and those when there are bankruptcies. In the Japanese market, when debt ratio (debt /asset) exceeds 70%, the bankruptcy risk premium rises rapidly under certain assumptions. With the debt ratio of 80%, the cost of equity is about 20% and the bankruptcy risk premium within the cost of equity is about 10%. Our empirical result matches our experience.

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Introduction

Theoretical research about the firm value derived from the option theory is gaining at tension recently. The first purpose of this research is to construct a stochastic process of firm value from the stochastic process of the free cash flow. And the second purpose is to understand the relation among the cost of capital, the cost of equity, the cost of debt, and the credit risk premium using Leland model ([Optimal Capital Structure Endogenous Bankruptcy, and the Term Structure of Credit Spreads] ,The Journal of Finance ,JULY1996) . In addition, applying the theory to the actual Japanese market would test the usefulness of the formulation. This paper assumes an ideal world where there are no taxes. The problem of taxes would be thought as the subject in the future because the influence of taxes is minor, although existent, compared to the effect of bankruptcy.

1. The firm value model in which the bankruptcy is not considered

Modigliani and Miller (1958) showed that in a perfect market, the firm value would be as follows.

(MM's proposition I in the absence of taxes)

: The firm value is not affected by its choice of capital structure. In other words, financial leverage has no effect on shareholders.

(MM's proposition II in the absence of taxes)

: The expected rate of return on the common stock of a levered firm increases in proportion to the debt-equity ratio, expressed in market values; the rate of increase depends on the spread between the expected rate of return on the equity and the expected return on the debt.

The firm value theory of MM asserts that the future profitability and the business risk decide the firm value if there were no taxes or bankruptcy. This is quite understandable if we think of a single person providing all the necessary funds of the firm as the stockholder and the corporate bondholder. Even if the composition of the stock and the bond investment is devised, investment profits in total can't be increased when bankruptcy and taxes are not in. But, as for the actual firm activities, bankruptcy is an important problem. There are researches such as Leland (1996) which addresses this

problem explicitly.

2. The traditional relation between the free cash flow discount model and the discount rate

At first, traditional free cash flow discount model, which is foundation of the MM theory, should be considered. The free cash flow discount model calculates the value of the firm by using the total present value of the stream of the future cash flows of the firm. At this stage, the free cash flow is not stochastic but deterministic.

(notation)

V_t: The firm value at t when the possibility of bankruptcy is not con,

$EBIT_t$: Earnings before interest and taxes at t

FCF_t: Free cash flow to the firm at t. The amount of depreciation is assumed to be equal to the amount of investment, and the increase and decrease of the necessary working capital would be zero. No taxes are assumed. Therefore, EBITt=FCFt.

w_t : The cost of capital. The firm value at time t is discounted by this cost. The cost of capital doesn't depend on the structure of finance, and it is fixed.

r_e: The expected return on equity

r_d: The expected return on debt

V_e: The equity value

V_d: The debt value

The firm value can be shown as the following equation, in which the future free cash flow is separated into the cash flow for the stockholder and for the creditor.

$$V = \sum_{t=1}^{\infty} \frac{FCF_t}{(1+w)^t} = \sum_{t=1}^{\infty} \frac{EBIT_t - V_d r_d}{(1+r_e)^t} + \sum_{t=1}^{\infty} \frac{V_d r_d}{(1+r_d)^t}$$

If we fix w, FCF, and EBIT we get the simple constant free cash flow model.

$$V = \frac{FCF}{w} = \frac{EBIT}{w}$$

$$V_e = \frac{EBIT - V_d r_d}{r_e}$$

$$V = V_d + V_e$$

The above three equations give us r_e. The next equation is related to MM's proposition Ⅱ.

$$r_e = w + (w - r_d)\frac{V_d}{V_e}$$

Rearranging the above equation for w, we get

$$w = \frac{V_e}{V} r_e + \frac{V_d}{V} r_d$$

This equation is not a necessarily natural equation. Generally this expression of weighted average is called the weighted average cost of capital (WACC). In finance theory point of view, w should be fixed first. The result of r_e ,r_d , V_e and V_d are decided by the struggle for profits between equity holder and debt holder.

3.Leland Model about the firm value to take bankruptcy into consideration

So far, the risk of bankruptcy has been ignored in evaluating the equity value.
Due to Leland model, the firm value with bankruptcy is thought as the value of the derivatives of the underlying asset. The underlying asset is the firm value without bankruptcy. On this assumption, Leland expressed the debt value and equity value consistently.
The outline of the Leland model is as follows.
(Notation)

T : Maturity year

V_t : Firm value at t without considering of bankruptcy

v_t: Firm value at t by the Leland model considering bankruptcy

P: Total outstanding debt principal

D_t: Debt value by the Leland model considering bankruptcy

E_t: Equity value without considering bankruptcy

e_t: Equity value by the Leland model considering of bankruptcy

r: Risk free interest rate

w: The cost of capital of the firm without considering bankruptcy

r_e: The coat of equity without considering bankruptcy

s_w: The default risk premium of the cost of capital

s_e: The default risk premium of the cost of equity

s_r: The default risk premium of the cost of debt

C: Annual coupon by the risk free interest

μ : Expected rate of return of EBIT and firm value without considering bankruptcy

σ : Expected volatility of EBIT and firm value without considering bankruptcy

δ : The constant fraction of value paid out to security holders

d Z: The increment of a standard Brownian motion

As in Merton(1974),Black and Cox(1976),and Brennan and Schwartz(1978), it is assumed that a firm has productive assets whose unleveraged value V follows a continuous diffusion process with constant proportional volatility σ.

$$\frac{dV}{V} = (\mu - \delta) \cdot dt + \sigma \cdot dZ$$

(Assumptions about bankruptcy)

: A firm is bankrupt at the moment when the firm value V without considering bankruptcy became equal to or less than VB.

: f (i;V, VB) : The density of the first passage time i to VB from V when the drift rate is (r- δ) using risk-neutral valuation.

: F (i;V, VB) : The cumulative distribution function of the first passage time to bankruptcy ;At that time, the equity value equals zero, and the debt value is (1- α) VB. The α is the fraction of firm asset value lost in bankruptcy.

The conditions of the debt are set as follows. For simplicity, at first it is described based on discrete time, and it would be made in continuous time later. The firm continuously borrows a constant amount of new debt annually. P is the total principal value. T is maturity years, and coupon interest rate C. The firm does the new borrowing of the same amount with dividing the amount of money of P/T and paying it back continuously with dividing the borrowing amount of money into every single year.

The debt total amount of period (0,T) is always fixed with P. By adding this assumption, the firm activities can be thought as perpetual. The present value d (V;VB, t) of one debt divided into every one year which has t periods to maturity, is computed as follows.

$$d(V;V_B,t)=\sum_{i=1}^{t}\frac{1}{(1+r)^i}\cdot\frac{C}{T}\cdot(1-F(i;V,V_B))$$
$$+\frac{1}{(1+r)^t}\cdot\frac{P}{T}\cdot(1-(F(t;V,V_B))+\sum_{i=1}^{t}\frac{1}{(1+r)^t}\cdot\frac{(1-\alpha)V_B}{T}\cdot f(t;V,V_B)$$

The first term in the above equation represents the discount expected value of the coupon flow when it doesn't go bankrupt (which will be paid at t with probability (1-F(t)). The second term represents the expected discounted value of repayment of principal; and the third term represents the expected discounted value of the fraction of the assets which will go to debt at time t, if bankruptcy occurs.

The continuous time expression is

$$d(V;V_B,t)=\frac{C}{Tr}+e^{-rt}(\frac{P}{T}-\frac{C}{Tr})(1-F(i;V,V_B))+(\frac{(1-a)V_B}{T}-\frac{C}{Tr})\int_{\tau=0}^{t}e^{-r\tau}\cdot f(\tau(\tau;V_B)d\tau$$

By the above, the evaluation of the debt value D (V;VB, T) in total is

$$D(V;V_B,T)=\int_{t=0}^{T}d(V;V_B,t)\,dt$$
$$=\frac{C}{r}\cdot+(P-\frac{C}{r})(\frac{1-e^{-rt}}{rT}-I(T))+((1-\alpha)V_B-\frac{C}{r})J(T)$$

where

$$I(T)=\frac{1}{rT}(G(T)-e^{-rt}F(T))$$

$$J(T)=\frac{1}{z\sigma\sqrt{T}}(-\left(\frac{V}{V_B}\right)^{-a+z}N[q_1(T)]q_1(T)+\left(\frac{V}{V_B}\right)^{-a-z}N[q_2(T)]q_2(T))$$

$$F(t)=N[h_1(t)]+\left(\frac{V}{V_B}\right)^{-2a}N[h_2(t)]$$

$$G(t)=\left(\frac{V}{V_B}\right)^{-a+z}N[q_1(t)]q_1(t)+\left(\frac{V}{V_B}\right)^{-a-z}N[q_2(t)]q_2(t)$$

$$q_1(T)=\frac{(-b-z\sigma^2t)}{\sigma\sqrt{T}}\quad,\quad q_2(T)=\frac{(-b+z\sigma^2t)}{\sigma\sqrt{T}}$$

$$h_1(T)=\frac{(-b-a\sigma^2t)}{\sigma\sqrt{T}}\quad,\quad h_2(T)=\frac{(-b+a\sigma^2t)}{\sigma\sqrt{T}}$$

$$a = \frac{(r - \delta - \frac{\sigma^2}{2})}{\sigma^2} \quad , \quad b = Ln\left(\frac{V}{V_B}\right) \quad , \quad z = \frac{[(a\sigma^2)^2 + 2r\sigma^2]^{\frac{1}{2}}}{\sigma^2}$$

According to Leland, the evaluation of the firm value with consideration of the bankruptcy is as follows.

$$v(V;V_B) = V - a \cdot V_B \left(\frac{V}{V_B}\right)^{-a-z}$$

Where V is the firm value without consideration of bankruptcy and arrival point is VB.

Therefore, the equity value with consideration of the bankruptcy becomes the next when the above is given.

$$e(V;V_B) = v(V;V_B) - D(V;V_B)$$

The above is the outline of the process for reducing the firm value, the debt value and equity value in consideration of the bankruptcy by the model Leland,.

4. The introduction of the stochastic cash flow process model

Here, how to calculate the credit risk premium should be considered. Two types of discount rate are introduced. One is for explaining the present value of the firm that doesn't consider bankruptcy. The other is for explaining the present value of the firm considering bankruptcy.

The difference of the two discount rates can be thought as credit risk premium.

To consider the method which defines the bankruptcy risk premium, the formulated relations among the firm value, free cash flow, and discount rate should be constructed. Therefore, the following basic assumption is introduced.

The firm value Vt at t, which doesn't take bankruptcy into consideration, could be decided by EBITt and the constant cost of capital w.

In other words, the related basic equation of EBIT and the firm value is defined as

follows. This is the simplest constant free cash flow discount model with the expected future EBIT fixed at the current EBIT.

$$V_t = \frac{EBIT_t}{w}$$

Therefore

$$\frac{dEBIT}{EBIT} = \frac{dV}{V}$$

It is assumed that the stochastic process of V is decided by the stochastic process of EBIT, which is the same assumptions in the LELAND model I mentioned above.

5. The bankruptcy risk premium *of* the capital cost

When EBITt is given as a stochastic process, the calculated firm value, Vt, without considering bankruptcy can be calculated by using the cost of capital w.
On the other hand, the spread of the cost of capital as the bankruptcy risk premium can be derived from the Leland model.

$$v(V;V_B)_t = \frac{EBIT_t}{w + s_{wt}}$$

The bankruptcy risk premium in the cost of capital is

$$s_{wt} = \frac{EBIT_t}{v(V;V_B)_t} - w$$

6. The bankruptcy risk premium of the cost of equity

First, the firm value V without bankruptcy is considered. In the absence of bankruptcy risk, the coupon C of the debt D can be thought to be equal to the risk free rate, and it is D=P at that time.
The firm value and the cost of equity without bankruptcy can be shown as follows.

$$V_t = \frac{EBIT_t}{w} = \frac{EBIT_t - rP}{r_{et}} + \frac{rP}{r} = \frac{EBIT_t - rP}{r_{et}} + P$$

The cost of equity without bankruptcy is

$$r_{et} = \frac{EBIT_t - rP}{V_t - P} = \frac{EBIT_t - rP}{E_t}$$

On the other hand, the cost of equity with bankruptcy by using debt value D from the Leland model and the EBIT is

$$v(V;V_B)_t = \frac{EBIT_t}{w + s_{wt}} = \frac{EBIT_t - rP}{r_{et} + s_{et}} + D_t$$

Therefore, the bankruptcy risk premium of the cost of equity is as follows.

$$r_{et} + s_{et} = \frac{EBIT_t - rP}{v_t - D_t} = \frac{EBIT_t - rP}{e_t}$$

$$s_{et} = \frac{EBIT_t - rP}{e_t} - r_{et}$$

7. The bankruptcy risk premium of the debt

The bankruptcy risk premium of the debt is the difference between the yield with bankruptcy from the Leland model and that without bankruptcy, that is, the coupon interest.

The yield with bankruptcy from the Leland model can be thought to be the same as a perpetual bond. So the difference is

$$s_{rt} = \frac{C}{D_t} - \frac{C}{P} = \frac{C}{D_t} - r$$

8. Relations between the debt ratio and each cost

(Basic assumptions)

Debt total amount: 100

The risk free interest rate: 4%

The total capital cost when bankruptcy isn't taken into consideration: 6%

EBIT: firm value without bankruptcy* total capital cost

Volatility of firm value: 20%

The bankruptcy loss rate of the debt: 0.5

The rate of total payment of dividend and the interest against the firm value: 6%

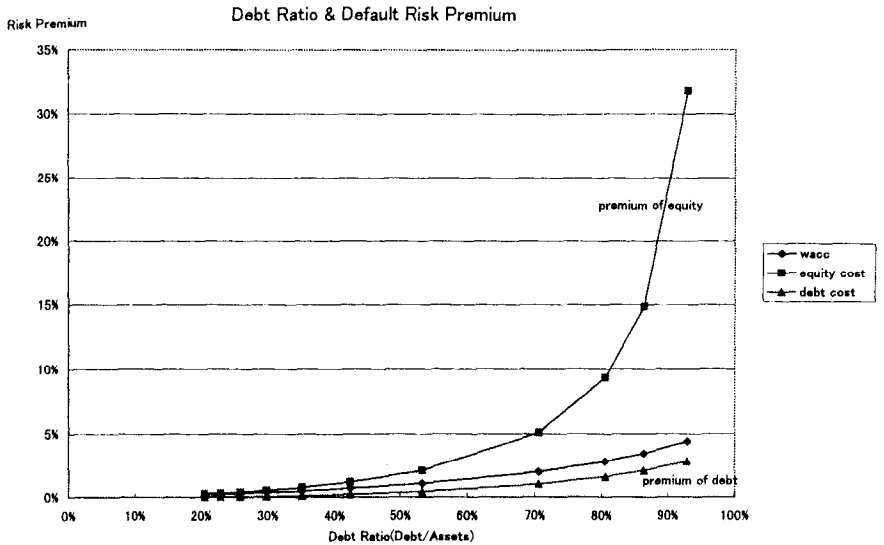
Under these assumptions, all of the cost of capital, the cost of debt, and the cost of equity rise with the increase of the debt ratio (debt/ firm value) as shown in figure 1.
Specially, when the debt ratio exceeds 70%, the cost of equity increases rapidly.
Using financial statements of 1318 companies in Japanese March 1998, the model estimates that the debt ratio (debt/assets) in book value is 68%. So if P/B is 2, the debt ratio (debt/assets) in market value is about 50%.

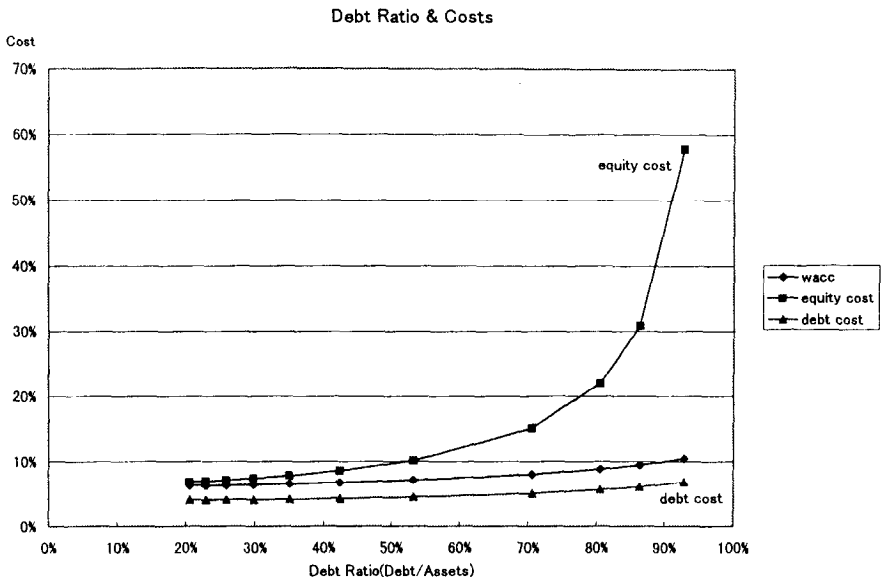
9. Debt ratio and bankruptcy risk premium

The assumptions are the same as above.
Similar to the above result, all of the bankruptcy risk premium in the cost of capital, the cost of debt, and the cost of equity rises with the increase of the debt ratio (debt/assets) as shown in figure 2.
Specially. when the debt ratio exceeds 70%, the bankruptcy risk premium of the cost of equity explains the rapid increment of the cost of equity shown above.
With the debt ratio of 80%, the cost of equity and the bankruptcy risk premium are 20% and 10% respectively.

10. Summary

By using the Leland model and the simplest formulation, assuming no taxes, the relations among the profit, the equity value, and the bankruptcy risk premium could be recognized in actual Japanese market. The result of research is simple but intuitive in actual market. These are not presented in the typical textbook of finance theory. The research in this area of study has been expanding in Japan, and the research of Professor Shirakawa in Tokyo Institute of Technology is a good example. The improvement of this area is rapid.


Debt Ratio & Default Risk Premium
Risk Premium
35%
30%
25%
20%
15%
10%
5%
0%
premium of equity
premium of debt
wacc
equity cost
debt cost
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%
Debt Ratio(Debt/Assets)


Debt Ratio & Costs
Cost
70%
60%
50%
40%
30%
20%
10%
0%
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%
Debt Ratio(Debt/Assets)
equity cost
debt cost
wacc
equity cost
debt cost