On Empirical Heteroskedastic Properties of Japanese Stock Price Changes

CHU Wen-Tseng
Doctoral Program in Policy and Planning Sciences, University of Tsukuba
Tennoudai 1-1-1, Tsukuba, Ibaraki, 305-8573 Japan
Telephone(& FAX): 0298-51-9498
E-mail: chuwen@shakosv.sk.tsukuba.ac.jp

KISHIMOTO Kazuo
Institute of Policy and Planning Sciences, University of Tsukuba
Tennoudai 1-1-1, Tsukuba, Ibaraki, 305-8573 Japan
Telephone: 0298-53-5085
E-mail: kishimot@shako.sk.tsukuba.ac.jp

Abstract

By estimating the volatility persistence of indivisual stock returns in the Tokyo Stock Exchange, we found that the estimated persistence negatively depends on the length of sample period. The computer simulation results indicate that this seeming decay of persistence is quantitively consistent with the assumption of stationarity if we take both the small sample effect and the 'model misspecification effect' into account. We also showed that a similar phenomenon observed in Lamoureux and Lastrapes (1990) is also explained without introducing structural changes through vast and thourough simulations.

Keywords : GARCH, stochastic volatility, volatility persistence

On Empirical Heteroskedastic Properties of Japanese Stock Price Changes

CHU Wen-Tseng
(Doctoral Program in Policy and Planning Sciences, University of Tsukuba)
KISHIMOTO Kazuo
(Institute of Policy and Planning Sciences, University of Tsukuba)

Keywords : GARCH, stochastic volatility, volatility persistence

1 Introduction

The temporal behavior of stock market volatility has undergone an extensive empirical investigation, it is generally found that volatility is highly persistent. In particular for daily data one may find evidence of near unit root behavior of the conditional variance process, as has been surveyed in Bolloerslev, Chou, and Kroner(1992), Bollerslev, Engle, and Nelson(1994), Diebold and Lopez(1995) etc. Models such as the autoregressive conditional heteroskedasticity (ARCH) model (Engle, 1982) and the generalized autoregressive conditional heteroskedasticity (GARCH) model (Bollerslev, 1986), have proven to be useful tools for capturing this phenomenon. Engle and Bollerslev(1986), on estimating a GARCH(1, 1) model for weekly exchange rate of U.S. dollar vis-a-vis Swiss Franc, found estimated parameter measuring persistence in variance is close to one, providing a motivation for building the Integrated GARCH(1, 1) model. Nelson(1990) also found evidence of persistence using an EGARCH formulation. Likewise, estimation of stochastic volatility(SV) models shows similar patterns of persistence. (See, e.g., Jacqier et al. 1994 and Shephard, 1996.).

In the present paper we first report estimated parameter values for sequences of stock returns for 160 actively traded stocks included in the Nikkei 225 index of the Tokyo Stock Exchange. By changing the subperiod under consideration, we find that for most stocks, the parameter values suggest strong persistence for long subperiods while it shows less strong persistence for shorter subperiods.

How can we explain this phenomenon? By computer simulation, one can check that the estimated values have downward bias for persistence parameters for shorter subperiods. Hong(1987) reported this phenomenon for IGARCH(1, 1) and Lumsdaine(1995) did for GARCH(1, 1). This downward bias is, however not large enough to explain fully the above decay of persistence observed in the actual data of small samples.

The finding in this paper is that a substantial decay of estimated persistence is observed if we fit the GARCH(1, 1) model for the time series generated by the SV or EGARCH(1, 1) models, and vis-a-vis. This fact suggests that a large proportion of reduction of persistence of small samples may come from fitting an inadequate model to stock returns.

A similar empirical phenomenon is observed in Lamoureux and Lastrapes (1990)(hereafter, abbreviated to LL) who added exogenous variables to the variance equation of the GARCH(1, 1) model . They suggested a modified GARCH(1, 1) model in which the volatility level varies on different predetermined subperiods by introducing structural jumps. They found that such jumps of volatility cause substantial drop of the estimated persistence. The longer the sample period, the higher the probability that structural shifts will be present. They concluded that the high degree of persistence in GARCH models might be due to the misspecification of the variance equation. This interprets why persistence of small-samples becomes much less pronounced.

The finding by our computer simulation suggests that their methodology may not be the only approach on this problem. Thus, another Monte Carlo experiment along the LL was conducted with the same sample size(4,228).

If we fit the LL type GARCH(1, 1) model to the data generated by the genuine GARCH(1, 1) model, the fitted parameters decrease for about 6%. When fitting the LL type GARCH(1, 1) model to another genuine heteroskedastic models, the downward bias becomes obviously larger. The experiment is plausible, because fitting the LL type GARCH(1, 1) to any time series gets the downside bias of estimation unexceptionally. Thus it seems more theoretical and empirical evidence is needed for the LL setting.

The rest of this paper begins in section 2 with the empirical investigation of the stock returns of the Tokyo Stock Exchange. In section 3, the Monte Carlo experiments are conducted. Some concluding remarks are contained in section 4.

2 Data and empirical results

2.1 Data and Estimation

The data were obtained from the Nikkei Historical Stock Price NM-A provided by the Nikkei Databank Bureau. We first take up all stocks enrolled in Nikkei 225 index, and then discard the stocks whose nontraded days are equal or more than 20 days during the period from January 4, 1977 through December 29, 1995. The monthly, weekly, and daily returns are calculated by the difference of the logarithms of closing prices (adjusted by stock dividends and stock splits) of months, weeks, and days over all the sample periods. The monthly and weekly returns are calculated and estimated for the whole period only. The daily returns are calculated and estimated for the whole period and about 9-year, 5-year, 3-year, 2-year subperiods. The sample sizes and the partitioning of the data are reported in Table 1.

GARCH(1, 1) , SV, and EGARCH(1, 1) time series models are chosen to fit the stock returns in this study.

GARCH(1, 1) Model (Bollerslev, 1986) is described as follows:

$$\begin{aligned} y_t &= \epsilon_t = \nu_t\sqrt{h_t} \quad (t = 1, 2, \dots, T) && (1) \\ h_t &= \alpha_0 + \alpha_1\epsilon_{t-1}^2 + \beta_1 h_{t-1}, && (2) \\ \alpha_0 &> 0, \quad \alpha_1 \geq 0, \quad \beta_1 \geq 0, \\ \alpha_1 &+ \beta_1 < 1, && (3) \end{aligned}$$

where y_t denotes the stock returns, ν_t is independent and identically distributed (iid) with mean zero and variance one. T represents the sample size, h_t is a time-varying, positive, and measurable function of the time $t-1$ information set, and $\alpha_1 + \beta_1 < 1$ to ensure y_t satisfies covariance-stationarity. Nelson(1990) showed that if $\mathrm{E}[\log(\beta_1 + \alpha_1\nu_t^2)] < 1$ and $\alpha_0 > 0$, y_t is strictly stationary even the fourth moment does not exist.

SV Model (Jacquier et al., 1994) is described as follows:

$$\begin{aligned} y_t &= \epsilon_t = \nu_t\sqrt{h_t} \quad (t = 1, 2, \dots, T) \\ \log h_t &= \gamma + \phi \log h_{t-1} + \sigma_\eta \eta_t, && (4) \end{aligned}$$

where $\eta_t \sim NID(0, 1)$ is assumed to be independent of ν_t. In SV model, the log-volatility $\log h_t$ unlike the GARCH(1, 1) case, is not explicitly observed, so must be estimated using the observations. If $|\phi| < 1$, $\log h_t$ is strictly stationary, for y_t is the product of two stationary processes, it must also be strictly stationary.

EGARCH(1, 1) Model (Nelson, 1991) is described as follows:

$$\begin{aligned} y_t &= \epsilon_t = \nu_t\sqrt{h_t} \quad (t = 1, 2, \dots, T) \\ \log h_t &= \omega + \beta \log h_{t-1} + \theta\{\frac{|\epsilon_{t-1}|}{\sqrt{h_{t-1}}} - \sqrt{2/\pi} + \delta\frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}}\}. && (5) \end{aligned}$$

The parameters in (5) need not be restricted to be positive, and the existence of strict and covariance stationary crucially depends on the distribution of $\epsilon_{t-1}/\sqrt{h_{t-1}}$, which is a rather unappealing property of EGARCH.

In the present paper, the estimation of the GARCH(1, 1) and EGARCH(1, 1) is performed by the method of maximum likelihood using the BHHH(1974) numerical optimization algorithm. For the SV model, the estimation is performed by the quasi-maximum likelihood method (QML), computing using the Kalman filter advocated by Harvey et al.(1994) and later modified by Breidt(1996).

2.2 Results

Persistence of shocks to volatility decays at a constant rate and the speed of decay is measured by the estimate of $\alpha_1 + \beta_1$ for GARCH(1, 1) model, ϕ for SV model, and β for EGARCH(1, 1) model.

The estimated results are given in Table 1. By the way, Engle and Gonzalez-Rivera(1991), and Engle and Mustafa(1992) reported that the persistence in variance seems to be related to the size of the firm, with small firms having a lower persistence than the larger firms. However, we here calculate ignoring the firm size effect.

For daily returns one will see the followings.

1. For the full period, and the 9-year subperiod, the estimated parameters are close to one except for $\hat{\beta}$ for EGARCH of the 77-85 subperiod. The degree of persistence is very high.

2. The following inequalities are observed: $\hat{\alpha_1} + \hat{\beta}_1 > \hat{\beta} > \hat{\phi}$ except for the full period case.

3. In the case of 4-year, 3-year, and 2-year subperiods, the persistence generally decreases as the periods grow shorter. In most cases, the estimated parameters are no more so close to one, particularly for the SV model.

4. Inequalities $\hat{\alpha_1} + \hat{\beta}_1 > \hat{\beta} > \hat{\phi}$ are observed except merely for the 90-91 case.

For monthly and weekly returns, we observe the following.

1. The persistence of the monthly returns and weekly returns is generally lower than that of the daily returns. This is consistent with many reports up to now (See Bolloerslev, Chou, and Kroner(1992) for reference.).

2. Except for few exceptions, $\alpha_1 + \beta_1 > 1$ holds.

3. $\hat{\phi}$ in the SV model is lower than the corresponding $\hat{\beta}$ in the EGARCH model, and $\hat{\alpha_1} + \hat{\beta}_1$ in the GARCH model. The estimated parameters are found to have an interesting stylized fact : $\hat{\alpha_1} + \hat{\beta}_1 > \hat{\beta} > \hat{\phi}$.

In the next section we conduct Monte Carlo simulations to try to explain the above empirical results.

3 Monte Carlo Simulations

3.1 Experiment 1

Artificial time series are generated based on the GARCH(1, 1) , SV, and EGARCH(1, 1) models which are defined in the previous section. We take up the cases of $T = 200$, $T = 500$ (10,000 replications) and $T = 5,000$ (1,000 replications). When generating the artificial series, the first 1000 values are discarded and then next T values are taken to form the time series. These samples were chosen approximately to be representative of the sizes of monthly, weekly, daily returns of this study. For each time series, the three types parameter fittings are conducted. The average and the standard deviation of the estimated persistence parameters are given in Table 2. We observe the followings.

1. When we estimate the parameters based on the correct model fitting, we have the following results.

(a) When $T = 200$, the estimated parameters are clearly downward biased about 6% for GARCH(1, 1) , 13% for SV, and 6% for EGARCH(1, 1) .
When $T = 500$, the biases are shrinked to about 2% for GARCH(1, 1) , 6% for SV, and 1% for EGARCH(1, 1) .
When $T = 5,000$, the biases are small enough to be unrecognized.

(b) Even if we use the downward biases calculated from Table 2 to up-correct the results in Table 1, the estimated persistence parameters of shorter subperiods remain to be smaller than those of longer subperiods.

2. When we estimate the parameters based on the wrong model fittings, we have the following results.

(a) When we estimate the parameters assuming that the target model follows GARCH(1, 1) , the downward bias in $\hat{\alpha}_1 + \hat{\beta}_1$ decreases in the following order: SV-GARCH(1, 1) > GARCH(1, 1) -GARCH(1, 1) > EGARCH(1, 1) -GARCH(1, 1) for $T = 200$ and $T = 500$. Some cases in EGARCH(1, 1) -GARCH(1, 1) even result in upward bias.

(b) When we estimate the parameters assuming that the target model follows SV, the downward bias in $\hat{\alpha}_1 + \hat{\beta}_1$ decreases in the following order: GARCH(1, 1) -SV > EGARCH(1, 1) -SV > SV-SV for $T = 200$, and $T = 500$.

(c) When we estimate the parameters assuming that the target model follows EGARCH(1, 1) , the downward bias in $\hat{\beta}$ decreases in the following order: SV-EGARCH(1, 1) > GARCH(1, 1) -EGARCH(1, 1) > EGARCH(1, 1) -EGARCH(1, 1) for $T = 200$ and $T = 500$.

(d) In general, we found when the estimation is conducted based on the wrong model, the downward bias is more remarkable.

(e) Again, if we multiply the downward rates to the values of the 2-year and 3-years' cases listed in Table 1 for each models respectively for correction, the majorities of the cases can not be corrected to be close to that of 9-year and the full periods' cases satisfactorily, and worse than the previous correct models' case.

(f) When $T = 5,000$, the downward biases almost vanish even if we estimate the parameters based on the wrong models. It means that the misspecification does not matter if the sample period is long enough.

Thus we concluded that the decline of persistence parameters in Table 1 should be attributed to the downward bias in the small sample data. A large proportion of the decline comes from the estimation based on wrong models (i.e., model misspecification).

3.2 Experiment 2

Our results up to now urges us to the rechecking of the LL's conclusion. Thus, we perform the similar experiment in the setting of the LL. That is, we take $T =$

4280 and estimate parameters by assuming that the target series is a realization of GARCH(1, 1) model including dummy variables:

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1} + \delta_1 D_{1t} + \ldots + \delta_k D_{kt}, \tag{6}$$

where $D_{it}(i = 1, \ldots, k)$ are dummy variables that correspond to periods over which the GARCH process is stationary. For observations ranging from $(1 + 302i)$ to $[(1 + i)302]$, $D_{it}(i = 1, \ldots, 13)$ is 1, 0 otherwise. The full sample contains 4,228 observations; thus the dummy variables allow for 14 nonoverlapping subsamples.

The LL advocates there is high probability that deterministic, or structural shifts would occur in the conditional variance of the stochastic process particularly for the long period. Such shifts, if unaccounted, may cause upward bias in GARCH estimates of persistence in variance. After using the model (2) and (6) to estimate 30 daily stocks returns selected from the CRSP, he found the average of $\hat{\alpha}_1 + \hat{\beta}_1$ estimated by (2) is 0.978, but declines to 0.817 when (6) is applied. So it might be misleading to take current evidence of IGARCH, or in general evidence of strong persistence, at face value.

As shown in Table 3, $\hat{\alpha}_1 + \hat{\beta}_1$ declines in most of the cases as the LL discovered. All the three data generating processes (DGPs) have high persistence, but when applying (6) to them, the estimated results will be not so high. Therefore, the LL's methodology may not be in effect. The decline in persistence may be due to the downward bias in estimation discussed in the above section, and is just a common phenomenon. Indeed, it really needs both theoretical and empirical investigation more thoroughly.

In fact, the diagnostic check on the standardized residuals of model (6) does not substantially improve, especially on the nonnormality problem as noted by the LL themselves. A recent paper by Choudhry(1996) also argues that when applying this method to the monthly data or small size subsamples, the results obtained were empirically weak and meaningless.

4 Conclusions

We have found the persistence parameters in GARCH(1, 1), SV, and EGARCH(1, 1) are downward biased for small sample data. This bias is particularly large if we conduct the parameter estimation based on a wrong time series model. Thus, strong persistence of the conditional variances of individual stock returns is consistent with our results. These results cast a doubt to the conclusion in Lamoureux and Lastrapes(1990).

References

[1] Berndt, E.R., B.H.Hall, R.E.Hall, and J.A. Hausman(1974), "Estimation and Inference in Nonlinear Structural Models," *Annals of Economic and Social Measurement*, Vol.3, pp.653-665.

[2] Bollerslev,T.(1986), "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics*, Vol.51, pp.307-327.

[3] Bollerslev,T., Y.C. Chouand K.Kroner(1992), "ARCH Modelling in Finance: A Review of the Theory and Empirical Evidence," *Journal of Econometrics*, Vol.52, pp.5-59.

[4] Bollerslev,T., R. Engle and D. Nelson(1994), "ARCH Models," in R.F. Engle and D. McFadden eds., *Handbook of Econometrics,* Volume IV. North-Holland, Amsterdam.

[5] Breidt, F.J. and A.L. Carriquiry(1996), "Improved Quasi-Maximum Likelihood Estimation for Stochastic Volatility Models," in J.C. Lee, W.O. Johnson, and A. Zellner eds., *Modelling and Prediction,* Sringer-Verlag, N.Y.

[6] Choudhry, Taufiq(1996), "Stock Market Volatility and the Crash of 1987: Evidence from Six Emerging Markets," *Journal of International Money and Finance*, Vol.15, pp.969-981.

[7] Diebold, F.X. and J.A. Lopez(1995), "Modelling Volatility Dynamics," in K. Hoover ed., *Macroeconomics: Developments, Tensions and Prospects.*

[8] Engle, R.F.(1982), "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation," *Econometrica*, Vol.50, pp.987-1008.

[9] Engle, R.F. and T. Bollerslev(1986), "Modelling the Persistence of Conditional Variances," *Econometric Review*, Vol.5, pp.1-50.

[10] Engle, R.F. and G. Gonzalez-Rivera(1991), "Semiparametric ARCH Models," *Journal of Business & Economic Statistics*, Vol.9, pp.345-359.

[11] Engle, R.F. and C. Mustafa(1992), "Implied ARCH models from option prices," *Journal of Econometrics*, Vol.52, pp.289-311.

[12] Harvey, A., Ruiz, E., and N. Shephard(1994), "Multivariate Stochastic Variance Models," *Review of Economic Studies*, Vol.61, pp.247-264.

[13] Hong, C.H.(1987), "The Integrated Generalized Autoregressive Conditional Heteroskedasticity Models: The Process, Estimation and Some Monte Carlo Experiments," unpublished manuscript, University of California, San Diego, Dept. of Economics.

[14] Jacquier, E., Polson, N., and P.E. Rossi(1994), "Bayesian Analysis of Stochastic Volatility Models," *Journal of Business & Economic Statistics*, Vol.12, pp.371-389.

[15] Lamoureux, G. Christopher and William D. Lastrapes(1990), "Persistence in Variance, Structural Change, and the GARCH Model," *Journal of Business & Economic Statistics*, Vol.8, pp.225-234.

[16] Lumsdaine, R.L.(1995), "Finite-Sample Properties of the Maximum Likelihood Estimator in GARCH(1, 1) and IGARCH(1, 1) Models: A Monte Carlo Investigation," *Journal of Business & Economic Statistics*, Vol.13, pp.1-10.

[17] Nelson, D.B.(1990), "Stationarity and Persistence in the GARCH(1, 1) Model," *Econometric Theory*, Vol.6, pp.318-334.

[18] Nelson, D.B.(1991), "Conditional Hereroskedasticity in Asset Returns: A New Approach," *Econometrica*, Vol.59, pp.347-370.

[19] Shephard, N.(1996), "Statistical Aspects of ARCH and Stochastic Volatility," in D.R.Cox and D.V. Hinkley eds., *Time Series Models,* Chapman & Hall, London.

Table 1: Estimates of Volatility Persistence of Stock Returns of the TSE

Data Types	Period	T	Averaged Estimated Parameters' Values and Standard Deviations[a]			Numbers of Stocks with Following Relationships		
			$\hat{\alpha}_1 + \hat{\beta}_1$	$\hat{\phi}$	$\hat{\beta}$	$\hat{\alpha}_1 + \hat{\beta}_1 > 1$	$\hat{\phi} > 1$	$\hat{\beta} > 1$
Daily Returns	77-79	873-878	0.837 (0.291)	0.518 (0.277)	0.740 (0.215)	2	0	0
	80-81	511-574	0.838 (0.212)	0.487 (0.277)	0.778 (0.212)	6	0	0
	82-83	//	0.803 (0.199)	0.229 (0.268)	0.745 (0.184)	4	0	3
	84-85	//	0.782 (0.244)	0.328 (0.307)	0.760 (0.203)	4	1	1
	86-87	//	0.766 (0.202)	0.124 (0.229)	0.692 (0.177)	4	0	0
	88-89	//	0.757 (0.286)	0.397 (0.199)	0.733 (0.242)	1	0	0
	90-91	//	0.902 (0.121)	0.250 (0.367)	0.916 (0.118)	1	0	0
	92-93	//	0.899 (0.178)	0.308 (0.291)	0.865 (0.186)	0	0	0
	94-95	//	0.870 (0.234)	0.561 (0.166)	0.786 (0.162)	3	0	7
Daily Returns	77-80	1137-1140	0.864 (0.243)	0.650 (0.244)	0.798 (0.153)	4	1	0
	81-85	1421-1423	0.905 (0.208)	0.659 (0.271)	0.855 (0.121)	4	6	0
	86-90	1320-1323	0.937 (0.049)	0.694 (0.257)	0.913 (0.078)	1	5	0
	91-95	1232-1235	0.950 (0.072)	0.535 (0.322)	0.921 (0.131)	0	1	0
Daily Returns	77-85	2561-2563	0.921 (0.172)	0.920 (0.087)	0.876 (0.080)	2	5	0
	86-95	2553-2556	0.956 (0.027)	0.939 (0.123)	0.944 (0.027)	0	5	0
Daily Returns	77-95	5109-5123	0.965 (0.083)	0.980 (0.011)	0.934 (0.040)	4	1	0
Weelky Returns	77-95	977	0.893 (0.112)	0.635 (0.184)	0.891 (0.102)	2	0	0
Monthly Returns	77-95	228	0.792 (0.194)	0.555 (0.162)	0.729 (0.230)	3	0	0

[a] Averaged Estimates=$\sum$(estimated parameters' values)/160, $\hat{\alpha}_1 + \hat{\beta}_1$ for GARCH(1,1), $\hat{\phi}$ for SV, and $\hat{\beta}$ for EGARCH(1,1). Standard Deviations are also obtained listed in parentheses.

Table 2: Simulation Experiment 1

True Models	Models Used in Estimation	T	Parameters' Values in the True Models				Averaged Estimated Parameters' Values and Standard Deviations[a]
			α_1	β_1	ϕ	σ_η	$\hat{\alpha}_1 + \hat{\beta}_1$
			θ	δ	β		
GARCH(1,1)	GARCH(1,1)	200	0.05	0.9			0.885
		//					(0.156)
		//	0.1	0.85			0.882
		//					(0.142)
		//	0.05	0.94			0.932
		//					(0.138)
		//	0.1	0.89			0.937
		//					(0.109)
		//	0.05	0.945			0.940
		//					(0.134)
		//	0.1	0.895			0.945
		//					(0.099)
SV	GARCH(1,1)	200			0.9	0.018	0.475
		//					(0.402)
		//			0.95	0.009	0.493
		//					(0.457)
		//			0.99	0.002	0.483
		//					(0.489)
		//			0.9	0.132	0.786
		//					(0.193)
		//			0.95	0.068	0.895
		//					(0.178)
		//			0.99	0.014	0.864
		//					(0.267)
EGARCH(1,1)	GARCH(1,1)	200	0.25	-0.5	0.9		0.913
		//					(0.164)
		//	0.5	-0.5	0.9		0.934
		//					(0.214)
		//	0.25	0	0.9		0.613
		//					(0.334)
		//	0.5	0	0.9		0.892
		//					(0.163)
		//	0.25	0.5	0.9		0.844
		//					(0.200)
		//	0.5	0.5	0.9		0.845
		//					(0.187)

[a] Averaged Estimates=$\sum$(estimated parameters' values)/10,000, Standard Deviations are also obtained listed in parentheses.

Table 2: (Continued)

True Models	Models Used in Estimation	T	Parameters' Values in the True Models				Averaged Estimated Parameters' Values and Standard Deviations[a]
			α_1	β_1	ϕ	σ_η	$\hat{\phi}$
			θ	δ	β		
GARCH(1,1)	SV	200	0.05	0.9			0.823
		//					(0.146)
		//	0.1	0.85			0.859
		//					(0.159)
		//	0.05	0.94			0.864
		//					(0.226)
		//	0.1	0.89			0.872
		//					(0.239)
		//	0.05	0.945			0.835
		//					(0.283)
		//	0.1	0.895			0.840
		//					(0.269)
SV	SV	200			0.9	0.018	0.768
		//					(0.173)
		//			0.95	0.009	0.843
		//					(0.154)
		//			0.99	0.002	0.867
		//					(0.207)
		//			0.9	0.132	0.823
		//					(0.149)
		//			0.95	0.068	0.875
		//					(0.142)
		//			0.99	0.014	0.871
		//					(0.225)
EGARCH(1,1)	SV	200	0.25	-0.5	0.9		0.872
		//					(0.063)
		//	0.5	-0.5	0.9		0.851
		//					(0.067)
		//	0.25	0	0.9		0.713
		//					(0.381)
		//	0.5	0	0.9		0.680
		//					(0.294)
		//	0.25	0.5	0.9		0.840
		//					(0.151)
		//	0.5	0.5	0.9		0.607
		//					(0.359)

[a] Averaged Estimates=$\sum$(estimated parameters' values)/10,000, Standard Deviations are also obtained listed in parentheses.

Table 2: (Continued)

True Models	Models Used in Estimation	T	Parameters' Values in the True Models				Averaged Estimated Parameters' Values and Standard Deviations[a]
			α_1	β_1	ϕ	σ_η	$\hat{\beta}$
			θ	δ	β		
GARCH(1,1)	EGARCH(1,1)	200	0.05	0.9			0.305
		//					(0.671)
		//	0.1	0.85			0.622
		//					(0.537)
		//	0.05	0.94			0.423
		//					(0.642)
		//	0.1	0.89			0.736
		//					(0.462)
		//	0.05	0.945			0.451
		//					(0.624)
		//	0.1	0.895			0.747
		//					(0.443)
SV	EGARCH(1,1)	200			0.9	0.018	0.298
		//					(0.668)
		//			0.95	0.009	0.347
		//					(0.658)
		//			0.99	0.002	0.274
		//					(0.700)
		//			0.9	0.132	0.727
		//					(0.401)
		//			0.95	0.068	0.772
		//					(0.401)
		//			0.99	0.014	0.647
		//					(0.573)
EGARCH(1,1)	EGARCH(1,1)	200	0.25	-0.5	0.9		0.884
		//					(0.108)
		//	0.5	-0.5	0.9		0.885
		//					(0.084)
		//	0.25	0	0.9		0.709
		//					(0.409)
		//	0.5	0	0.9		0.859
		//					(0.155)
		//	0.25	0.5	0.9		0.852
		//					(0.218)
		//	0.5	0.5	0.9		0.797
		//					(0.285)

[a] Averaged Estimates=$\sum$(estimated parameters' values)/10,000, Standard Deviations are also obtained listed in parentheses.

Table 2: (Continued)

True Models	Models Used in Estimation	T	Parameters' Values in the True Models				Averaged Estimated Parameters' Values and Standard Deviations[a]
			α_1	β_1	ϕ	σ_η	$\hat{\alpha}_1 + \hat{\beta}_1$
			θ	δ	β		
GARCH(1,1)	GARCH(1,1)	500	0.05	0.9			0.921
		//					(0.081)
		//	0.1	0.85			0.926
		//					(0.060)
		//	0.05	0.94			0.971
		//					(0.045)
		//	0.1	0.89			0.975
		//					(0.031)
		//	0.05	0.945			0.978
		//					(0.041)
		//	0.1	0.895			0.981
		//					(0.023)
SV	GARCH(1,1)	500			0.9	0.018	0.490
		//					(0.381)
		//			0.95	0.009	0.535
		//					(0.401)
		//			0.99	0.002	0.513
		//					(0.422)
		//			0.9	0.132	0.880
		//					(0.133)
		//			0.95	0.068	0.937
		//					(0.111)
		//			0.99	0.014	0.934
		//					(0.194)
EGARCH(1,1)	GARCH(1,1)	500	0.25	-0.5	0.9		0.923
		//					(0.081)
		//	0.5	-0.5	0.9		0.934
		//					(0.122)
		//	0.25	0	0.9		0.803
		//					(0.214)
		//	0.5	0	0.9		0.947
		//					(0.055)
		//	0.25	0.5	0.9		0.917
		//					(0.064)
		//	0.5	0.5	0.9		0.909
		//					(0.064)

[a] Averaged Estimates=$\sum$(estimated parameters' values)/10,000, Standard Deviations are also obtained listed in parentheses.

Table 2: (Continued)

True Models	Models Used in Estimation	T	Parameters' Values in the True Models				Averaged Estimated Parameters' Values and Standard Deviations[a]
			α_1	β_1	ϕ	σ_η	ϕ
			θ	δ	β		
GARCH(1,1)	SV	500	0.05	0.9			0.880
		//					(0.101)
		//	0.1	0.85			0.921
		//					(0.069)
		//	0.05	0.94			0.948
		//					(0.108)
		//	0.1	0.89			0.961
		//					(0.059)
		//	0.05	0.945			0.946
		//					(0.132)
		//	0.1	0.895			0.958
		//					(0.080)
SV	SV	500			0.9	0.018	0.822
		//					(0.120)
		//			0.95	0.009	0.889
		//					(0.104)
		//			0.99	0.002	0.934
		//					(0.149)
		//			0.9	0.132	0.875
		//					(0.064)
		//			0.95	0.068	0.929
		//					(0.044)
		//			0.99	0.014	0.963
		//					(0.064)
EGARCH(1,1)	SV	500	0.25	-0.5	0.9		0.897
		//					(0.033)
		//	0.5	-0.5	0.9		0.893
		//					(0.032)
		//	0.25	0	0.9		0.852
		//					(0.165)
		//	0.5	0	0.9		0.843
		//					(0.075)
		//	0.25	0.5	0.9		0.888
		//					(0.048)
		//	0.5	0.5	0.9		0.819
		//					(0.118)

[a] Averaged Estimates=$\sum$(estimated parameters' values)/10,000, Standard Deviations are also obtained listed in parentheses.

Table 2: (Continued)

True Models	Models Used in Estimation	T	Parameters' Values in the True Models				Averaged Estimated Parameters' Values and Standard Deviations[a]
			α_1	β_1	ϕ	σ_η	$\hat{\beta}$
			θ	δ	β		
GARCH(1,1)	EGARCH(1,1)	500	0.05	0.9			0.479
		//					(0.654)
		//	0.1	0.85			0.842
		//					(0.323)
		//	0.05	0.94			0.705
		//					(0.552)
		//	0.1	0.89			0.953
		//					(0.144)
		//	0.05	0.945			0.779
		//					(0.471)
		//	0.1	0.895			0.953
		//					(0.162)
SV	EGARCH(1,1)	500			0.9	0.018	0.460
		//					(0.599)
		//			0.95	0.009	0.544
		//					(0.589)
		//			0.99	0.002	0.501
		//					(0.640)
		//			0.9	0.132	0.868
		//					(0.139)
		//			0.95	0.068	0.921
		//					(0.137)
		//			0.99	0.014	0.928
		//					(0.253)
EGARCH(1,1)	EGARCH(1,1)	500	0.25	-0.5	0.9		0.899
		//					(0.016)
		//	0.5	-0.5	0.9		0.899
		//					(0.026)
		//	0.25	0	0.9		0.863
		//					(0.163)
		//	0.5	0	0.9		0.893
		//					(0.036)
		//	0.25	0.5	0.9		0.895
		//					(0.022)
		//	0.5	0.5	0.9		0.877
		//					(0.112)

[a] Averaged Estimates=$\sum$(estimated parameters' values)/10,000, Standard Deviations are also obtained listed in parentheses.

Table 2: (Continued)

True Models	Models Used in Estimation	T	Parameters' Values in the True Models				Averaged Estimated Parameters' Values and Standard Deviations[a]
			α_1	β_1	ϕ	σ_η	$\hat{\alpha_1} + \hat{\beta_1}$
			θ	δ	β		
GARCH(1,1)	GARCH(1,1)	5000	0.05	0.9			0.949
		//					(0.009)
		//	0.1	0.85			0.950
		//					(0.009)
		//	0.05	0.94			0.989
		//					(0.003)
		//	0.1	0.89			0.989
		//					(0.004)
		//	0.05	0.945			0.993
		//					(0.003)
		//	0.1	0.895			0.995
		//					(0.004)
SV	GARCH(1,1)	5000			0.9	0.018	0.882
		//					(0.097)
		//			0.95	0.009	0.943
		//					(0.023)
		//			0.99	0.002	0.989
		//					(0.005)
		//			0.9	0.132	0.919
		//					(0.018)
		//			0.95	0.068	0.968
		//					(0.010)
		//			0.99	0.014	0.996
		//					(0.003)
EGARCH(1,1)	GARCH(1,1)	5000	0.25	-0.5	0.9		0.925
		//					(0.025)
		//	0.5	-0.5	0.9		0.934
		//					(0.038)
		//	0.25	0	0.9		0.908
		//					(0.016)
		//	0.5	0	0.9		0.964
		//					(0.013)
		//	0.25	0.5	0.9		0.935
		//					(0.015)
		//	0.5	0.5	0.9		0.927
		//					(0.014)

[a] Averaged Estimates=$\sum$(estimated parameters' values)/1,000, Standard Deviations are also obtained listed in parentheses.

Table 2: (Continued)

True Models	Models Used in Estimation	T	Parameters' Values in the True Models				Averaged Estimated Parameters' Values and Standard Deviations[a]
			α_1	β_1	ϕ	σ_η	$\hat{\phi}$
			θ	δ	β		
GARCH(1,1)	SV	5000	0.05	0.9			0.949
		//					(0.021)
		//	0.1	0.85			0.952
		//					(0.012)
		//	0.05	0.94			0.987
		//					(0.004)
		//	0.1	0.89			0.985
		//					(0.005)
		//	0.05	0.945			0.993
		//					(0.003)
		//	0.1	0.895			0.989
		//					(0.004)
SV	SV	5000			0.9	0.018	0.880
		//					(0.015)
		//			0.95	0.009	0.942
		//					(0.011)
		//			0.99	0.002	0.988
		//					(0.004)
		//			0.9	0.132	0.901
		//					(0.003)
		//			0.95	0.068	0.950
		//					(0.002)
		//			0.99	0.014	0.989
		//					(0.004)
EGARCH(1,1)	SV	5000	0.25	-0.5	0.9		0.908
		//					(0.009)
		//	0.5	-0.5	0.9		0.917
		//					(0.008)
		//	0.25	0	0.9		0.910
		//					(0.019)
		//	0.5	0	0.9		0.909
		//					(0.011)
		//	0.25	0.5	0.9		0.908
		//					(0.012)
		//	0.5	0.5	0.9		0.908
		//					(0.011)

[a] Averaged Estimates=$\sum$(estimated parameters' values)/1,000, Standard Deviations are also obtained listed in parentheses.

Table 2: (Continued)

True Models	Models Used in Estimation	T	Parameters' Values in the True Models				Averaged Estimated Parameters' Values and Standard Deviations[a]
			α_1	β_1	ϕ	σ_η	$\hat{\beta}$
			θ	δ	β		
GARCH(1,1)	EGARCH(1,1)	5000	0.05	0.9			0.948
		//					(0.015)
		//	0.1	0.85			0.947
		//					(0.011)
		//	0.05	0.94			0.988
		//					(0.004)
		//	0.1	0.89			0.984
		//					(0.005)
		//	0.05	0.945			0.992
		//					(0.005)
		//	0.1	0.895			0.989
		//					(0.004)
SV	EGARCH(1,1)	5000			0.9	0.018	0.893
		//					(0.045)
		//			0.95	0.009	0.943
		//					(0.022)
		//			0.99	0.002	0.989
		//					(0.005)
		//			0.9	0.132	0.901
		//					(0.016)
		//			0.95	0.068	0.950
		//					(0.009)
		//			0.99	0.014	0.989
		//					(0.003)
EGARCH(1,1)	EGARCH(1,1)	5000	0.25	-0.5	0.9		0.903
		//					(0.005)
		//	0.5	-0.5	0.9		0.906
		//					(0.006)
		//	0.25	0	0.9		0.902
		//					(0.013)
		//	0.5	0	0.9		0.905
		//					(0.009)
		//	0.25	0.5	0.9		0.899
		//					(0.006)
		//	0.5	0.5	0.9		0.898
		//					(0.009)

[a] Averaged Estimates=$\sum$(estimated parameters' values)/1,000, Standard Deviations are also obtained listed in parentheses.

Table 3: Simulation Experiment 2

True Models	Models Used in Estimation	T	Parameters' Values in the True Models				Averaged Estimated Parameters' Values and Standard Deviations[a]
			α_1	β_1	ϕ	σ_η	$\hat{\alpha}_1 + \hat{\beta}_1$
			θ	δ	β		
GARCH(1,1)	GARCH(1,1)	4228	0.05	0.9			0.901
	+ Dummies	//					(0.019)
		//	0.1	0.85			0.911
		//					(0.013)
		//	0.05	0.94			0.926
		//					(0.011)
		//	0.1	0.89			0.931
		//					(0.006)
		//	0.05	0.945			0.934
		//					(0.006)
		//	0.1	0.895			0.937
		//					(0.006)
SV	GARCH(1,1)	4228			0.9	0.018	0.611
	+ Dummies	//					(0.181)
		//			0.95	0.009	0.618
		//					(0.181)
		//			0.99	0.002	0.739
		//					(0.218)
		//			0.9	0.132	0.876
		//					(0.049)
		//			0.95	0.068	0.911
		//					(0.057)
		//			0.99	0.014	0.901
		//					(0.117)
EGARCH(1,1)	GARCH(1,1)	4228	0.25	-0.5	0.9		0.912
	+ Dummies	//					(0.027)
		//	0.5	-0.5	0.9		0.933
		//					(0.040)
		//	0.25	0	0.9		0.793
		//					(0.017)
		//	0.5	0	0.9		0.853
		//					(0.014)
		//	0.25	0.5	0.9		0.824
		//					(0.016)
		//	0.5	0.5	0.9		0.816
		//					(0.014)

[a] Averaged Estimates=$\sum$(estimated parameters' values)/1,000, Standard Deviations are also obtained listed in parentheses.