Insuring corporate failure: credit default swaps

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ABSTRACT

This paper has three aims:

1. to inform actuaries of recent developments in the analysis of credit risk, and the growing market in credit derivatives,
2. to highlight potential opportunities for insurance companies as writers of default swaps,
3. to identify the risks in default insurance.

We describe the form of historical data relating to default and recovery on corporate bonds. We discuss the pricing of corporate bonds, and the form and pricing of typical default insurance or default swap contracts. An appropriate pricing model for default insurance is described. Historical data and, separately, market data are used to price default contracts: the difference between the results is compared. The market driven ('arbitrage-free') pricing approach generally leads to much higher prices than the historical approach. Furthermore, insurers pricing on the latter approach are offering capital market investors almost risk-free profits at the insurer's opportunity cost.

Keywords

credit risk, credit derivatives, credit default swaps, arbitrage-free pricing, insurance pricing, historical pricing

Introduction

Recent years have seen a growing market in credit derivatives and, in particular, credit default swaps (CDS). The market is large - probably in excess of $1bn per day - with some individual trades of that size.

A simple CDS contract may pay $100 on default of a company in exchange for 100 nominal of a specific defaulted bond issued by that company. An alternative form (a digital or binary option) is one which pays $100 on default of the company with no exchange of defaulted bond. The former is similar to a car insurance policy, the latter closer to life insurance.

Some insurance companies ('monoline' insurers) have been offering default insurance for many years. Typically the form of the contract is more complicated than described here, including embedded options, but the principles we describe are still vaild and can be extended to handle more complicated structures.

Contents

PART I: HISTORICAL DEFAULT DATA

Corporate default: historical default rates

In the credit environment we find - based primarily on US experience - that default rates depend primarily on rating; that default rates vary a little over time; that default is generally a progression from higher rating to lower rating before finally defaulting; that recovery rates do not seem to depend on rating but on seniority of debt; that recovery rates also depend on industry (Altman); and that recovery rates have a very high standard deviation. For example, we may be fairly sure that over time and with a large number of senior unsecured debt, we will recover about 50% on the defaults. But on any individual security the uncertainty is such that all we know is that recovery will be between 0 and 100%!

Incidentally, the level of variability of default rates over time has been used in CreditMetrics to show that the default correlation in any year between two names chosen at random is very likely in to 0 to 0.06 range. Bearing in mind that a credit derivative will be settled on average within three months of the default event, then coincident default between two parties (name and counterparty) is very small unless these have an unusually strong business relationship. (Spotting the existence of this relationship is another problem.)

Historical data is generally given in terms of a rating transition matrix (Table 1). The matrix shows, for example, that a BBB has a 88% chance of still being a BBB in one year's time, and a 0.15% chance of default. By taking matrix powers we can find the default (and hence survival) probability for any number of years (and part years) in the future. These default rates (together with recovery rates) are themselves sufficient to 'value' credit derivatives. Note that data for rating transitions - particularly for transitions to the same or adjacent ratings - is generally extensive, but that default is a rare event. Historically derived default rates are therefore subject to a significant margin of error.

Table 1: default-transition matrix (based on Moody's historical default rate studies)

	AAA	AA	A	BBB	BB	B	C	default
AAA	0.9338	0.0594	0.0064	0	0.0002	0	0	0.0002
AA	0.0161	0.9055	0.0746	0.0026	0.0009	0.0001	0	0.0002
A	0.0007	0.0228	0.9237	0.0463	0.0045	0.0012	0.0001	0.0007
BBB	0.0005	0.0026	0.0551	0.8848	0.0476	0.0071	0.0008	0.0015
BB	0.0002	0.0005	0.0042	0.0516	0.8691	0.0591	0.0024	0.0129
B	0	0.0004	0.0013	0.0054	0.0635	0.8422	0.0191	0.0681
C	0	0	0	0.0062	0.0205	0.0408	0.6920	0.2405
default	0	0	0	0	0	0	0	1

Note: some figures have been changed to give default probabilities greater than zero.

Corporate default: historical recovery rates

On default, senior bonds get repaid first, filtering down until the money starts to run out and a particular debt seniority gets only a proportion of claim value, and lower seniorities get none. This is born out historically. Table 2 show how average recovery rate declines as seniority worsens.

Table 2: Defaulted Bond and Loan Recoveries: 1989-1996 (Moody's)

Seniority	**Number of defaults**	**Mean Recovery (%)**	**Standard Deviation (%)**
Senior secured bank loans	59	71	21
Senior secured public debt	57	63	26
Senior unsecured public debt	156	48	26
Senior subordinated public debt	166	38	25
Subordinated public debt	119	28	20
Junior Subordinated public debt	8	15	9

Note for comparison: the mean of a uniform distribution is 50% and standard deviation is 29%.

The 'mean recovery rate' is often used as an expected proportion prior to default. Post-default all bonds above a certain seniority should get 100% of claim amount back; below that seniority they should get zero; at that seniority they will get a percentage which may be above or below the expected figure.

PART II: PRICING MODEL

A pricing model for credit and credit default swaps

We shall illustrate the pricing model through a simple specific example rather than general formulae. The example is sufficiently complex to illustrate the main features, and capture the main risks, of credit derivative pricing. In this part we define the form of the model, we than apply the pricing model in two alternative ways:

(i) an insurance or historical data driven pricing approach (part III), and

(ii) an arbitrage-free pricing approach (part IV).

Form of the Model

We require a form of model and a set of assumptions. There are many: Jarrow, Lando and Turnbull use a default-transition matrix approach but modify historical rates to get agreement with the market prices of bonds; Jarrow and Turnbull just use default rates; Duffie and Singleton use default rates but calibrate to a theoretical rather than a realistic claim type (the gain being a simple practical model, albeit a less realistic one); other approaches make recovery rates dynamic. The model outlined below captures some of the main features and is cast in a continuous time format. We assume credit derivatives can be modelled in a world in which risk-free interest rates follow a stochastic process, in which default is a 'rare' event - described by a Poisson process - and in which the default rate itself is also a stochastic process.

Assumptions behind the Model

i) the risk-free curve and form of the stochastic process is known: we are modelling credit risk so lets make life simple and assume we have all the information we need on the risk-free world

ii) the form of the default rate stochastic process is known: for some products all we need to know is the expected default rate at any time - effectively a deterministic (time-dependent) default rate

iii) default rates and interest rates are independent: this is needed to enable us to easily value a payment of 1 in (say) a year's time subject to the survival of a credit name.

iv) name default rates are independent: without a similar assumption we cannot calibrate our model

v) trading is costless etc.

vi) recovery rates are known

vii) it is possible to short credits

Basic Model Formulae - Bond Price

We shall begin by defining the model for a simple cash-flow (a 'bond'). Let's suppose the credit has promised to pay 6% coupon plus 100 redemption amount in one year's time. Lets suppose the bond is not special on repo, then the value is B where

$$B = 106 \times v(1) \times p(1) + \int_0^1 R \times (100 + A(t)) \times v(t) \times p(t) \times q(t) dt \qquad \text{(formula 1)}$$

where $v(t)$ is the risk-free discount factor for period t, $q(t)$ is the default rate and $p(t)$ is the survival probability for time t, R ($0 <= R <= 1$) is the recovery rate, and $A(t)$ is the accrued at time t. We have supposed that recovery is based on 'par plus accrued'[i].

(The incorporation of repo is straightforward so not included here. It directly affects only the pricing of the bond. However, it has several important indirect effects. If a bond is special, the implied default rate (which we calculate below) will be different. In a practical sense repo is a big problem: it may not be possibly to borrow bonds easily, and the repo rate may increase dramatically just at the time when it is desirable to borrow the debt.)

Interpretation of the Formula

The above formula (and those below) are quite general in that all the unknowns can be regarded as stochastic variables. But what we want to do is replace the B by the market price - an expected value - and/or the individual terms on the right hand side by their individual expected values. We can do this if we assume interest rates and default rates are independent, if we assume recovery rates and default rates are independent, and if default rates are deterministic. In the case of stochastic default rates the situation is mathematically a little more complicated. Rather than assume an explicit form of stochastic model for the default rate a much simpler approach is to keep the same formulae but change the interpretation. $p(t)$

represents the expected survival probability, and $q(t)p(t)$ is the expected default rate at a date t given only that the name has not defaulted now, and p and q can still be related by the usual Poisson formula. Knowledge of q is sufficient. But q is no longer the default rate and we never need to know the default rate function!. This is a much easier approach and one we adopt. We shall still *refer* to q as the default rate even in the stochastic case.

In practice, generally assume a recovery rate based on historical data. (There are exceptions to this: either when we have a rich data set of bond information for the underlying name, or when the default rate is very high.)

Basic Model Formulae - Credit Derivative Formula

Now suppose XYZ Insurance Company (AAA rated, say) promises to buy the bond for a price of 100 plus accrued interest in the event that the underlying name defaults. The value of this payment is the chance that the name defaults and the counterparty survives at time t, times the discount factor, times the amount paid, less the value of the defaulted bond. So the value of the default protection is

$$D = \int_0^1 (1 - R) \times (100 + A(t)) \times v(t) \times P(t) \times p(t) \times q(t) dt \qquad \text{(formula 2)}$$

where $P(t)$ is the probability that the counterparty survives to time t. Again we can replace the terms by their expected values if we have independence of name and counterparty default rates, and of risk-free interest rates.

If there is no exchange of the bond on default, then the option pays 100 (there is typically no underlying bond so no accrued) and is called a 'binary' or 'digital' option. The value is

$$binary = \int_0^1 100 \times v(t) \times P(t) \times p(t) \times q(t) dt \qquad \text{(formula 3)}$$

PART III: INSURANCE/HISTORICAL PRICING

Bond prices

We are not directly interested in bond prices but the results from the historical pricing approach are sufficiently disturbing that they should be noted at the outset. We use formula 1 with historical default rates: we suppose the credit is a BBB senior unsecured bond, $q(1)$ = 0.0015, R=0.48. Lets suppose $v(1)$ = 0.95, $v(0)$ = 1 (of course), and $v(t)$ is obtained by log-linear interpolation. A risk-free bond ($p(t)$ = 1) would be priced at 100.70. Let's make a further simplifying assumption to evaluate the formula: we suppose default occurs at the half-way point only. Then the value of this bond is:

$$106 \times 0.95 \times 0.9985 + 0.48 \times 103 \times 0.9747 \times 0.9985 \times 0.0015 = 100.621$$

corresponding to a yield spread of roughly 8bp above risk-free rates. (We could have guessed this from the default rate (15bp) times 1-0.48.)

Note that, on this approach, all BBB bonds of the same seniority (and same coupon, maturity and frequency) have the same price and spread - irrespective of issuer.

Pricing a CDS with exchange of defaulted bond

We have made no explicit allowance for practical matters such as expenses and profit margin. This will affect the results of the calculation and comparison of the insurance pricing approach with the arbitrage-free pricing approach. A full analysis should of course include these terms: they only apply to the insurance pricing approach. Applying formula 2 with the same timing assumption as above, the value of the default protection from a risk-free counterparty is

$$0.52 \times 103 \times 0.9747 \times 1 \times .9985 \times .0015 = 0.078.$$

Pricing a binary CDS

Applying formula 3 the value of the binary option is

$$100 \times 0.9747 \times 1 \times .9985 \times .0015 = 0.146$$

PART IV: ARBITRAGE-FREE PRICING

The arbitrage-free approach

The key feature of the arbitrage-free approach is to price credit derivatives in terms of the cost of completely hedging the risks in the derivative. We can hedge a long position in a CDS on a particular bond by buying the underlying bond and, in principle, similarly for a short position. Our pricing model for bonds should therefore be used to derive the unknown data rather than using historical data alone.

The credit bond price contains information on future default rates, reward for risk, recovery rate, interest rates, and possibly some information on volatility. However, some of these factors are so tightly bound together it is impossible to disentangle them: default rate and reward for risk appear as a single 'risk-neutral' default rate in the terminology of financial economists. Others may only be partially disentangled: default rate and recovery rate for example. The process of obtaining implied model values from the observed bond price data is called 'calibration'.

The arbitrage-free approach then enables us to

- value a credit derivative ('mark-to-market') if we know the values of some other instruments
- price an instrument to reflect the cost of hedging our risks.

The approach is not concerned with history or with actual default rates.

Calibration

In the case of the above model we can calibrate the model to the market if we assume

- a functional form of the default curve (an 'interpolation method')
- the recovery rate.

We need to assume one figure (either default or recovery) and calculate the other making sure it satisfies the constraints:

a) $0 <= R <= 1$

b) $0 <= q <=$ infinity.

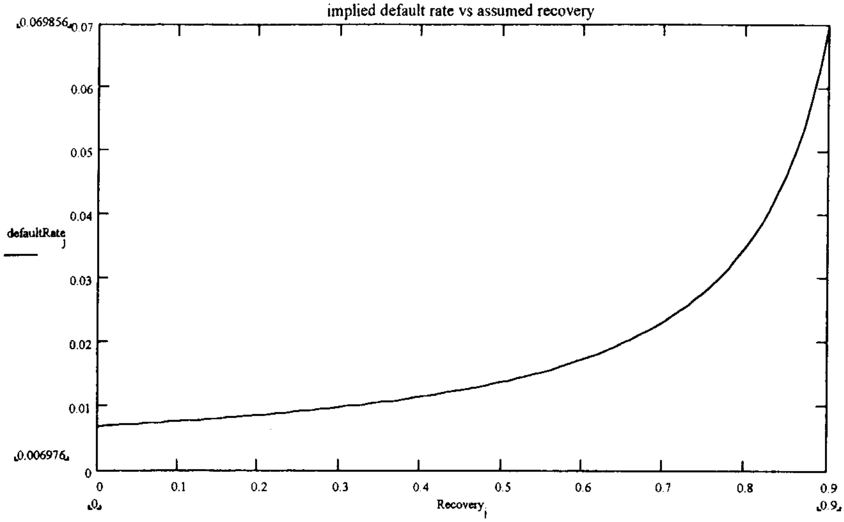
We use the same interest rate assumptions as before. Suppose our risky bond is priced at 100.

Table 3 shows the results of calibrating using formula 1 for a range of assumed recovery rates; graph 1 shows the results in graphical form. Note that the default rate goes up as the recovery rate increases. Note also that the bond price does not depend on the recovery rate - as along as we use the default rate consistent with that assumed recovery rate.

Table 3: Calibrated Default Rates for a Range of Assumed Recovery Rates

Recovery Rate	**Default Rate**
0.0	0.006976
0.1	0.007751
0.2	0.00872
0.3	0.009966
0.4	0.011628
0.5	0.013955
0.6	0.017445
0.7	0.023265
0.8	0.034908
0.9	0.069856

Graph 1: Calibrated Default Rates for a Range of Assumed Recovery Rates



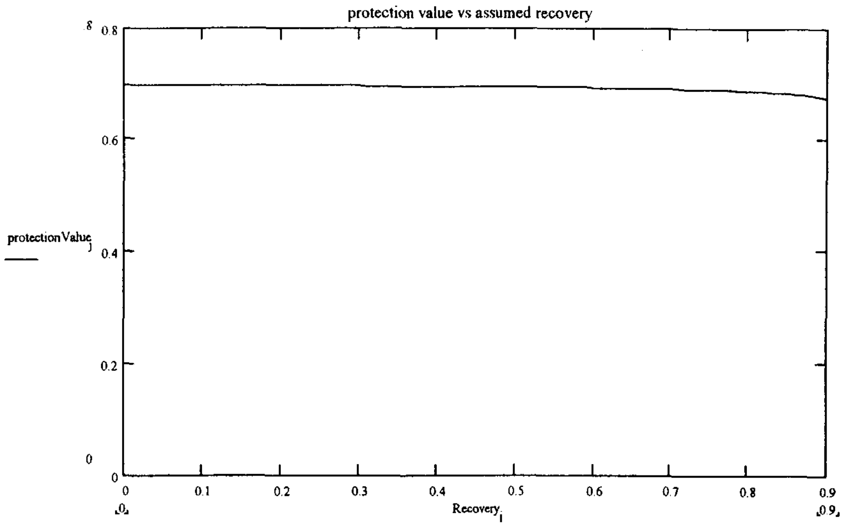
Pricing a CDS with exchange of defaulted bond

We can now take the assumed recovery rate and implied default rate to use formula 2 and calculate the price of a default swap where payment is in exchange for a defaulted bond. Table 4 gives the results for both a risk free and a risk counterparty (counterparty survival probabilities are estimated in the same way using counterparty bonds in principle); graph 2 gives the results for a risky counterparty in graphical form.

Table 4: Premium for Default Protection as a function of assumed recovery rate.

Recovery Rate	Risk-free counterparty	High quality risky counterparty
0.0	0.697753	0.697404
0.1	0.697502	0.697154
0.2	0.697190	0.696841
0.3	0.696787	0.696439
0.4	0.696250	0.695902
0.5	0.695497	0.69515
0.6	0.694367	0.694021
0.7	0.692479	0.692134
0.8	0.688691	0.688348
0.9	0.677239	0.676904

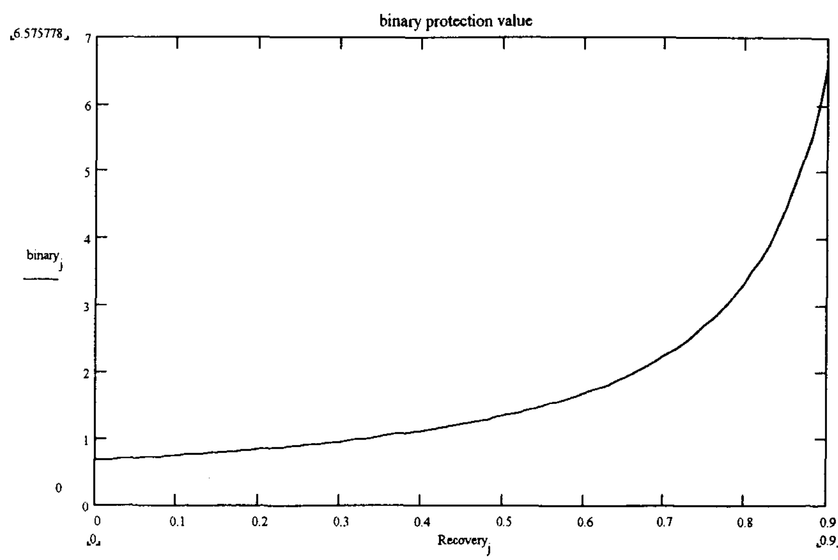
Graph 2: Premium for Default Protection from risky counterparty as a function of assumed recovery rate.



Pricing a binary CDS

We can also use formula 3 to price a digital default option. However the results shown in graph 3 for the binary default option is very different:

Graph 3: Premium for binary default protection as a function of assumed recovery rate.



Because there is no offset for the value of the defaulted bond, the payoff on the option is the same no matter what the recovery rate or default rate. Hence a low assumed recovery rate in the calibration leads to a high default rate and high binary option value. (The standard default option payoff is of course a much lower amount in this case, which offsets the high default rate to produce a roughly constant cost.)

PART V: COMPARISON OF ARBITRAGE-FREE AND INSURANCE PRICING

The major differences between models described are not to the models themselves but the data driving, and calibration of, those models. The difference specifically is:

a) the historical model uses historical data on default and recovery rates rather along the lines of general insurance pricing;
b) the arbitrage-free approach uses market data to derive (where possible) implied default and recovery rates: these default rates are not 'anticipated' default rates but the sum of anticipated rates and the market reward for risk - 'expected' default rates, in the jargon of financial economics.

As a pricing model the historical approach is not consistent with the market prices of bonds: it opens up arbitrage opportunities. The arbitrage-free models lead to a greater understanding of the risks. They (barring practical difficulties and market incompleteness) are intimately tied to the instruments and procedures used to hedge the risks via creation on synthetic replicating strategies. The shortcomings of the arbitrage-free approach are non-trivial (see below), but are a subset of the shortcomings of the historical approach.

The results of the calculations are startlingly different: these differences persist even after making allowance for expenses etc. in the insurance pricing approach. The differences are typical of real world differences in premiums. Table 5 summarises the results:

	Insurance Pricing Premium	**Arbitrage-free Premium range (0<=R<=0.9)**
CDS on a bond	0.078	0.698-0.678
binary CDS	0.146	0.698-6.500

In addition, the arbitrage-free approach highlights the risk in binary options.

PART VI: RISKS

The aim of this section is to identify the risks inherent in pricing and hedging credit derivatives. Prior to the modelling of these products there are legal and documentation risks. These risks are less for default insurance on corporate names than on sovereigns, but are still present. Standardisation of CDS documentation and product, under the auspices of ISDA, greatly improves the situation of what it was only a few years ago. This section concentrates on market and model risks.

Risks fall into two groups:

i) those captured and handled by the model and

ii) those which are real but not captured by the model.

An example of the former is default rate or recovery rate risk, of the latter correlation between default and risk-free interest rates. The second group requires a careful examination of the assumptions used in building the model and some greater ingenuity in devising methods to estimate these risks.

We define three sources of risk:

Event Risk: primarily the event of default or non-default.

Market Risk: by which I mean sensitivity of the instrument we are looking at to various variables in our model - interest rates, default rates, recovery rate, etc. Market risk is the risks in the model world. Risks captured by the model.

Hidden Risk: these risks are external to the model but present in the real world. They are related to our simplifying assumptions but are as real as any other risk. We just don't have a working model which captures them. Examples are: relationship between risk-free rates and default rates (correlation) - what is the impact of a positive rather than zero correlation? Name and counterparty default - default insurance from the banking subsidiary of a motor company on the parent may not be worth a lot!

Hidden risk is present in any modelling process - we always make some assumptions. The model itself can handle event risk and the market risks it perceives; we then need to devise methods for estimating, and handling or coping with the hidden risks.

We can identify the following risks in credit derivatives

1. (Event) default and non-default
2. (Model) risk-free interest rates
3. (Model) bond repo rates
4. (Model) default rate
5. (Model) recovery rate
6. (Hidden) correlation between interest rates (and repo rates) and default rates
7. (Hidden) interpolation method for default rate at any time given value(s) at certain time(s).
8. (Hidden) name/counterparty default correlation
9. (Model) name and counterparty default *rate* correlation.
10. (Model) recovery rate on counterparty trade debt.
11. (Hidden) arbitrage-free pricing approach for credit

Model and event risks are handled within the model: raw data (risk-free rates, credit spreads, repo rates) plus the assumed data (recovery rates and correlations) are used to calibrate the model and obtain model parameters to allow books to be marked-to-market. Hidden risks have to be analysed by building a better, but usually less tractable, model. This is typically used to find - in general terms - the magnitude of the risks and to establish appropriate levels of reserves.

Finally we made assumptions regarding the appropriateness of the arbitrage free approach. The method is applicable if positions can be hedged as assumed. This is generally *not* the case for a range of reasons: costs make the strategy more expensive than the model would suggest; lack of repo may make it impossible; counterparty risk may not be hedged. If the hedging strategy is not followed is the arbitrage free approach appropriate? Where a wide spread of risks is taken on board the approach still makes sense. But it loses its appeal where large concentrations of risk arise, and utility based pricing approaches become more interesting.

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[1] What can be claimed in the event of default is the 'claim amount'. This may be specified in the issue document (generally the case for corporate borrowers). Claim amounts are usually 'par plus accrued' or the cash-flows that were promised (which, equivalently, have a value based on risk-free discounting factors) generally referred to as 'treasury' claim amount. For sovereign debt it is often assumed that recovery is based on pre-default market value.