

**IMPLICIT OPTIONS IN LIFE INSURANCE CONTRACTS**  
**The case of lump sum options in deferred annuity contracts**

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**Abstract**

Options in life insurance contracts are being offered more and more frequently. This is for reasons of competitiveness as well as (in particular under German legislation) for tax reasons. Up to now, such options were not taken into account when the policy was priced. In the present paper, we look at a specific example of an implicit option, the lump sum option in a deferred annuity contract. The underlying of this option is a portfolio of bonds. We quantify the value of the option within a Hull-White model framework and perform extensive sensitivity analysis.

**Keywords**

Implicit option, lump sum option, Hull-White model, bond option pricing.

# 1 Introduction

Options in life insurance contracts give the insured person the right to change some product features or to choose between some alternatives at certain times or events. Hence, the contract can be adapted to changing circumstances. Such flexible policies are of course more competitive. Furthermore, in Germany such implicit options are getting very popular for tax reasons as life insurance policies are only privileged with respect to taxes, if they have a term of at least 12 years. If the contract is changed, it is only privileged, if there is a term of 12 years both before and after the change. However, if the right to change the contract is included as an option, a total term of 12 years is sufficient.

As such options can have an extreme impact on future cash flows, they can bear financial risks, which up to now have not been taken into account at the pricing of the policy. Hence, if some insured persons exercise these options in an advantageous way, other insured persons will implicitly pay for it. This leads to a transfer of risk that is not desired. Therefore, such options should be taken into account when the policy is priced.

In [Ge 97], some implicit options have been priced using a very simple model. In particular, it was assumed, that the underlying of each option follows a geometric Brownian motion. This assumption is of course less than satisfactory, especially when the option is an interest rate derivative.

In the present paper, we look at a simple example of an implicit option, the so-called lump sum option in a deferred annuity contract. In [Ge 97], Gerdes has examined this option, as well, and found that its value can be substantial. The underlying of this option is the expected annuity payment, i.e. a portfolio of zero-coupon bonds. We use the widely used Hull-White model for our analysis. This is a one-factor no-arbitrage model for the short rate.

Our paper is organized as follows: In Section 2, we describe the insurance contract with the included lump sum option. In Section 3, we introduce the Hull-White model.<sup>1</sup> We

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<sup>1</sup>Cf. [Hu/Wh 90].

give closed form solutions for the prices of the included bonds and bond options. In Section 4, we calibrate our model to actual market data and give empirical results for a specific insurance contract. We furthermore perform extensive sensitivity analysis. Section 5 concludes with a summary and an outlook.

## 2 The Lump Sum Option

Many deferred annuities include a so-called lump sum option that can be exercised by the insured person at the end of the deferment period. Exercising the option means that the insured person chooses to receive a lump sum rather than a lifelong annuity payment.<sup>2</sup> If the insured person chooses the lump sum, this amount includes surplus that was created during the deferment period.

If we regard the expected annuity payment as a cash flow, the lump sum option is obviously equivalent to a European put option that gives the insured person the right to sell this cash flow for the lump sum (at the end of the deferment period). Usually, this option is not taken into account when the policy is priced. Hence, if some insured persons exercise this options in a profitable way, the resulting costs are carried by those insured persons who do not use the option to their advantage.

In what follows, we will quantify the value of such an option. We let  $t = 0$  denote the start of the policy and  $x$  the age of the insured person at  $t = 0$ . We assume the deferment period to be  $n > 0$  years, i.e. the first annuity payment is due at time  $t = n$ , the beginning of year  $n + 1$ .<sup>3</sup> We assume a lump sum of  $K^n$  to be paid at time  $n$  if the option is exercised. Also, we let  $R_j$  denote the annuity that is paid at time  $j$ . Hence, we

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<sup>2</sup>The annuity payment includes surplus. Usually, in Germany a guaranteed rate of interest of 4% and an additional surplus is earned on the net premiums during the deferment period as well as during the period of annuity payment.

<sup>3</sup>We assume all annuities to be paid in advance. In the case of payment in arrear, our results can be applied analogously by a simple adjustment of the indices. Furthermore, the annuity is of course only paid, if the insured person is still alive at time  $n$ .

get

$$v_n K^n = \sum_{j=n}^{\infty} R_j v_j {}_{j-n}p_{x+n}.$$

Here,  $v_k$  denotes the discount rate from time 0 to  $k$ , and  ${}_k p_x$  is the probability that an insured person aged  $x$  survives the next  $k$  years.<sup>4</sup> In case of a constant annuity, we have  $R_j \equiv R$ , and thus

$$R = \frac{v_n K^n}{\sum_{j=n}^{\infty} v_j {}_{j-n}p_{x+n}}.$$

If we furthermore assume  $v_j = v^j$ , we get

$$\begin{aligned} R &= \frac{K^n}{\sum_{j=n}^{\infty} v^{j-n} {}_{j-n}p_{x+n}} \\ &= \frac{K^n}{\sum_{j=0}^{\infty} v^j {}_j p_{x+n}}. \end{aligned} \tag{1}$$

Hence, for given  $K^n$ , we can determine  $R$  and thus the expected cash-flow. Usually,  $K^n$  is not guaranteed. It consists of a guaranteed part and a part resulting from surplus. However,  $K^n$  can be predicted rather well, because the surplus rates are very stable in Germany.

Letting  $L_j$  denote the expected annuity payment at time  $j$ , given that the insured person is still alive at time  $n$ , we get

$$L_j = R_j {}_{j-n}p_{x+n}, \quad \text{for } j = n, n+1, \dots$$

As mentioned above, the lump sum option is the right to sell the annuity for  $K^n$  at  $t = n$ . Hence, the strike of the option is  $K^n$ , the maturity date is  $t = n$ , and the underlying

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<sup>4</sup>In our empirical analysis, we use the mortality table of the German Society of Actuaries (DAV).

is the expected cash flow of the annuity.<sup>5</sup> This is equivalent to a coupon bond with annual coupon payment of  $L_j$  at time  $j$ . In [Hu/Wh 90] and [Ja 89], it was shown how the pricing of a European call option on a coupon bearing bond can be reduced to the pricing of a portfolio of call options on one zero-coupon bond each. We will now apply those ideas to the case of a put option. In what follows, it is crucial that bond prices are decreasing functions in the short rate  $r$ .<sup>6</sup>

We analyze the general case of a European put option with strike  $X$  and maturity  $T$  on a coupon bond that pays  $c_k$  at time  $s_k \geq T$ ,  $k = 1, \dots, m$ . Let  $B(r, t_1, t_2)$  denote the price at time  $t_1$  of a zero bond maturing at  $t_2$  as a function of the short rate  $r = r(t_1)$ . Then, we define  $r^*$  by

$$\sum_{k=1}^m c_k B(r^*, T, s_k) = X. \quad (2)$$

Hence, the option is exercised, whenever  $r(T) > r^*$ ,<sup>7</sup> since the payoff of the put option is given by

$$\max \left[ 0, X - \sum_{k=1}^m c_k B(r, T, s_k) \right]. \quad (3)$$

This is equal to

$$\sum_{k=1}^m c_k \max [0, X_{s_k} - B(r, T, s_k)], \quad (4)$$

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<sup>5</sup>From the insured person's point of view, the underlying is, of course, the actual annuity payment. However, if the portfolio of policies is not too small, it makes sense from the insurance company's point of view, to regard the expected cash flow as the underlying of the option.

<sup>6</sup>Hence, the following arguments can in particular be applied to all one-factor models for the short rate.

<sup>7</sup>Here, we assume that the investor acts rationally, meaning that he exercises the option if and only if its value is positive. In reality, the decision of an insured person to exercise the lump sum option will additionally depend on other factors. Hence, the values derived within our model are upper bounds for the real value of the lump sum option.

with

$$X_{s_k} = B(r^*, T, s_k). \quad (5)$$

Obviously, (4) is the payoff of a portfolio of  $m$  put options, each on a zero bond with strike  $X_{s_k}$  and maturity  $s_k$ . Hence, the price of a put option on a coupon bond equals the price of a portfolio of put options, each on one zero bond.

We will now apply these ideas to the described lump sum option: The coupons are paid at  $t = n, n + 1, \dots$ , paying  $L_j$  at time  $j$ . The maturity of the put option is  $t = n$  and the strike is  $K^n$ .<sup>8</sup> However, the option can only be exercised, if the insured person is still alive at time  $n$ . This is considered in (6).

For  $j \geq n$ , we now let  $V_j$  denote the price at time 0 of a European put option with maturity  $n$  and payoff  $\max\{0, X_j - B(r, n, j)\}$ . Therefore, the price of the lump sum option is given by

$$p = {}_n p_x \sum_{j=n}^{\infty} L_j V_j. \quad (6)$$

To determine the prices  $V_j$ , we need a model for the economy. In the next section, we introduce the Hull-White model and derive explicit pricing formulas for our options.

### 3 The Hull and White Model

The Hull-White model is a no-arbitrage model and a generalization of both the Vasicek model (cf. [Va 77]) and the Ho-Lee model (cf. [Ho/Le 86]).

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<sup>8</sup>Hence, in (2), (3), (4), and (5) we have to replace  $c_k$  by  $L_{n+k-1}$ ,  $s_k$  by  $n+k-1$ ,  $T$  by  $n$ , and  $X$  by  $K^n$ .

Hull and White assume the short rate to follow the process

$$\begin{aligned} dr(t) &= (\theta(t) - ar(t))dt + \sigma dz(t) \\ &= a \left[ \frac{\theta(t)}{a} - r(t) \right] dt + \sigma dz(t). \end{aligned} \tag{7}$$

Here,  $a$  and  $\sigma$  are some constants  $> 0$ .<sup>9</sup>

The Hull-White model includes mean reversion: At time  $t$ , the short rate reverts to  $\frac{\theta(t)}{a}$  with mean reversion rate  $a$ . If we let  $a = 0$ , we get the Ho-Lee model, if  $\theta$  is constant, we get the Vasicek model.

The model fits any given term structure if we let

$$\theta(t) = F_t(0, t) + aF(0, t) + \frac{\sigma^2}{2a}(1 - e^{-2at}).^{10} \tag{8}$$

Usually, the last term of this equation is rather small. Hence the drift of the process is approximately  $F_t(0, t) + a[F(0, t) - r(t)]$ , implying that on average,  $r$  approximately follows the slope of the initially given curve of instantaneous forward rates. When it moves away from that curve, it reverts back at rate  $a$ .<sup>11</sup>

Within this model, the bond prices  $B(r, t, T)$  are given by (cf. [Hu 97]):

$$B(r, t, T) = A(t, T) e^{-C(t, T)r(t)} \tag{9}$$

<sup>9</sup>For all that follows, we take a filtered probability space  $(\Omega, \Sigma, P)$  with a filtration  $\mathcal{F}_t$  as a basis. We furthermore assume a complete and arbitrage-free market. This is essentially equivalent to the unique existence of a so-called equivalent martingale measure  $Q$ . We assume the process in (7) to be the so-called risk neutral process, i.e. the process under  $Q$ . Here,  $z(t)$  denotes an adapted Wiener process under  $Q$ . For a detailed overview, cf. e.g. [Du 96]. We furthermore assume the financial markets to be independent of mortality, and the insurance company to be risk neutral with respect to mortality. For a detailed overview of these aspects, cf. e.g. [Aa/Pe 94].

<sup>10</sup>Here, we denote by  $F(t, t_1, t_2)$  the forward rate at time  $t$  for the period of time  $[t_1, t_2]$ . The so-called instantaneous forward rate  $F(t, t_1)$  at time  $t$  for  $t_1$  is then given by  $F(t, t_1) = \lim_{t_2 \rightarrow t_1} F(t, t_1, t_2) = \lim_{t_2 \rightarrow t_1} \frac{\ln B(t, t_1) - \ln B(t, t_2)}{t_2 - t_1}$ . Furthermore,  $F_t$  denotes the partial derivative of  $F(t_1, t_2)$  with respect to  $t_2$ .

<sup>11</sup>Cf. [Hu 97].

with

$$C(t, T) = \frac{1 - e^{-a(T-t)}}{a} \quad (10)$$

and

$$\begin{aligned} \ln A(t, T) = \ln \frac{B(r, 0, T)}{B(r, 0, t)} - C(t, T) \frac{\partial \ln B(r, 0, t)}{\partial t} \\ - \frac{1}{4a^3} \sigma^2 (e^{-aT} - e^{-at})^2 (e^{2at} - 1). \end{aligned} \quad (11)$$

Hence, for given  $r(t)$ , bond prices at time  $t$  can be determined from (9), (10), and (11) using today's bond prices. The partial derivative  $\frac{\partial \ln B(0, t)}{\partial t}$  that is needed for the calculation of  $A(t, T)$  in (11) can e.g. be approximated by

$$\frac{\ln B(r, 0, t + \delta) - \ln B(r, 0, t - \delta)}{2\delta}, \quad (12)$$

for some small length  $\delta > 0$ .

If we assume  $\theta(t) \equiv \theta$ ,  $A(t, T)$  is given by

$$A(t, T) = \exp \left[ \frac{(C(t, T) - T + t) \left( a \theta - \frac{\sigma^2}{2} \right)}{a^2} - \frac{\sigma^2 C(t, T)^2}{4a} \right].$$

The price at time  $t$  of a European call option with maturity  $T$  on a zero bond maturing at  $S$  ( $t \leq T \leq S$ ) is given by<sup>12</sup>

$$B(r, t, S)N(h) - X B(r, t, T)N(h - \sigma_B), \quad (13)$$

where  $X$  denotes the strike price of the option. Furthermore,  $h$  and  $\sigma_B$  are given by

$$h = \frac{1}{\sigma_B} \ln \frac{B(r, t, S)}{X B(r, t, T)} + \frac{\sigma_B}{2}$$

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<sup>12</sup>Cf. [Hu/Wh 90]. Here, we only give the result for an option on a bond with face value 1, as this is sufficient for our analysis, cf. (4).



and

$$\begin{aligned}\sigma_B &= \left( \int_t^T \sigma^2 [C(\tau, S) - C(\tau, T)]^2 d\tau \right)^{\frac{1}{2}} \\ &= \frac{\sigma}{a} \sqrt{\frac{1}{2a} \left[ (1 - e^{-a(S-T)})^2 - (e^{-a(T-t)} - e^{-a(S-t)})^2 \right]}.\end{aligned}$$

In particular, for  $t = 0$  we get

$$\begin{aligned}\sigma_B &= \frac{\sigma}{a} \left[ 1 - e^{-a(S-T)} \right] \sqrt{\frac{1 - e^{-2aT}}{2a}} \\ &= \sigma C(T, S) \sqrt{\frac{1}{2} C(0, T)}.\end{aligned}$$

The price of the corresponding put option on that bond is then given by

$$X B(r, t, T) N(-h + \sigma_B) - B(r, t, S) N(-h). \quad (14)$$

Applying these ideas to the pricing of the lump sum option, we get  $V_j$  as the price at time 0 of a European put option with strike  $X_j$  and maturity  $n$  on a zero bond maturing at  $j$ . Hence, it follows from (14) that

$$V_j = X_j B(r, 0, n) N(-h_j + \sigma_B^j) - B(r, 0, j) N(-h_j),$$

with

$$h_j = \frac{1}{\sigma_B^j} \ln \frac{B(r, 0, j)}{X_j B(r, 0, n)} + \frac{\sigma_B^j}{2},$$

and

$$\begin{aligned}\sigma_B^j &= \frac{\sigma}{a} \left[ 1 - e^{-a(j-n)} \right] \sqrt{\frac{1 - e^{-2an}}{2a}} \\ &= \sigma C(n, j) \sqrt{\frac{1}{2} C(0, n)}.\end{aligned}$$

## 4 Empirical Results

For our following analysis, we use market data from June 24, 1998. In particular, the term structure of interest rates is given by the prices  $B(0, t)$  of discount bonds. Some of them are given in Table 1.<sup>13</sup>

$t$	0.5	1	2	3	4
$B(0, t)$	0.98232	0.96345	0.92316	0.88269	0.84275
$t$	5	6	7	8	9
$B(0, t)$	0.80251	0.76166	0.72510	0.68908	0.65485
$t$	10	15	20	25	30
$B(0, t)$	0.62453	0.47465	0.35320	0.25911	0.19563

Table 1: Discount bond prices from June 24, 1998 ( $t$  in years)

The parameters of the Hull-White model were given by  $a = 0.0001$  and  $\sigma = 0.6306\%$ .<sup>14</sup> We furthermore assume the policy to be defined as follows: The insured person is male and aged  $x$  years. The deferment period is  $n$  years. The guaranteed rate of interest on the net premium is 4%. An additional surplus of  $u_1$  and  $u_2$  is paid during the deferment period and during the annuity payment, respectively.<sup>15</sup>

The value of the lump sum option for different values of  $x$ ,  $n$ , and  $u = u_1 = u_2$  is given in Table 2. We assumed the insured person to pay a single premium of 100,000 DM at  $t = 0$ .<sup>16</sup> For the sake of simplicity, we did not allow for any costs.

The values of Table 2 for  $u = 3.5\%$  are shown in Figure 1.

The option price is higher for younger insured persons. This results from the fact that

<sup>13</sup>Any bond price  $B(0, t^*)$  that was needed but not given was derived by interpolation: We calculated the corresponding spot rates from the neighboring bond prices  $B(0, t_1)$  and  $B(0, t_2)$ ,  $t_1 < t^* < t_2$ . The spot rate for  $t^*$  was then derived by linear interpolation and the bond price  $B(0, t^*)$  was calculated from this spot rate. Extrapolations were performed analogously.

<sup>14</sup>The calibration was done using a 2-year and a 5-year at the money cap.

<sup>15</sup>Hence, in (1), we let  $v_j = 4\% + u_2$ .

<sup>16</sup>Hence,  $K^n = 100,000(1.04 + u_1)^n$ .

	$n = 5$			$n = 10$		
	$x = 20$	$x = 40$	$x = 60$	$x = 20$	$x = 40$	$x = 60$
$u = 2.0$	5465.95	4941.27	3417.38	9915.33	8655.09	5726.85
$u = 2.5$	3319.80	3020.77	2078.47	7483.18	6566.44	4349.74
$u = 3.0$	1874.02	1711.45	1171.87	5461.64	4816.02	3192.82
$u = 3.5$	979.39	897.03	610.54	3849.85	3409.20	2261.38
$u = 4.0$	472.12	433.20	292.91	2617.17	2325.86	1543.37
	$n = 20$			$n = 30$		
	$x = 20$	$x = 40$	$x = 60$	$x = 20$	$x = 40$	$x = 60$
$u = 2.0$	12787.00	10169.70	5294.96	9645.29	5927.35	1397.41
$u = 2.5$	11204.64	8965.36	4697.21	8926.14	5518.48	1300.85
$u = 3.0$	9661.36	7777.40	4102.03	8222.97	5075.18	1196.52
$u = 3.5$	8194.93	6637.01	3524.37	7449.20	4609.88	1087.29
$u = 4.0$	6835.78	5568.50	2977.81	6662.20	4135.12	975.87

Table 2: Value of the lump sum option in DM ( $x$ ,  $T$  in years,  $u$  in %)

the probability to survive the deferment period is smaller for older persons, cf. (6). A longer deferment period  $n$ , in general increases the values  $V_j$  in (6). However, the  ${}_n p_x$  are decreasing in  $n$ . The latter effect is the stronger, the older the insured person. Thus, the option value is first increasing and then decreasing in  $n$ , cf. Figure 1.

Furthermore, the value of the option depends heavily on  $u$ . There are, however, two contrary effects: On the one hand, a high rate of surplus during the deferment period leads to a higher value of  $K^n$ , and hence to a higher annuity payment. Thus, the value of the option increases proportionally. On the other hand, a high surplus rate during the time of annuity payment leads to a higher annuity compared to  $K^n$ , and therefore decreases the value of the option. Table 3 shows the value of the lump sum option for  $x = 40$  and  $n = 20$  for different values of  $u_1$  and  $u_2$ . Here, we can see as expected, that the value is increasing in  $u_1$  and decreasing in  $u_2$ . Figure 2 visualizes these effects.

Table 4 shows the value of the lump sum option for  $x = 40$ ,  $T = 20$ , and  $u = 3.5\%$  for different market scenarios. We performed interest rate and volatility shifts of  $\Delta r^{17}$  and  $\Delta \sigma$ , respectively. The values are also shown in Figure 3.

<sup>17</sup>All interest rate shifts were performed by shifting the spot rates, cf. footnote 13.

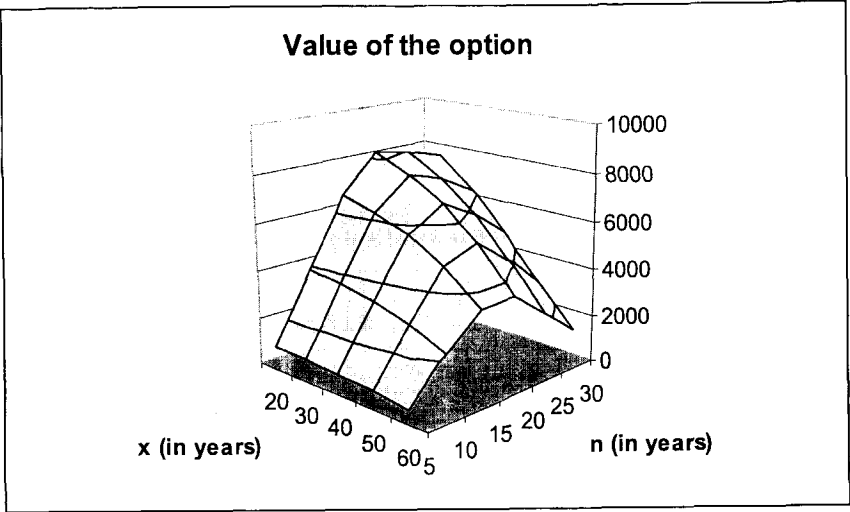


Figure 1: Value of the lump sum option as a function of  $x$  and  $n$  for  $u = 3.5\%$

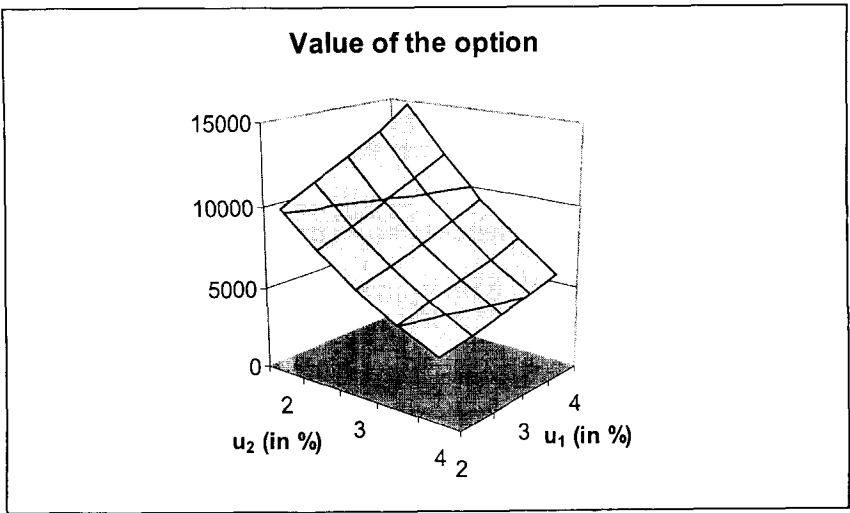


Figure 2: Value of the lump sum option for different values of  $u_1$  and  $u_2$

	$u_2 = 2.0$	$u_2 = 2.5$	$u_2 = 3.0$	$u_2 = 3.5$	$u_2 = 4.0$
$u_1 = 2.0$	10169.70	8160.05	6445.79	5010.95	3831.60
$u_1 = 2.5$	11173.34	8965.36	7081.92	5505.48	4209.74
$u_1 = 3.0$	12270.62	9845.80	7777.40	6046.15	4623.16
$u_1 = 3.5$	13469.77	10807.99	8537.45	6637.01	5074.96
$u_1 = 4.0$	14779.72	11859.07	9367.72	7282.46	5568.50

Table 3: Value of the lump sum option in DM for different values of  $u_1$  and  $u_2$  (in %)

	$\Delta\sigma = -0.4$	$\Delta\sigma = -0.2$	$\Delta\sigma = 0$	$\Delta\sigma = 0.2$	$\Delta\sigma = 0.4$
$\Delta r = -2$	0.67	454.75	3028.02	7667.01	13527.35
$\Delta r = -1.5$	3.97	767.34	3810.09	8561.83	14193.14
$\Delta r = -1$	19.12	1227.15	4685.67	9444.33	14780.54
$\Delta r = -0.5$	74.96	1863.46	5636.46	10294.79	15281.14
$\Delta r = 0$	241.09	2692.14	6636.65	11093.49	15687.99
$\Delta r = 0.5$	641.32	3707.06	7654.74	11820.23	15994.28
$\Delta r = 1$	1427.88	4878.19	8656.08	12459.86	16198.77
$\Delta r = 1.5$	2697.96	6150.31	9605.33	12998.38	16300.42
$\Delta r = 2$	4401.15	7450.84	10469.29	13425.70	16300.85

Table 4: Value of the lump sum option in DM for different market scenarios ( $\Delta r$ ,  $\Delta\sigma$  in %)

As expected, the value of the option is increasing in  $\sigma$ . Furthermore, it usually increases in  $\Delta r$ , as the underlying becomes cheaper for higher levels of interest rates.<sup>18</sup>

Our results show, that the value of the option can be substantial. Hence, it should be taken into account in the pricing of the policy.

## 5 Summary and Outlook

In the present paper, we quantified the value of the lump sum option in deferred annuity contracts. We have shown that the value of this option can be substantial. Therefore, the option has to be considered in the pricing of the policy.

As options in insurance contracts will become more and more important in the future,

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<sup>18</sup>Furthermore, the value is slightly decreasing in  $a$ , but as this effect is rather small, we do not quote any values.

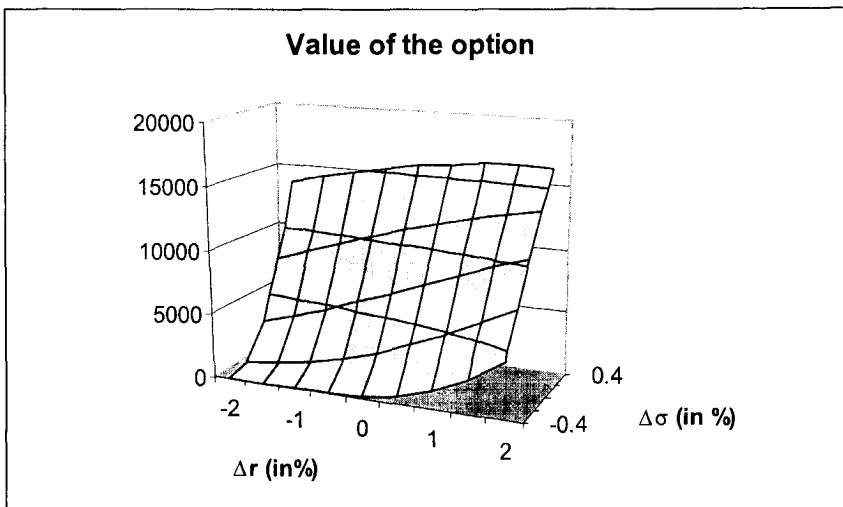


Figure 3: Value of the lump sum option for different market scenarios

a systematic classification and analysis of such implicit options is required. The concept that was used in the present paper could be applied to other interest rate sensitive options as well. However, for more complicated options no explicit pricing formulas exist and numerical methods are required.

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