Optimization of portfolios with longer investment period

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ABSTRACT

We investigate the optimisation of portfolios with the investment *I* done periodically (n-times) with a period Δt_1, and the investment is been hold after the last investment for a time Δt_2 much larger than *n* Δt_1. We show that, when using the $\mu - k\sigma$ optimisation for the portfolio one has to consider, that σ is time dependent. Considering different assets (shares) with the same $\sigma(\Delta t_2)$ the investment in the asset is preferable with the highest $\sigma(\Delta t_1)$. That means, that portfolio optimisation with the measure of risk as $\mu - k\sigma$ and the cost average effect holds best for assets with $\sigma(\Delta t_1)$ large and $\sigma(\Delta t_2)$ small. Also this shows, that one should add a measure of risk for the investment process.

Keywords: cost-average, investment, measure of risk

1 INTRODUCTION

For optimisation of portfolio several measure of risks are been used, as value at risk, standard deviation [1-3], or as in our last works lower partial moment [4-6] or tail value at risk.

For the very often used μ - $k\sigma$ optimisation process for portfolios, one has to consider that the standard deviation σ and the expected rate of return μ are derived from frequencies measured at different points of time. Because of that one has to consider, that these values are time dependent, i.e. a function of the time difference Δt between two measures.

2 DESCRIPTION OF INVESTMENT AND VALUE OF THE PORTFOLIO

We discuss this on a portfolio with investments done n times, starting without loss of generality at time $t = 0$. The distance between two investments is Δt_1. The whole time between the first and the last investment, we call investment period Δt_{inv}

$$\Delta t_{inv} = (n-1)\Delta t_1$$

After the last investment or the investment period, we assume, that the holding period starts, for a time Δt_2. Also we assume that:

$$\Delta t_2 >> (n-1)\Delta t_1 .$$

Therefore we investigate the portfolio after a time of Δt_{fin} with

$$\Delta t_{inv} = \Delta t_2 + (n-1)\Delta t_1 \cong \Delta t_2 .$$

We optimise our portfolio with respect to μ - $k\sigma$ at the end of the holding period.

In our model the investment is done in one asset only, called asset A. Without loss of generality we call it share A. The amount of money invested at every investment point is *I*. The price of share A at time 0 is called A(0), after Δt_1 the price is A(Δt_1), and at the last point of time of the investment the price is A((Δt_1 (n-1)).

Using the rate of return $r(t)$ as measure of the change of A between t and 0, we define the prices (or values) of the asset A at the stock exchange:

$$\begin{aligned} A(\Delta t_1) &= A(0)(1 + r(\Delta t_1)) \\ A(\Delta t_1) &= A(0)(1 + r(\Delta t_1(n-1))) \end{aligned}.$$

With this at the end of the investment process the portfolio consists of N_A shares. N_A is calculated as follows:

$$N_A = \sum_{i=0}^{n-1} \frac{I}{A(\Delta t_1 \cdot i)} = \sum_{i=0}^{n-1} \frac{I}{A(0)(1 + r(\Delta t_1 \cdot i))}.$$

And the value $V(\Delta t_{fin})$ of this portfolio at the end of the holding process, at Δt_{fin} after $t = 0$ is:

$$V(\Delta t_{fin}) = N_A A(\Delta t_{fin}) = N_A A((n-1)\Delta t_1 + \Delta t_2) = \sum_{i=0}^{n-1} \frac{I}{A(\Delta t_1 \cdot i)} A((n-1)\Delta t_1 + \Delta t_2).$$

Or using *r(t)* we can conclude:

$$V = N_A A(\Delta t_{fin}) = N_A A(0)(1 + r((n-1)\Delta t_1 + \Delta t_2)) = \sum_{i=0}^{n-1} \frac{I}{1 + r(\Delta t_1 \cdot i)} (1 + r((n-1)\Delta t_1 + \Delta t_2)).$$

And using a $\mu - k\,\sigma$ optimisation, one would say, that the following utility function U should be maximal

$$U = N_A(\mu - k\sigma)$$

With μ as expected rate of return.

3 TIME DEPENDENCE OF CHARACTERISTICS OF THE SHARES AND EXAMPLES

The standard deviation σ is calculated from frequencies taken at time intervals Δt_a. Therefore using a $\mu - k\,\sigma$ optimisation, means, that we optimise our portfolio with respect to $\mu - k\,\sigma(\Delta t_2)$. Because the asset A, which is acquired during the investment period, is also the asset called asset A, which is kept during the hold period, we categorize the shares within the following four cases 1-4:

- 1: $\sigma(\Delta t_1)$ and $\sigma(\Delta t_2)$ are small

- 2: $\sigma(\Delta t_1)$ is small, and $\sigma(\Delta t_2)$ is large
- 3: $\sigma(\Delta t_1)$ is large, and $\sigma(\Delta t_2)$ is small
- 4: $\sigma(\Delta t_1)$ and $\sigma(\Delta t_2)$ are large

We can not simply conclude a dependence between $\sigma(\Delta t_1)$ and $\sigma(\Delta t_2)$. Therefore we will give below some numerical discussions of these values based on shares of some well known concerns. We use the following shares:

- Allianz, DE0008404005
- Daimler, DE0007100000
- Siemens, DE0007236101
- BASF, DE0005151005

The data are from the kassa prices of these titles from the years 1999 to 2006. Our investment period $\Delta t1$ is one day, n is 5, and the holding period is 1 year. In our example the investment was on the days between 16.10.2000. and 20.10.2000.

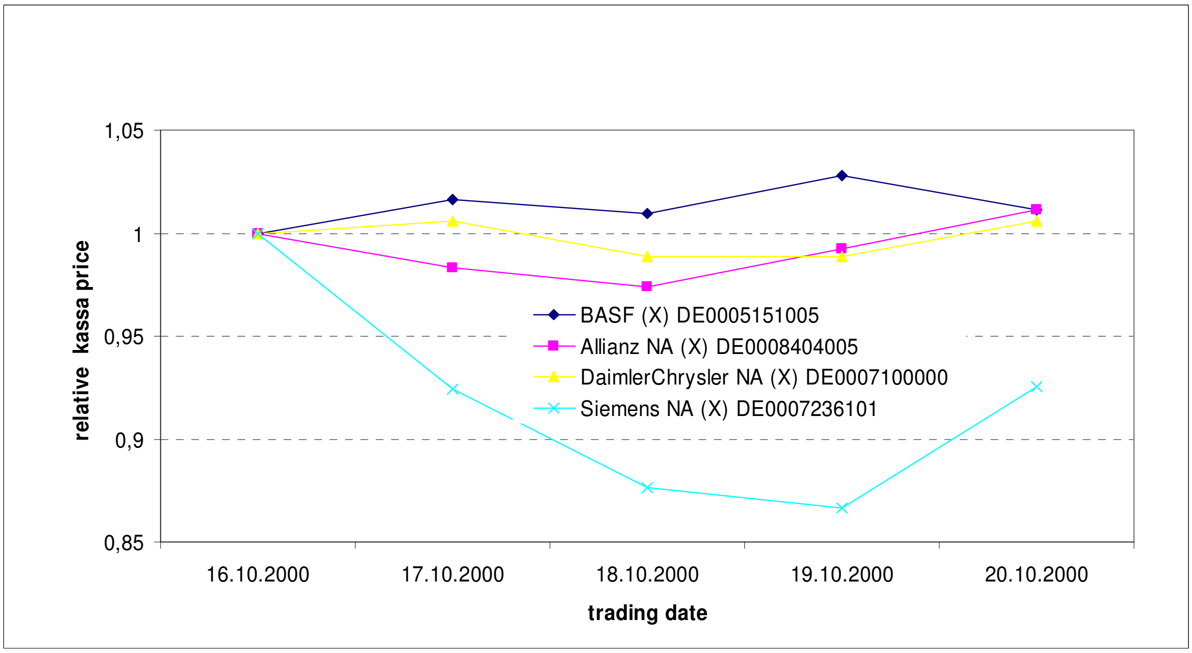

Fig 1: Kassa price of 4 DAX-shares beetween 16.10. and 20.10.2000.

Using simply a μ - $k\sigma$ optimisation one would prefer an investment in a share of categorie 1.

We see from our numerical simulations, that this is not true.

Instead of this portfolios of the asset Allianz and especially Siemens are preferable with respect to the number of shares N_A. (fig.2). This is caused by the higher variation of the asset price value $\sigma(\Delta t_1)$, when buying the assets (fig.1).

Concerning the expected value V of the portfolio; i.e. the number of shares times the expected value of the share, the share of Siemens is preferable because of high expected rate of return and high N_A (fig.2 and fig 3.)

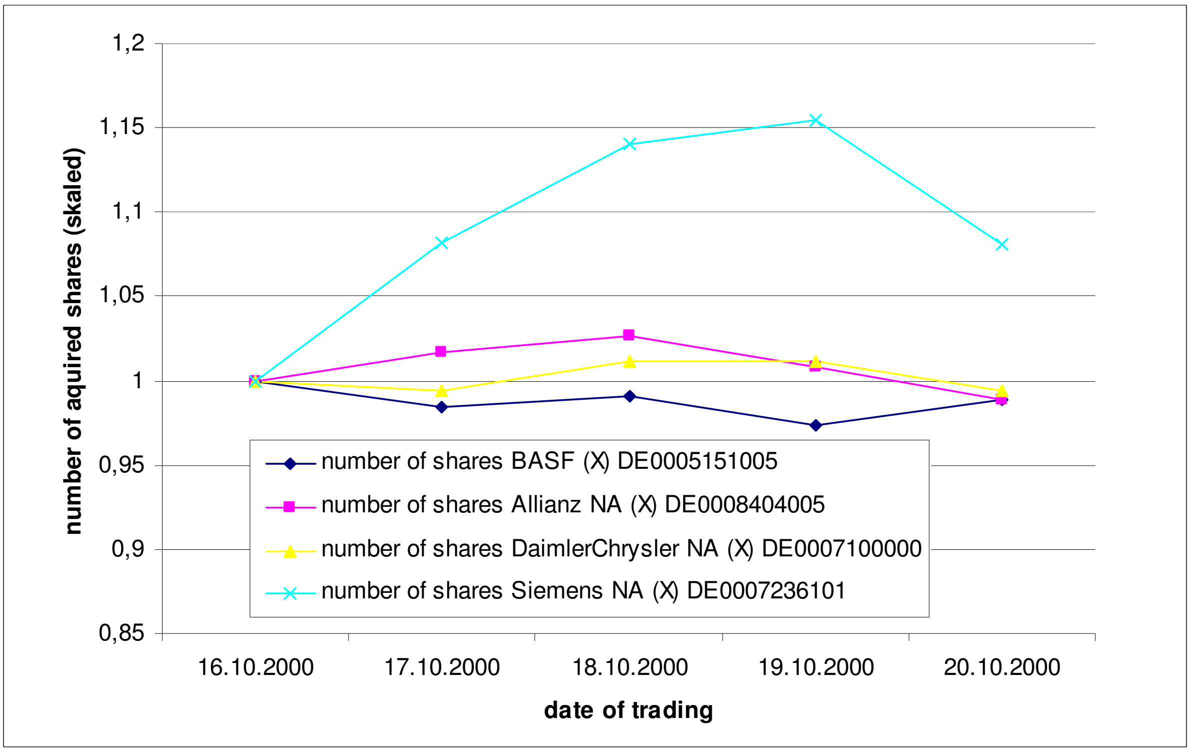

Fig. 2: Number of shares acquired on every trading day beetween 16.10. and 20.10.2000.

Concerning the measure of risk $\mu - k\,\sigma$ - depending on the value of k - the portfolios of BASF and Daimler are preferable, because on high expected rate of return and low $\sigma(\Delta t_2)$ (fig.3). Concerning the utility function U – the portfolios – especially Siemens - with higher values of measure of risk are similar to the portfolios with lower values for the measure of risk , because on higher values of NA.

We think we can give the following explanations and summary for that:

- For the shares (Siemens and also Allianz) of category 3 or 4 the number of shares N_A acquired during the investment period is (much) higher than for shares of category 1or 2. This means, that during the investment, a large $\sigma(\Delta t_1)$ is preferable. This can be called a cost average effect.
- With this (much) higher number of shares in portfolios of category 3 and 4 also the expected value of the portfolio at Δt_{fin} and the value of U at Δt_{fin} is high.
- An optimisation of the portfolio with respect to $\mu - k\sigma(\Delta t_2)$, which does not consider the investment process is insufficient and the number of shares acquired in the investment process.

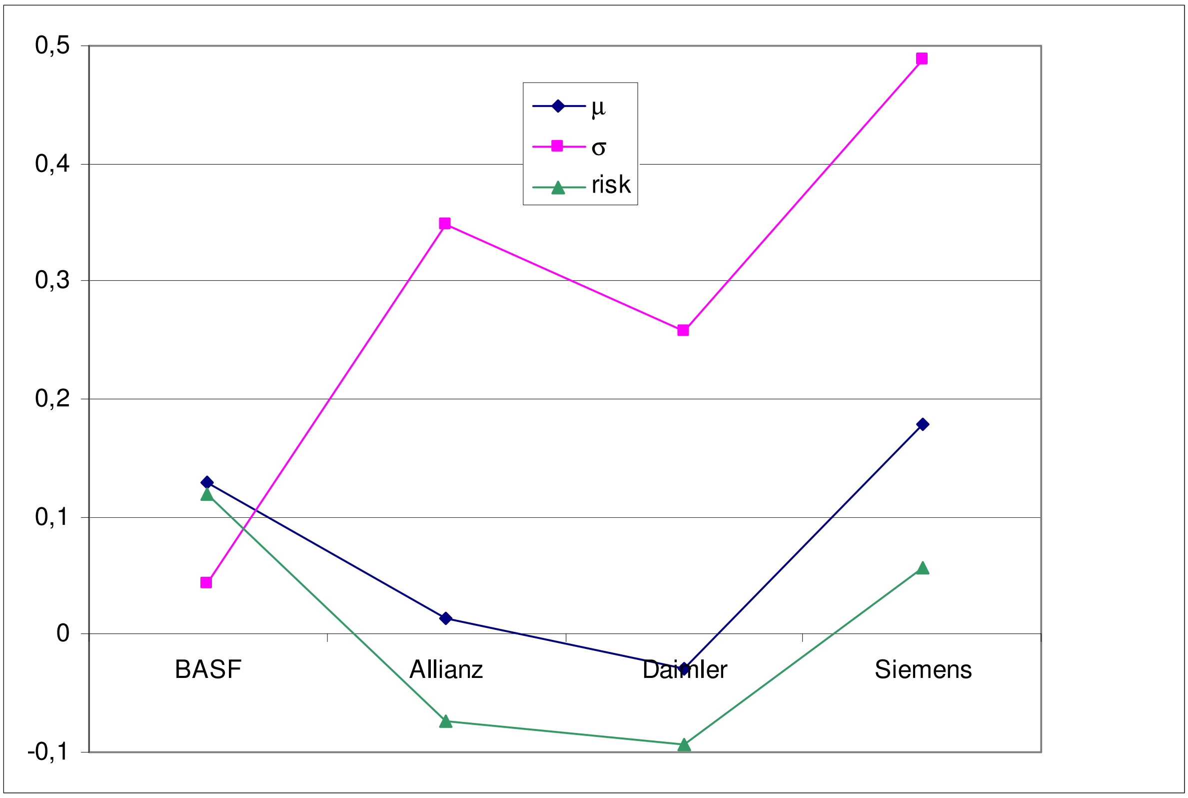

Fig 3: expected rate of return (per year), standard deviation (per year) and measure of risk of the 4 portfolios.

4 CONCLUSION

As we see, this strategy of average cost in combination with an optimization based on $\mu - k\sigma$ gives high values for the portfolio, especially when one notices, that σ depends on the time difference of the single measure points and uses well suited shares.

An optimisation based only on $\mu - k\sigma$ is insufficient, if it does not include the time dependence of s or if it does not include a suitable measure of risk for the investment process or if it even uses the same measure of risk for the investment process, which it uses for the holding process.

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