Financial Engineering and the Japanese Financial Industry

—Toward Finansurance—

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From a viewpoint of a financial engineering paradigm, we develop an argument on recent movements in finance and insurance in Japan. Our argument is made in view of risk and functional finance for the efficiency of capital. Among others, we foresee a convergence of finance and insurance to "finansurance" in near future, in which households and firms optimize their overall risk positions in life-cycle and business. It is also argued that financial technology together with information technology accelerates the trend of functional finance and will provide products to complete an incomplete system for risk optimization. Analytically we observe that a discrete time approach to credit risk analysis in financial engineering makes a bridge between finance and insurance and demonstrate the valuation of the premium of a life insurance and the price of an earthquake bond via the no-arbitrage concept.

All the views and opinions in this paper belong to the author .

1 Introduction

I am very much honored to be invited here as a keynote speaker at the occasion of the ASTIN&AFIR Meetings and the Centennial Anniversary of the Institute of Actuaries of Japan. I would like to congratulate the Institute for the wonderful achievements. I suppose I am invited here

A because I am the founder of the JAFEE, the Japanese Association of Financial Econometrics and Engineering(FE).

B because I am a strong advocate of the importance of financial technologies for Japanese financial industries to be functionally efficient and

C probably because I work for the IBJ-DL Financial Tecnology, the first joint venture of banking and insurance in Japan, where IBJ and DL respectively stands for Industrial Bank of Japan and Dai-ichi Life Insurance.

The themes of this paper are in fact associated with these reasons.

As a promoter of FE, I founded the Jafee in 1993 to make a forum for people in academia, industry and government to communicate for research, information, opinions and so on in financial engineering. Jafee was started with about 250 individual members and 25 corporate members. Now the size is more than doubled. Almost all the big financial institutions in Japan including insurance companies are the corporate members of Jafee.

As for B, let me briefly state about the Japanese system and the situation of the financial industries briefly. First of all, I was the first professor of social sciences in top national universities that moved from academia to industry. This implies that Japan is still at this level in the mobility of labor from academia to industry, which is associated with the inflexible structure of the society including life-time employment and pension systems. I acted as a deviate from our social norms that govern us strongly in mentality.

In fact, the Japanese society is basically or at least traditionally a harmonious community governed by the social norms and orders. The social norms are so stubborn that the three co-governing bodies of Japan: politicians, bureaucrats and management people, known as the "iron-triangle", all believed and still believe to some extent the existence of Japanese own economic system and have been relying on their judgements, not on the capitalistic mechanism of the dynamic process of capital associated with dynamic evolution of people's preferences. Naturally it led us to an inefficient and high cost society and hence to the loss of economic competitive power. We also wasted capital through the bubbling economy. With severe suffers from depressions after the crash of bubble, we are now gradually recognizing the economic functions of capital and capital market, and we will make our economy more efficient. But we have accumulated a huge deficit in the government budget, which the future generations have to pay. It was Prime Minister Hashimoto that gave a strong clue to a dramatic change for reformation and deregulation in 1996, known as the Big Bang in Japan.

About financial industries, due to the bankruptcy of banks, insurance companies and securities companies, financial institutions are now sensitive to credit risks in their loans and contracts. The Government seems to have given up keeping the strong protection for financial industries that it transfers the risks of the industries into people through taxes. This will make the industries more exposed to international competitions, through which they will have to be functionally efficient due to the demands of the efficiency of capital. Financial technologies with the FE paradigm will promote the trend of functional finance together with information technology. After all, capital, information and technology, which brings productivity in finance , will go over sovereignty.

As for the IBJ-DL Financial Technology, the first joint venture of banking and insurance, in October 1998, the Industrial Bank of Japan and the Dai-ichi Life Insurance made a comprehensive or total alliance for future business. This is a notable movement in the Japanese financial and insurance history. The institutional walls between the financial industries, i.e., banking, securities, insurance, commodities, used to be highly built up for segregation not only by the Government but by each industrial organizations. However, the depression following the crash of the bubble and the BIS requirement on the capital adequacy made a significant change on the situation. From a viewpoint of capitalism and the FE paradigm of no-free lunch, some types of insurance risks will be better treated with the financial risk in the capital market. Besides, the credit risk theory will make a bridge between the credit risk and insurance risk, which have been already implemented in Japan. This will in turn lower the wall between finance and insurance further. We predict a convergence of finance and insurance in near future according to the demands of capital, behind which there are firms' and households' risk optimizations, as has been pointed out by e.g., Shimpi.

The content of this paper is as follows.

Section 2 Financial engineering for functional finance

Section 3 From institutional finance to functional finance

Section 4 Financial technology as a competitive power

Section 5 No free lunch theory and insurance

Section 6 Credit risk analysis makes a bridge between finance and insurance

Section 7 Convergence of finance and insurance to finansurance

2 Financial Engineering for Functionally Efficient Finance

Financial engineering (FE) is in my definition the science of creating thoughts and knowledge on the functional efficiency of finance in view of capital. It enables us to make a most efficient utilization of capital and hence it promotes the efficiency of capital allocation and movement in the world via risk, returns and cost. FE is an interdisciplinary science associated with economics, accounting, finance, insurance, law, statistics, mathematics, computer science, engineering, etc. It consists of four main areas: Investment, Derivatives, Risk Management and Legal engineering. Financial business is now being recognized as risk business (not risky business!) in which risks in broad sense are transformed, transferred, pooled and interchanged for optimalities.

The driving force of functionally efficient finance is the greatly accumulated capital in the world that seeks its efficient use relative to cost, risk and return. Naturally capital is a main source of creating future productivity and employment and therefore it has to be used efficiently for future generations. Besides, the concept of efficiency from this viewpoint will lead to a business success in financial industries. In particular, it is important to strategically understand the relations among the functions of finance, the concept of risk and the financial technology for a successful business.

As is well known, there are two approaches to finance or the science of finance: namely,

(1) institutional approach

(2) functional approach.

In the institutional approach, such institutional system as banking, insurance, securities is the given concept the approach relies upon, hence it cannot have the conceptual and analytical framework to treat the change of the institutional system, though in reality it is quickly evolving. On the other hand, in the functional approach the analysis is focussed upon the functions of finance and it treats the change of the financial system along its evolution. In this respect, financial engineering takes the functional approach and provides as means not only individual technologies such as pricing methods for various risks or risky assets including derivatives but also the conceptual framework for functional finance or power of developing concepts for efficiency. The no-arbitrage concept and the associated paradigm on which the Black-Scholes Option theory is derived is so powerful that it will push us to a unification and standardization of the segmented capital markets in the world in terms of the rules of the game and systemic risk and to a unification of the segregated industries via functional efficiencies for risk allocation. Another implication of the paradigm is given in Section 5.

In view of the functional approach, let us review some notable movements in financial engineering in the US and in Japan.

1972 CME introduced the foreign exchange futures

1973 Black-Scholes Stock Option Theory (Foundation of Financial Engineering)

Establishment of the New Financial Paradigm :No-Free Lunch Paradigm

1980's Financial deregulation under the Regan's regime and developments and practices of financially engineered business with interactions between industry and academia

1990 H. Markowitz, W. Sharpe, and M. Miller shared the economic Nobel Prize

1997 R. Merton and M. Scholes became the Nobel Laureates (Worldwide recognition of Financial Engineering as a science)

1993 The Jafee (Japanese Association of Financial Econometrics and Engineering) founded. Public recognition of financial engineering :Holds two domestic meetings and one international meeting a year, publishes international journal "Asian-Pacific Financial Markets (formally Financial Engineering and the Japanese Markets)" (4 issues per year) and domestic journal "Jafee Journal" in Japanese.

1997-1999 Jafee organized with Columbia University "Columbia=JafeeConference" once a year

1998 IBJ Financial Technology was established : first visible recognition of FE in the big banks in Japan

1999 Tokyo Institute of Technology made a research center and Tokyo University launched a course in FE

2000 Hitotsubashi University and Kyoto University plan to launch a course in FE

3 From Institutional Finance to Functional Finance

First we shall observe actual movements in a big tide of functionally efficient finance and the promoting techniques due to the strong demands of capital.

(1) New financial products

Derivatives on market risk, credit risk and insurance risk (via application of financial engineering), credit swap, earthquake bond, CBO(collateralized bond obligations), weather derivatives. These are made possible by the development of risk pricing techniques, by the demand of capital for new risk structure and return enhancement and by the deregulations. However, the Government often mentions the existence of the Gambling Law to intimidate financial institutions. Still derivatives could be intentionally regarded as gambles by this law in Japan.

(2) Sophistication of investment and money-managing techniques

Development and applications of new models and techniques: complex system model (neural network model, hereditary model, nonlinear time series model, pattern recognition model), which has already been used in Japan.

(3) Innovations in infrastructure of finance and financial system (efficiency of capital market) : futures, options, standardization of financial products, exchange-traded commodities, mortgage securities to be introduced. Since late 80's, the Japanese infrastructures are gradually improved. Internet financial schemes are being developed.

(4) Financial business associated with life-cycle: mutual funds, pension, principal-guaranteed products associated with equity. We are now introducing the 401K type pension system in which the individuals take the market risks. This introduction is due to a crisis of private pension system. The liability of pension funds in firms is now required to be listed on the balance sheets in 2001.

(5) Engineered financing: CBO (Collateralized Bond Obligations), ABS (Asset-Backed Securities), CB (Convertible Bond) with some options, etc. The Fuji Securities and the Sakura Securities made CBO finances in 1998 and 1999. The Tokyo Metropolitan Government is planning to set up a CLO (Collateralized Loan Obligations) scheme of finance for small firms, because banks are now sensitively aware of credit risk and are subject to the BIS requirements.

(6) Convergence of financial market and insurance market: weather derivatives, earthquake bond, alliance of Industrial Bank of Japan and Dai-ichi Life Insurance,

(7) Competition of different systems in the world

Next to understand these movements, we discuss on what finance should be to capital. In the following arguments I share a lot with Professor Merton. Financial products and instruments carry as assets the three basic elements: time, risk, and return. Through financial instruments and products with these elements, the function of finance is to make cashflows between individual households and firms by combining the decisions of consumption and saving with the decisions of production and capital accumulation.

Capital market should promote the efficiency of this function of finance, provide opportunities of risk allocation, risk sharing and risk protection for households and firms and combine the internationally diversified decisions efficiently. The efficiency of the capital market is related to the capital cost including the information of the market, trading cost and tax, to the risk involving technology and regulations (rules of the game) and country risk, and to the depth and scope of the markets. Financial institutions should exist to promote this function of capital market and should help the capital utilized efficiently. Financial system should exist for the same purpose and not for the controllability of the financial policy by the government. Regulations based on the political and institutional purpose will delay the development of the financial industries and the industries will lose business competitive power in the international markets. Japan is a typical example. Financial technology promotes the efficiency of the functions of capital market and finance together with information technology. It creates more opportunities for risk sharing, decreases trading costs and improves on the asymmetry of information in financial business together with decrease of agency cost. Through these opportunities households and firms can attain their far better optimalities in wider choices. Capitalism from a functional perspective is an open system which is never completed, and hence in its development, it is important to plan it or regulate it as an open dynamic system which should evolve along the need of people and firms.

On the basis of this viewpoint, we make

Comparisons of the Institutional Approach (a) and the Functional Approach (b)

(1a) The concept of institutions is given (static).Traditional finance as a science relies on the conceptual framework of institutions such as banking, securities, insurance etc.

(1b) The concept of function is given (dynamic). Financial engineering relies on the conceptual framework of financial functions and hence it can treat the change of a institutional system.

(2a) Firm Value Accounting measures static financial values for value allocation. Accountants give the values.

(2b) Risk Accounting measures the change of values for risk allocation. The financial institutions with financial technologies give the dynamic values of risks. The current (market) value evaluation is necessary.

(3a) In risk management, capital is a main source to absorb the risks from the viewpoint of accounting.

(3b) Risk management is viewed from the point of risk portfolios and diversification. Hence derivatives are natural means to control risks.

(4a) Institutional Regulations are made from a viewpoint of keeping the institutional system, which leads to itemized regulations and a high-cost system.

(4b) Functional Regulations are made from a viewpoint of promoting the functions of finance, leading to comprehensive regulations.

(5a) Prices are formed based on costs in financial institutions.

(5b) Prices are formed for risks in the markets.

(6a) Financial institutions are to provide the institutional financial services.

(6b) Financial institutions are to help agents allocate risks for capital efficiencies.

(7a) Assets are legally classified by itemization.

(7b) Assets are classified by the concept of risk.

(8a) Excess Profit (rent) is likely to be generated.

(8b) Only competitive profit will be generated.

4 Financial Technology as a Competitive Power

Implementation technologies for pursuing the functional finance are naturally means to financial innovations such as the refinement of business concept, the planning of financial systems in government and the development of new commodities in financial business. They form a basis for the internationally competitive power of financial institutions and a financial system. There the technologies require a high level and broad scope of knowledge base in FE and its related fields.

From a macroeconomic viewpoint, due to the global nature of capital associated with the reversibility of financial decisions, financial systems in different countries are destined to compete with each others for seeking the functional efficiency of finance. This is because capital is a strategic element for the development of each economy. Inefficient financial systems fail to generate incomes in financial industries and invite capital inflow. In addition, financial decisions between firms and households are not optimal in the inefficient systems. Furthermore inefficiencies may be exploited by institutional arbitrage through derivatives. There FE plays a basic role in designing an efficient and innovative system. As has been experienced in the US, there will be three basic elements for competitive and efficient systems.

(1) Standardized Instruments corresponding to different risks

(2) Deregulated and Standardized Markets

(3) Informative and Fair Infrastructure for financial system

For example, mortgage securities such as GNMA and FNMA in the US are an additional instrument with prepayment risk which yields return-enhancement and risk-diversification for investors. The instrument is made credit-risk-free by the governmental corporations so that it is of the almost same credit quality as the T-bond. The market is well designed in order that the securities are continuously well supplied and the mortgages collateralizing the securities are well diversified in area and size. Furthermore the information on the prepayment history in each pool of mortgagors is accurately provided for investors in the secondary market. This makes the market liquid and attractive. FE has been supporting the valuation of mortgage securities though the prepayment risk is hard to model. In Japan, a few big banks privately issued the securities on the OTC basis in recent years and hence the default risk is not necessarily well controlled and the diversification of the collateralized mortgages is not well made. The government is now planning to open an official market based on the mortgage loans made through the Nippon Jutaku Kinyu Kouko (Japan Housing Loan Corporation). But unfortunately it has not well accumulated necessary information on prepayment. Besides, the values of the collateralized mortgages might be smaller than the loan values because the land prices went down significantly after the crash of the Bubble. Hence unless the Banks takes the credit risk, the investors will have to take a part of it. Assuming the US scheme, theoretical and empirical research on the prepayment is being done in Japan (e.g., Kariya and Kobayashi (1999)).

From a microeconomic viewpoint, financial technologies are also a strategic element in financial manufacturing, typically in investment banking. The technologies are classified according to the areas of FE: (1) investment and money management, (2) derivatives, (3) risk management and (4) structured finance. According to the needs of each area, different technologies and models are used.

In investment (1), the market risk is a main object to cope with to generate returns. The difficulty to cope with the market risk may arise from the fact that it includes game-theoretic uncertainty as well as probabilistic uncertainty. Prices change according to the change of behaviors of investors and the behaviors also change when prices change. Hence various approaches and models are used to extract unfound information contained in data. Among others, such nonlinear models as threshold autoregressive model, neuro model, chaotic model, etc. are recently applied in Japan as has been stated in Section 2.

In the area of derivatives, new products have been created associated with credit risks and insurance risks. As will be discussed in Section 5, this can be viewed as a process of the completion of the incomplete system. It is well known that derivatives function as instruments for risk transformation, change of cashflows and payoffs, risk sharing, reduction of cost due to the asymmetry of information and institutional arbitration. Most recent creations in Japan are made on credit risk and insurance risk. They are event-triggered derivatives, which may be regarded as insurance or guaranty. Among others, weather derivatives and earthquake derivatives are notable. For example, a temperature derivative is made as a put on the average of daily temperatures in a summer period with a fixed exercise temperature, say $24^{\circ}C$. A formal valuation for such a derivative is made possible by the development of the theory on credit risk though there will be a variety of models on temperature variations.

In risk management, the Japanese big banks are now in a stage to use an internally developed integrated risk management system. Among others, the Sakura bank, IBJ and Asahi bank systems on credit risk management are for sales though the CreditMetrics of J.P. Morgan and the CREDIT RISK+ of the Credit Suisse Financial Products are famous. Each system seems to describe the loss distribution of a loan portfolio. The default correlations seem to be treated differently in each system.

The loss distribution is basically described as the distribution of

(4.1) $$R_N = A_{1N}(1-\gamma_{1N})L_{1N} + \cdots + A_{MN}(1-\gamma_{MN})L_{MN}$$

where A_{in} is the loan exposure of the i-th firm at n, γ_{in} the (random) recovery rate of the i-th firm in default at n, which may be replaced by the expected value $\bar{\gamma}_{in}$, and L_{in} the accumulated default generation process with

(4.2) $$L_{in} = \begin{cases} 1 & \text{if the } i\text{-th firm default up to or on } n \\ 0 & \text{otherwise} \end{cases}$$

When A_{in} and $\bar{\gamma}_{in}$ are given, the distribution of R_N can be in principle derived if the joint distribution of $\vec{L}_N = (L_{1N},\cdots,L_{MN})$ is specified. If the defaults of the firms are independent, the distribution becomes the product of binomial distributions. The problem here is how to specify the correlations of the defaults and how to compute the probabilities of R_N for possible outcomes when the number of loans is large. The correlations are often associated with the ratings of the firms and the industry factors. On this point, there are a variety of formulations including the CREDIT RISK+. In life insurance, the distribution due to claims can be specified as in (4.1) with $\gamma_{in} = 0$ and A_{in} fixed, though it is often described through Poisson process. This approach can be used in the ALM of life insurance companies, and is used in the risk management by the PBGC (Pension Benefits Guaranty Corporation). Furthermore, it is a basis for CBO or CLO in structured finance.

The BIS regulations changed the Rules of the Games in Japan

Speaking of risk management, the BIS (Bank of International Settlement) regulation on the capital adequacy has been making significant impacts on the Japanese banking system as well as on the Japanese traditions and social mentality and order in banking business. First of all, the banks are set free in part from the strong bondage to the Ministry of Finance in pursuing their business. Secondly, the current (market) value accounting system has to be adopted, which will take place from 2000 on. This will reveal the current values of the banks and the risk structure. Thirdly, it promotes a trend of quantifying the risks and making objective judgements based on the measurements in management, which in turn creates more demands on new financial technologies. It also promotes the disclosure of banks. Fourthly, the local banks are completely separated from the city banks and have to stay as retail banks. Fifthly, the public are made to recognize the default risk of financial institutions and the bankruptcy of some big banks is leading us to forming a deposit payoff system, which will start in 2001. This will lead the public to following the rule of self-responsibility. Finally, the financial institutions in Japan have to seek their own expertized business for controlling risks and stop the business routines of expanding their balance sheets.

5 Implications of No Free Lunch Theory and Insurance

The no-arbitrage concept is the fundamental concept of valuing relative prices of financial assets and products and it gives a means to producing strategic concepts for business, to developing financial products associated with the concepts and to developing financial systems in promoting the functional efficiency of finance. This is an economic implication of the Black-Scholes mathematical result. The result implies a new recognition about risk and its price. In fact, this implication is confirmed by Harrison –Pliska (1981), where the concept of the no-arbitrage is exploited in terms of relative prices of assets and products via replicability. Since the no-arbitrage concept is a broad concept including the equilibrium concept in economics and the theory based on the concept need not assume formal behaviors of economic agents in pricing the values of assets, it is so persuasive and powerful in implementing the result in real world.

The theory states that the prices of financial assets and products are mutually adjusted to exclude arbitrage opportunities, when the market is perfect and complete. Here the perfectness of market is that there is no frictional cost such as trading cost and tax and any financial position of portfolios of the assets can be taken including arbitrarily large short positions. This is not realistic assumption. But when a market is very far from the assumption, the concept and the theory enables us to make an arbitration between markets or between institutions in different countries through derivatives or structured schemes. Hence an inefficient system has to adjust itself toward a more efficient one, leading to a standardization. What is more important is that the incompleteness due to the lack of sufficient instruments and products for important risks is eventually completed. The appearance of many new products is viewed as this process of the completion of the incomplete system. Among others, many derivatives appearing in insurance risks can be regarded as such a process. When capital is accumulated and globalized and when the level of life is improved, firms and households are more willing to optimize their risk positions for protection of achieved life and business levels in a more accurate manner.

The completion of an incomplete system due to additions of risk products is in fact in a process of making firms and people in the world form comprehensive and optimal risk positions over all risks significant to them. The process of course includes the insurance risks and makes the agents better off in business and life-cycle in their dynamic and integrated risk management.

As Brennan pointed out, a divide between finance and insurance has made insurance segregated from a behavior science as had been in the relation between economics and finance. Now finance and insurance will be simultaneously treated in terms of optimal allocation of risks of agents. In other words, finance and insurance will be definitely combined in view of optimal allocation of capital over risks. After all, all risks are associated in life and business in the globalized world through many channels.

6 Credit Risk Analysis Makes a Bridge between Finance and Insurance

The no-arbitrage theory on credit risk analysis enables us to make a bridge between finance and insurance. Risk factors behind an insurance risk are in general hard to identify uniquely and thus the market is incomplete in views of risk factors. Hence in the framework of the continuous time theory, the nonuniqueness of equivalent Martingale measures can be used to include the models used in insurance if necessary. Then the pricing techniques in insurance can be viewed as a special case of the no-arbitrage pricing techniques though the market is incomplete in the sense that long and short positions on the insurance risk are limited. In addition, the process of risk factors may not be such as required in the continuous time theory.

To overcome this situation, I have been proposing a discrete time approach to valuing assets. The theory keeps the no-arbitrage concept with self-financing rule but discards the replicability of a payoff or price. Then a sufficient condition for the no-arbitrage is that the relative prices are martingale under an equivalent measure, which is the same as in the case of continuous time setting. The theorem is given in Appendix for reference. The loss by doing so is that we lose the concept of hedgeability through replication. But the gains are great. Among others, the models in the theory can be out of the semi-martingale framework. Some models describing insurance or prepayment risks may not be in the semi-martingale and then they can not be treated in the continuous time framework. For example, a model for describing the variations of temperature or the prepayment behaviors in the mortgage loans will be such and hence the discrete time theory is useful for valuing these derivatives and assets.

Application of the technique to valuing life insurance.

Let $h = 1/12$ year be the time unit for analysis and let $n = 0,1,\cdots,N$ be monthly time points with $n = 0$ the contract month and $n = N$ the maturity. Also let A (yen) denote the amount of insurance for death and $J(y)$ the default time to death where y denotes the age of the insured. Then the default generation process for $J(y)$ is defined by $J(y)$ as

$$(6.1) \qquad L_n(y) = \begin{cases} 1 & \text{if } J(y) \le n \\ 0 & \text{otherwise} \end{cases}$$

just as in credit risk analysis. We assume that when $J(y) = n$, the amount A yen is paid at $n+1$. Then the payoff of an insurance company at n is described as

$$(6.2) \qquad \begin{aligned} U_n(y) &= x(1 - L_n(y)) - AL_{n-1}(y)(1 - L_{n-2}(y)) \\ U_{N+1}(y) &= -AL_N(y)(1 - L_{N-1}(y)) \end{aligned}$$

where x is the monthly premium. To value the premium x, let $\{r_n\}$ be the process of monthly interest rates and let

$$(6.3) \qquad d(m,n) = B_m / B_n \quad \text{with } B_m = \exp\left(\sum_{j=0}^{m-1} r_j h\right).$$

Then the no-arbitrage value of the payoff (6.2) at 0 is given by the martingale condition on $\{U_n(y)/B_n\}$;

$$(6.4) \qquad u_n(y) = E_0^*[\, d(0,n)U_n(y)\,]$$

where $E_0^*(\cdot)$ is the conditional expectation of $\cdot$ with respect to an equivalent martingale measure Q^*. Since the measure is not unique, we may regard the actual processes of $\{L_n(y)\}$ and $\{r_n\}$ as the processes under an equivalent measure. The total payoff of the insurance company is thus valued as

$$(6.5) \qquad V(0,N:y) = \sum_{n=1}^{N+1} u_n(y) \ .$$

Then assuming that $\{L_n(y)\}$ and $\{r_n\}$ are independent and imposing the fair value condition

$$V(0,N:y) = 0 \ ,$$

we obtain a fair value premium: $x = C_1 / C_2$ with

$$C_1 = A\sum_{n=1}^{N+1} D(0,n)Q^*(\, J(y) = n-1\,)$$

$$C_2 = \sum_{n=1}^{N+1} D(0,n)Q^*(\, J(y) > n\,)$$

where $D(0,n) = E_0^*[d(0,n)]$, the discount factor at n. In the continuous time setting, it is often the case that the default probability is expressed in terms of hazard rates via the Doob-Meyer's theorem. In the discrete time approach, we obtain the following approximation via the Doob's Theorem

$$Q(\, J(y) > n\,) \approx E_0\left[\exp\left(-\sum_{j=0}^{n-1} q_j(y)h\right)\right] ,$$

where $q_j(y)$ is the discrete time hazard rate at j. For the hazard rate process, we may assume such a model as

$$\Delta q_j(y) = \mu(y)h + \sigma(y)\sqrt{h}\varepsilon_j$$

with $\varepsilon_j \sim$ iid $N(0,1)$. The argument can be extended to a valuation problem for a person of age y with additional risk factor such as smoking.

7 Convergence of Finance and Insurance to Finansurance

Until 1997 no insurance company has ever provided an insurance against earthquakes. It is because not much data about the occurrence of catastrophic earthquakes has been accumulated and because the reinsurance markets for such catastrophic events are not big enough relative to the sizes of the losses caused by the rare events. Swiss Re Capital Markets and Goldman Sachs came in in 1997, structured the earthquake risk into earthquake bond and sold them in the US capital market. The investors in the capital market can and in fact are willing to take the risk in view of its coupon so that their portfolios are more diversified and more enhanced as the earthquake risk in Japan is independent of any other risks in their portfolio. Thanks to the scheme, the Tokyo Marine and Fire Insurance Co. can sell the insurance and those who need it enjoy the secured life. The Mitsui Marine and Fire Insurance Co. also sold earthquake insurance under a slightly different scheme though the Swiss Re again helped it. Now we intermediate the risk by derivatives.

What is important is that the catastrophic risk is taken in capital markets, not in the industry of insurance. The capacity of the world capital market is big enough to absorb various risks so long as financial products correspond to the need of investors. The investors, on the other hand, have different capacities for taking risks, which depend on the risk structure of their portfolios together with risk-return preferences. In the case of the earthquake bond, the collateralized principal is managed safely in the cash market by an SPC (or SPR). Hence in this case the amount of the principal in the capital market shift to the cash market, meaning that there will be neither inflow of new money into the capital market nor outflow from the capital market till a specified earthquake occurs. If the earthquake risk is pooled in the insurance industry, insurance companies have to make reserves for having the risk, which will be managed safely in the cash market. Hence no difference between the two schemes is basically observed in the capital market so long as no earthquake occurs, though the prices may be different.

Another notable feature in this example is that nobody knows about the default (occurance) probability of the earthquakes. Usually insurance products are constructed on the law of large numbers for each specific risk which repeats in a period, i.e., on the collective risk theory. But when a transparent scheme is given, the cat risk or rare event risk that is not necessarily based on the law of large numbers can be transformed into products in the capital market. This function of the globalized capital market will be paid a greater attention to for risk optimization of firms and households. In addition, combinations of different risks in finance and insurance are recently made into products in the capital market, leading us to finansurance.

The Structure of the Earthquake Bond (see Shimpi)

Swiss Re Capital Markets and Goldman Sachs placed $100 million of Catastrophic Bonds in November 1997.

(1) Sponsor: Tokyo Marine/ Swiss Re

(2) Issuer: Parametric Re (SPR, Cayman)

(3) Risk: Japanese Earthquake

(4) Cover: $90 million

(5) Term: 10 years

(6) Loss Index: JMA (Japan Meteorological Agency) reported EQ magnitude

Bond Structure:

A $80 Million Note: LIBOR+430bp, Ba2 Moody's
Graduated loss starting at 25% for JMA 7.1 and ending at 100% for JMA 7.7

B $20 Million Units: LIBOR+206bp, 50% Notes 50% Guaranteed Principal
Baa3 Moody's

To value bond A at n, let $n+m_j$ be the time points at which cashflows are generated, and let $C_n(m_j)$ be the cashflow at $n+m_j$, where $j=1,\cdots,M$. Assume that when an earthquake of magnitude greater than or equal to 7.1 occurs in the period $I_j = (\ (n+m_{j-1})h,\ (n+m_j)h\ ]$, the corresponding payoff is made at $n+m_j$. Let $\{L_n\}$ be the default (earthquake) generation process, $\{\gamma_n\}$ the recovery process linked with the size of the magnitude and $\{r_n\}$ an interest rate process and assume that these processes are independent. Then the payoff at $n+m_j$ is described as

$$G_n(n+m_j) = C_n(m_j)(1-L_{n+m_j}) + 100\,\gamma_{n+m_j}(1-L_{n+m_{j-1}})\,L_{n+m_j}$$

where

$$C_n(m_j) = 100\,(l_{n+m_j}+0.043)\,(m_j-m_{j-1})\,h + 100\,e_{jM}$$

with $\{l_n\}$ the LIBOR process and $e_{jM}=1$ if $j=M$ and 0 otherwise. Therefore in the same way as above, the earthquake bond with maturity N is valued as

$$V(n,N) = \sum_{j=1}^{M} E_n \left[d(n,n+m_j)\, G_n(n+m_j) \right]$$

Here the conditional expectation is

$$\begin{aligned} &100\, E_n \left[(l_{n+m_j} + 0.043)\, d(n,n+m_j) \right] (m_j - m_{j-1}) h\, Q(J > n+m_j) \\ &+ D(n,n+m_j) \left\{ 00 e_{jM}\, Q(J > n+m_j) + \bar{\gamma}_{n+m_j}\, Q(n+m_{j-1} < J \le n+m_j) \right\} \end{aligned}$$

where J is the first time to the default. In such a problem, the processes of $\{L_n\}$ and $\{\gamma_n\}$ are hard to specify and they will be out of the simi-martingale framework.

Appendix: Discrete time no-arbitrage theory

Here the discrete time version of the no-arbitrage theory we used in sections 5 and 6 is formulated in a general form and we give a sufficient condition for no-arbitrage that the processes of relative prices form Maringales with respect to an equivalent measure. This result is known in a continuous-time setting, which requires a set of regularity conditions to define stochastic integrals and differntials, i.e., it requires a semi-Martingale framework. Our approach does not require the specification of the processes of asset prices and hence it is free from the set of regularity conditions on the stochastic integrals and differentials and Markovness of price processes.

Let X_{in} be the price at n of the i-th asset ($i = 0,1,\cdots,M$), where $\vec{X}_n = (X_{0n},\cdots,X_{Mn})$ follows a stochastic process, and asset 0 is regarded as a standardizing asset with $X_{0n} > 0$. The time horizon is set as $n = 0,1,\cdots,N$, where the unit of time is fixed as h year in advance (e.g., $h = 1/365$). Let Q be the probability measure generating $\{\vec{X}_n : n = 1,\cdots,N\}$ given $\vec{X}_0$ where the distribution of $\vec{X}_0$ is not specified. We make a portfolio of the M assets at each time n $(n = 0,\cdots,N-1)$ so that the value of the portfolio at n is expressed as

(A.1) $$V_n(\vec{a}_n) = a_{0n}X_{0n} + a_{1n}X_{1n} + \cdots + a_{Mn}X_{Mn} ,$$

where $\vec{a}_n = (a_{0n}, a_{1n}, \cdots, a_{Mn})$ is the portfolio, and depends on the past paths of prices $\{\vec{X}_m : m \le n\}$. The process $\{\vec{a}_n\}$ formed from $\vec{a}_n$'s is called a trading strategy. At time $n+1$, the value of the portfolio changes due to the change of prices to

(A.2) $$V_{n+1}(\vec{a}_n) = a_{0n}X_{0n+1} + a_{1n+1}X_{1n+1} + \cdots + a_{Mn+1}X_{Mn+1} .$$

Definition 1: A trading strategy $\{\vec{a}_n : n = 1, \cdots, N-1\}$ is called a self-financing strategy if at each time n

$$V_n(\vec{a}_{n-1}) = V_n(\vec{a}_n) \quad (n = 1, \cdots, N).$$

The concept of self-financing guarantees the legitimacy of the concept of no-arbitrage.

Definition 2: The M assets are of an arbitrage relation if for some self-financing strategy $\{\vec{a}_n\}$,

(A.4) $\quad V_0(\vec{a}_0) \le 0$ and $V_N(\vec{a}_{N-1}) > 0$ with probability 1, or

(A.5) $\quad V_0(\vec{a}_0) < 0$ and $V_N(\vec{a}_{N-1}) \ge 0$ with probability 1.

When we can obtain neither (A.4) nor (A.5) for any trading strategy $\{\vec{a}_n\}$, we say that the M assets are of no arbitrage relation.

Theorem: The M assets are of no-arbitrage relation if the processes of relative prices $\widetilde{X}_{in} = X_{in} / X_{0n}$ become Martingales under some measure Q^* which is equivalent to the measure Q generating $\vec{X}_n$ $(n = 1, \cdots, N)$ with X_0 arbitrarily given;

$$E^*_{n-1}\left(\widetilde{X}_{in}\right) = \widetilde{X}_{in-1} \quad (n = 1, \cdots, N).$$

Proof: We only treat the case (A.4) as the other case is similar. First note (A.4) is equivalent to

(A.6) $\quad \widetilde{V}_0(\vec{a}_0) \le 0$ and $\widetilde{V}_N(\vec{a}_{N-1}) > 0$ with probability 1,

as $X_{0n} > 0$ where

(A.7) $\quad \widetilde{V}_n(\vec{a}_n) = a_{0n}\widetilde{X}_{0n} + a_{1n}\widetilde{X}_{1n} + \cdots + a_{Mn}\widetilde{X}_{Mn}.$

Here $\widetilde{X}_{0n} \equiv 1$. Also note that using the self-financing condition $\widetilde{V}_n(\vec{a}_{n-1}) = \widetilde{V}_n(\vec{a}_n)$,

(A.8) $$\widetilde{V}_N(\vec{a}_{N-1}) = \widetilde{V}_0(\vec{a}_0) + \sum_{n=1}^{N}\left[\widetilde{V}_n(\vec{a}_{n-1}) - \widetilde{V}_{n-1}(\vec{a}_{n-1})\right]$$

Hence if $\left\{\widetilde{X}_{in}\right\}$ is a Martingale for each i under some Q^*,

(A.9) $$E^*_0\left[\widetilde{V}_N(\vec{a}_{N-1})\right] = \widetilde{V}_0(\vec{a}_0)$$

since the conditional expectation under Q^* of the second term is

$$\sum_{n=1}^{N}\sum_{i=0}^{M} E^*_0\left[a_{in-1}\left(\widetilde{X}_{in} - \widetilde{X}_{in-1}\right)\right]$$
$$= \sum_{n=1}^{N}\sum_{i=0}^{M} E^*_0\left[a_{in-1}\left[E^*_{n-1}\left(\widetilde{X}_{in}\right) - \widetilde{X}_{in-1}\right]\right] = 0.$$

Here (A.9) means that $\widetilde{V}_N(\vec{a}_{N-1}) > 0$ with probability 1 under Q^* implies $\widetilde{V}_0(\vec{a}_0) > 0$ with probability 1 under Q^*. But since Q and Q^* are equivalent, (A.4) does not hold, and hence no-arbitrage relation among the M assets obtains. If the prices themselves of the M assets are all Martingales, they are of no arbitrage relation. In fact, the same proof holds for price processes themselves. But such a price as bank deposit $X_{0n} = \exp(rnh)$ where r is a fixed interest rate is not a Martingale under any equivalent measure, though relative prices may become a Martingale under some measure as we will see next.

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