Risk Perceptions and Rationality in Measures of Risk

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Abstract
Individuals may have biased perceptions of health and safety risks. We conjecture perceptions of mortality risk from various risk measures using parametric and non-parametric methods. We investigate how risk perceptions are measured and what rational explanations can be found for these measures.

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Individuals may have biased perceptions of health and safety risks. This is the issue addressed by many research studies concerning health and safety as well as mortality risk. These studies include among others Lichtenstein *et-al* (1978), Jones-Lee (1989), Benjamin and Dougan (1997), Hakes and Viscusi (1997), Luce and Weber (1986), Sarin (1987), Benjamin *et-al* (2001). Brachinger and Weber (1997) and Brachinger (2003) describe a spectrum of measures of risk that we may broadly divide into symmetric and asymmetric measures of risk.

A question we turn in our study is to ask whether people err in their perceptions of mortality risk. If yes, is it because they behave irrationally or they behave rationally but lack sufficient information when forming perceptions or rather, they have full information and behave rationally but make mistakes because they are subject to bounded rationality. If no, what evidence is there to verify bias in perceptions? The evidence has tended to be in the form of regression analysis as in Benjamin and Dougan (1997) and Benjamin *et-al* (2001) in which risk perceptions may be negatively skewed. Measures of risk that are symmetric like variance or standard deviation are not best measures of risk perception when perceived risks are skewed. Fishburn's (1982, 1984) measures of perceived risk are more appropriate in accounting for skewed risk perceptions. Following Fishburn (1982, 1984) and Unser (1998) we develop a model of lower partial moments as a measure of perceived risk.

In order to show skewed risk perception empirical studies look for any systematic errors in responses to survey questions concerning hazards. Tests involve the significance of dispersion of perceived hazard rates from actual hazard rates. The conclusion has tended to be either to accept or reject the common assertion that people's estimates of hazards to life appear to overstate low probability causes and to understate high probability causes. This conclusion is based on an underlying symmetric distribution of risks.

In order to find out how information acquisition is significant, two groups of individuals are usually considered. A group who has low cost in acquiring information or due to their careers/occupations finds information gathering economically justified. A second group on the other hand is provided with some relevant information to anchor their responses in order to reflect their perceptions. The maintained hypothesis in this regard is that people are well informed about the risks they face but may be uninformed about risks that population faces. Thus in gathering information individuals are assumed to look for the most relevant information and bear in mind the costs involved in acquiring this information. In this sense individuals may be assumed to be behaving rationally. What underlie the maintained hypothesis are the individual particular circumstances like age, occupation and education. Perceptions are more likely to be affected by information about one's own age group, occupation or circumstances than population wide information. People anchor their perceptions also on limited and costly information they have and can acquire. Whereas correction of their perceived image about a hazard or a risk could be adjusting slowly to new information as individuals get older, educate further or change occupations.

If risk perception is affected by individual specific information than population wide information then, the nature of skewness in risk perception may be defined.

In considering how rational individuals behave, attention is drawn to the point that information acquisition by individuals about certain events, are greater than their information gathering about uncertain events. This implies that rational agents' information gathering is subject to bounded rationality and may be less than perfect. Therefore Bayesian learning is cost constrained. This means our Bayesian updating is subject to costs and accordingly the conditional probabilities need to include cost parameters. Costs create inertia in learning and make risk minimization subject to information costs.

If it is true that there is a bias in risk perceptions, then can we conclude that people are irrational in forming perceptions? This bias is due to information asymmetry and decision-making subject to cost considerations. But based on a Bayesian learning process, individuals may be quite rational. If rationality imply judgment with respect to symmetric risk distributions, then people are irrational. But if we allow asymmetric risk distributions as in lower partial moments, then people are not irrational. Does this mean that the results on perceptions of risk and mortality have been path dependent? In this light we will consider regression techniques as well as Bayesian approaches to inference about public perception of hazard and mortality risks.

1. A model of rational risk perception

We start by making the following basic assumptions:

Assumption 1- Individuals gather information efficiently. Costs of obtaining information imply individuals collect the most relevant information concerning them and their cohort.

Assumption 2- Individuals' evaluation of information is rational.

We need to be clear here about what definition of rationality we are using:

a) Define an objective function.
b) Allow asymmetric risk distributions.
c) Allow Bayesian learning with cost considerations.
d) Define decision-making with respect to above.

Proposition 1. Risk perception is negatively skewed when perception is affected by individual specific information and is positively skewed when risk perception is affected by population wide information.

This proposition can be tested empirically to see the effect of information category on risk perception formation and will be tested later.

Proposition 2. Information acquisition is subject to event certainty.

Public perceptions may be formed both by the probabilities of risk and the magnitude of risks which may include other criteria than simple probability to measure risk. Probabilities of risks and in fact the crude rates in many instances are more accessible than any other risk measure. A number of risk measures have been developed in the literature as in Viscusi *et-al* (1997), Viscusi (1993) that involve some measure of life lost in order to quantify the magnitude of risks. Such measures of risks do in fact change the order of importance of risks to life but may not be so obvious intuitively.

In order to estimate some measure of perception of risks faced by individuals, it is necessary to find some quantitative measures of risks faced by them. These measures could be estimates of the hazard rates for any of the ailments individuals are exposed to or the current probability of hazard.

Probability of hazard is explained in terms of the conditional probability of hazard given exposure to risk and the probability of exposure, i.e. $P(D) = P(D|E)P(E)$ where $P(D)$ is the probability of hazard, $P(D|E)$ is the conditional probability and $P(E)$is the probability of exposure to risk. The probability of hazard given exposure is estimated by the frequency of hazard j for individuals aged x, D_{jx} divided by the total number of individuals aged x, N_x i.e. $P(D|E) = D_{jx}/N_x$. But as $P(E) = N_x/N$ where N is total population, a tabular hazard rate can be estimated as the current probability of hazard or $P(D) = D_{jx}/N$. But $P(D)$ calculated this way does not take into account individual specific characteristics and only considers frequencies of hazards. Crude rates do not show clearly hazards to life as individual characteristics like age are ignored. For simplicity we now only consider age as a defining factor between different individuals.

The conditional probability, i.e. the probability of hazard j given individual was exposed aged x, is $\rho_{jx} = D_{jx}/D_x$ where D_x is the total number of individuals aged x exposed to hazard j. Whereas, the probability of hazard for an individual aged x, given the individual was exposed to hazard j is $\lambda_{jx} = D_{jx}/D_j$ in which D_j is the total number of individuals who are subject to hazard j.[1]

Life Lost Expectancy (LLE) and Expected Years of Life Lost (EYLL) are two measures of risk that show how various hazards to life are ranked according to each degree of risk. Based on LLE and EYLL we can calculate a relative measure of risk in order to estimate perceptions of mortality. This means finding a relative number of lives saved based on the mortality

[1] Note that $p_j = \sum \rho_{jx} = EYLL_j / LLE_j$ where EYLL is the Expected Years of Life Lost and LLE is the Life Lost Expectancy.

measure for particular conditions. For example taking auto accidents as a reference point, i.e. anchoring the information given to individuals in their assessment of hazards, the LLE and EYLL for all causes may be compared with the corresponding LLE and EYLL for auto accidents in order to provide a measure of relative risk and hence perception.

Note, in forming perceptions, public knowledge of illnesses is significant. What is a particular version of neoplasm to the medical profession is just cancer to the ordinary public. Perceptions of health risks thus are labeled as one or the other of some well-known ailments. These public labels i.e. the "labeling effect" may not exceed more than a few names about which perceptions are formed.

In Viscusi *et-al* (1997) using regressions to show how public perception may be affected by the mortality measures, death perception is regressed on total number of death as well as life lost expectancy and expected years of life lost variables. The variable representing public perception, in the absence of a survey to provide direct estimates of perception, is the actual fatality data for an ailment. This variable is assumed to represent a proxy variable for perception. This proxy variable is then regressed on total number of death as well as a combination of life lost expectancy and expected years of life lost variables.

The inclusion of the product of life lost expectancy and total death, and total death as an independent regressors, appear to improve the explaining power of the regression. But what is worth noting is not the improved adjusted coefficient of correlation; it is the change in the sign of the total death regressors. While for an additional 22 total deaths, perception increases by one, for more deaths, public perception falls. Hence the more death there is, the less the public perceives it for any condition.

However, perceptions are formed by a learning process, through the probability of exposure to a hazard that can be written as

$$P(E) = \frac{P(D)}{P(D|E)} \tag{1}$$

On the other hand there are risks taking time to cause a death, i.e. there is duration between exposure and death. Duration related risks are those risks taking effect in time, thus lagged for a number of years and thought to explain public perception of mortality as in Viscusi *et-al* (1997). Some illnesses last for a period during which the individual is facing a health risk. For example, most cancer ailments affect the individual for many years. While other conditions, for example motor accidents resulting death take effect immediately. However, it is possible to take note of factors that vary across individuals in a hazard function. The proportional hazard function introduced by Cox (1972) takes into account variations of hazards across individuals. This model has been studied further as in Elbers and Ridder (1982), Heckman and Singer (1984a) and Sickles and Taubman (1997). The probability that a death hazard takes place when the individual is aged t, i.e. during time t and $t + \Delta t$, can be decomposed into a heterogeneity factor, a duration probability, i.e. time dependent and a function of observed covariates. These covariates may include individual specific characteristics or costs of information acquisition. The hazard rate is then written as

$$h(t, x, \beta, \gamma) = \phi(x, \beta)\ p(t)\ \gamma \qquad (2)$$

Where γ is a scalar parameter, representing individual specific characteristics that may be unobservable i.e. heterogeneity factor. $p(t)$ denote probability of hazard dependence on time (age) and $\phi(x, \beta)$ represents factors that depend on time invariant exogenous observed cofactors x and parameters β. The specification in (2) introduces a functional decomposition that describes hazard rates for time invariant hazards, duration dependency for risks that take effect in time and include dummy variables (indicator functions) to incorporate time invariance and duration dependence simultaneously. This decomposition of the hazard function using the proportional hazard technique is also applied in multivariate survival analysis as in Chen *et-at* (2002)

The hazard rate is typically written as:

$$h(t) = \frac{f(t)}{1 - F(t)} \qquad (3)$$

Where $F(t) = \int_0^t f(t)\,dt$ represents the probability distribution of hazard with density function $f(t)$ defined over time t and $S(t) = 1 - F(t)$ is survival function. The shape of the hazard function in (3) depends on the particular probability distribution $F(t)$ that best describes hazard probability from data. A number of probability distributions may be used; in particular the constant hazard $h(t) = h$, exponential function and log logistic have been used in the literature. The choice of these distributions depends usually on the goodness of fit criteria.

If $F(t) = \frac{\lambda t^p}{1 + \lambda t^p}$ then we can write (3) in log-logistic form as

$$h(t) = \frac{\lambda p t^{p-1}}{1 + \lambda t^p} \qquad (4)$$

Where, p and λ are parameters representing duration dependence and time invariance, accordingly. As $t \geq 0$ and $\lambda > 0$, whenever $0 < p \leq 1$ the hazard function in (4) exhibits negative time dependence (decreasing hazard) and for $p > 1$ it has positive time dependence (increasing hazard) which can be established by showing $\frac{d}{dt} h(t) < 0 \quad or \quad > 0$ respectively. In particular we are interested to use this functional form to show skewness of perception depending on information about risks. The p parameter is also known as the "shape parameter". This parametric hazard function allows us to model increasing and decreasing hazards that are typical of mortality experiences of people with hazard falling and then rising with age or other characteristics.

This method of estimation has also been adopted in finance literature as in DeYoung (1999) and Whalen (1991). While a rigorous treatment of properties of general hazard functions, identifiability and the existence of a non-parametric maximum likelihood estimator is given in Heckman and Singer (1984a, b). The hazard function in (4) is parametric and thus continuous as opposed to non-parametric hazard functions that are discrete. It is possible then, to include covariates, i.e. independent regressors that measure or explain characteristics that may vary across individuals but fixed, at least for a period for every individual. These covariates could be characteristics such as life-style, gender, location, etc. where they may be used in the form of dummy variables when considering risk that take effect in time.

We can expresses $\phi(x,\beta)$ in (2) as a function of covariates, i.e.

$$\phi(x,\beta) = e^{-\beta x_i} \quad for \quad i = 1, \cdots, M \tag{5}$$

Where, x_i are dummy variables representing covariates, M is the number of individuals and β is survival time parameter. Equation (5) can be estimated using the maximum likelihood method. For the log-logistic function we could write (5) as

$$L = \prod_{i=1}^{M} \left[f\left(t_i | p, \beta\right)\right]^{\delta_i} \left[S\left(t_i | p, \beta\right)\right]^{1-\delta_i} \tag{6}$$

Where $\delta_i = 0$ if individual survives and 1 otherwise. Now substituting for $f(t) = h(t)S(t)$ in (6) and taking logs and then maximizing the log likelihood, we get

$$\ln L = \sum_{i=1}^{M} \ln h\left(t_i | p, \beta\right) + \sum_{i=1}^{M} \ln S\left(t_i | p, \beta\right) \qquad where \quad i \le M \tag{7}$$

As all individuals die eventually, the likelihood function may be written as

$$L = \prod_{i=1}^{M} \left[\varphi f\left(t_i | p, \beta\right)\right]^{\delta_i} \left[(1-\varphi) + \varphi S\left(t_i | p, \beta\right)\right]^{1-\delta_i} \tag{8}$$

But when considering perception as a function of the implicit rate of time preference, we may not be able to estimate the implicit rate of time preference for individuals directly. However this rate can be measured as the marginal rate of substitution of health, now as opposed to in the future. Sunstein (1997) suggests that people may trade off quantity of life for the quality of life. The marginal rate of substitution then offers a way to estimate the rate of discount between quality of life versus quantity of life. In order to discount lives lost, the marginal rate of substitution can be taken as a proxy for this rate of discount. It is estimated in Viscusi *et-al* (1997) that a 3.3% to 12.4% rate of discount shows a best fit in their regression analysis. But the alternative rate of time preference that is derived from the marginal rate of substitution shows that this rate may be variable throughout the lifetime of individuals. In order to calculate the rate of discount, Viscusi *et-al* (1997) add a third parameter, the discount rate, r, to their perception equations and then search for the best rate, i.e. fit various percentages such that the best fit is obtained for the equations. The criteria chosen for best finding a rate of discount is the residual sum of squares, the standard error of the parameters and the $\overline{R}^2$ of the regression.

However, in the life cycle model of consumption as in Newman and McCulloch (1984) who adopt a hazard rate technique (the occurrence and timing of an uncertain event) a rate of discount is estimated in an economic context in which demand for children is considered. They use the maximum likelihood estimation method to explain variations in fertility rates in Costa Rica. While in Hurd (1989), the life cycle model of consumption is augmented to take into account the effects of mortality and bequests.

In regression models of mortality in contrast with the hazard rate models, that dominate the demographic and population economics literature, Woplin (1997) surveys studies that use the regression technique for a number of mortality risk measures. In these regression models, covariates include a number of explanatory variables in order to estimate infant and child mortality. Similarly, Sickles and Taubman (1997) survey estimates of mortality among the elderly. Here we see factors affecting health include: Education, ability, earnings, job satisfaction, gender, smoking, weight, hypertension, high cholesterol, nutrition with the community price of food as a proxy variable, occupation to indicate the level of physical activity, exit from labour force (i.e. retirement, disability, unemployment and death) and marital status. These socio-economic factors are used to explain risks faced at an individual level.

We have tried to highlight problems associated with measuring and estimating perceptions of an uncertain event: Life. While it is no uncertainty that all lives will end, but the timing is uncertain. Perceptions regarding this uncertainty are affected by a number of factors in particular age, occupation and education. The methods that are used in estimating perceptions are by no means definite. There appears to be a substantial number of issues yet to be resolved and I hope this discussion will help to answer some of them.

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