ON A ROBUST PARAMETER-FREE PRICING PRINCIPLE:

FAIR VALUE AND RISK ADJUSTED PREMIUM

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Abstract

For the space of all feasible risks with arbitrary mean and standard deviation and a fixed limit, the use of the rule of thumb "actuarial premium = mean + half of standard deviation" is justified. It is shown that this actuarial premium coincides with the maximum of the minimum risk adjusted premium obtained from a simple solvency model under the assumption that the supervising authority chooses the minimum level of "fair" actuarial premium and values the insolvency risk with the risk-neutral distortion risk measure. A case study using dynamic Monte Carlo simulation of the balance sheet of a portfolio of endowment life insurance shows that the introduced profit loading absorbs in average the random claims fluctuations.

Key words

axiomatic pricing models, minimax principle, coherent risk measure, solvency

1. Introduction.

In practice actuarial premiums are usually set using the expected value principle, the variance principle or the standard deviation principle, where different methods are applied to determine the unknown loading factor. From a theoretical perspective, pricing principles should take into account the whole probability distribution of the risk to be covered. However, in applied work often only a few characteristics of the insurance risk are known, for example the mean and variance. But, risks with identical first two moments may have very different probability distributions, and it is difficult to determine a unique price based solely on the knowledge of the mean and variance. In the presence of incomplete information, say the first few moments of the risk and the range, it is possible to compute stochastic bounds for the risk in its moment space. If actuaries agree to compute prices, not on the actual risk, but rather using these stochastic bounds, then compromise solutions at the interface between theory and practice can be determined.

Based on the statistical knowledge of the mean and coefficient of variation, as well as on a finite range of the risk, the stop-loss ordered extremal actuarial premiums for various plausible pricing principles can be determined. If one requires further that in the extreme situation of a maximum coefficient of variation, the price of a risk should be maximum and uniquely defined, then the obtained distribution-free actuarial premiums can be made parameter-free and compared. This idea has been thoroughly analysed in Hürlimann(2001).

In the present paper the focus lies on pricing risks in the space of random variables with arbitrary values of the mean and standard deviation but with a fixed limit. To fix ideas assume that actuarial premiums are set using the standard deviation principle with unknown but constant loading factor. Then, by fixed and known limit but unknown mean, the maximum variance on the space of all feasible risks will itself be maximum provided the mean and standard deviation equals half of the limit. A comparison with previous results shows that the loading factor must be equal to a half. This extreme situation justifies the use of the robust parameter-free pricing principle "actuarial premium = mean + half of standard deviation", which is often applied as a rule of thumb in setting actuarial premiums for (re)insurance captives. This is the content of Section 2.

Next, we ask if this pricing principle is consistent from a solvency point of view. It is assumed that the insolvency risk is valued using a distortion risk measure, which is a particular case of a coherent risk measure. Section 3 recalls these notions. Then, in Section 4, to determine the economic risk capital and the cost of capital of an insurance risk business, we apply the simple model of solvency first introduced by Dhaene et al.(2003), which yields a specific "optimal" risk adjusted premium formula. If a supervising authority chooses the minimum level of "fair" actuarial premium and values the insolvency risk with the risk-neutral distortion measure, then the maximum of the minimum risk adjusted premium coincides with the robust parameter-free premium derived in Section 2 and justifies its use for solvency purposes. Section 5 discusses a case study for a portfolio of endowment life insurance policies. It is shown that the introduced profit loading absorbs in average the random claims fluctuations.

2. A parameter-free actuarial pricing principle.

In the distribution-free approach of Hürlimann(2001) the focus is on the space of risks $D := D([0, L]; \mu, \sigma)$ of all random variables with fixed mean μ, standard deviation σ, and bounded support $[0, L]$. This apparent restriction takes into account that statistical knowledge beyond the mean and variance is seldom available, and the fact that an upper limit L on an insurance risk is often fixed by contract or via reinsurance.

The most common axioms a pricing principle should satisfy are the following ones:

(P1) $P[S] \geq E[S]$ for all $S \in D$

(P2) $P[S] \leq \sup[S] = L$ for all $S \in D$

(P3) $P[c] = c$ for all $c \geq 0$ (no unjustified loading)

(P4) $P[S+T] \leq P[S] + P[T]$ for all $S, T \in D$ such that $S + T \in D$ (subadditivity)

(P5) $P[S] \leq P[T]$ if $S \leq_{sl} T$ and $S, T \in D$ (stop-loss order preserving property)

(P5) means that actuarial premiums should be consistent with the risk preferences of risk-averse decision makers having a concave non-decreasing utility function. Based on the elementary axioms (P1)-(P5), the main pricing systems considered so far take into account further additive restrictions on pricing strengthening axiom (P4). To be able to compare distribution-free premiums of different pricing principles, a further normalizing axiomatic assumption is made:

(P6) In the extreme situation of maximum coefficient of variation $k_{\max}^2 = \dfrac{L-\mu}{\mu}$ for risks with support $[0, L]$, the actuarial premium should be maximum and uniquely defined.

As a main result one obtains the following uniquely defined distribution-free and parameter-free pricing principle.

Theorem 2.1. Let $S \in D([0, L]; \mu, \sigma)$ be an insurance risk with known mean μ and finite support $[0, L]$ but unknown standard deviation σ. Suppose actuarial premiums are set using one of four plausible pricing principles, namely the exponential principle, the truncated linear zero utility principle, the PH-transform principle and the Dutch principle. If the actuarial premium is based on the maximum variance $(L-\mu)\mu$ and the common axioms (P1)-(P6), then the actuarial premium is uniquely given by the formula

$$P[S] = \left[1 + \frac{k_{\max}^2}{1 + k_{\max}^2}\right] \cdot \mu . \tag{2.1}$$

Proof. This follows through comparison of the Propositions 3.3 to 3.4 in Hürlimann(2001) under the made assumptions. ◊

As an interesting and intriguing question, one may ask whether this result is of use in a more general setting, for example to set actuarial premiums for insurance risks in the space of random variables $D([0,L];\mu,\sigma)$ with arbitrary values of the mean and standard deviation. Suppose actuarial premiums are set using the standard deviation principle $P[S]=\mu+\theta\sigma$ with unknown but constant loading factor θ. By fixed and known limit L but unknown mean, the maximum variance $(L-\mu)\mu$ on the space $D([0,L];\mu,\sigma)$ will itself be maximum provided one has

$$\mu=\sigma=\tfrac{1}{2}L. \tag{2.2}$$

Then the maximum coefficient of variation equals $k_{\max}=1$ and a comparison with (2.1) shows that the loading factor must be equal to $\theta=\tfrac{1}{2}$. This extreme situation justifies the use of the robust parameter-free pricing principle

$$P[S]=\mu+\tfrac{1}{2}\sigma \tag{2.3}$$

for the space of all insurance risks $S\in D([0,L];\mu,\sigma)$.

Remark 2.1.

It is worthwhile and instructive to compare this robust and parameter-free pricing principle with similar pricing principles obtained from actuarial rules imposed by regulator authorities. For example, according to the "Grand Ducal Regulation" dated 31 December 2001, which specifies the terms of approval and operation of reinsurance companies with domicile in Luxemburg, any such company must, for all its activities, constitute a provision for fluctuation of claims. The so-called *multiple* for equalization reserves determines this provision as a multiple of the average of premiums received during the past five years. Roughly speaking the multiple of a risk or portfolio of risks is equal to the half-number above the sextuple of the standard deviation of the ratio of claims burden to premiums received. The analysis in Hürlimann(2006) of the theoretical maximum multiple of 17.5 for limiting cases shows that this value is attained provided the actuarial premium is set according to the robust parameter-free standard deviation principle

$$P[S]=\mu+\tfrac{12}{35}\sigma=\mu+0.343\sigma. \tag{2.4}$$

If instead actuarial premiums are set following the derived robust pricing principle (2.3), then the corresponding theoretical maximum multiple turns out to be 24 instead of 17.5.

3. The notion of coherent distortion risk measures.

The axiomatic approach to risk measures is an important topic, which has applications to premium calculation and capital requirements. Besides the *coherent risk measures* by Arztner et al.(1997/99), one is interested in the *distortion measures* by Denneberg(1990/94), Wang(1995/96). Under certain circumstances, distortion measures are coherent risk measures (e.g. Wang et al.(1997), Theorem 3). For this reason, they can be used to determine the capital requirements of a risky business, as suggested first by several authors including Arztner(1999), Wirch and Hardy(1999), Wang(2002) and Dhaene et al(2003).

Let (Ω, A, P) be a probability space such that Ω is the space of outcomes or states of the world, A is the σ-algebra of events and P is the probability measure. For a measurable real-valued random variable X on this probability space, that is a map $X:\Omega \to R$, the probability distribution of X is defined and denoted by $F_X(x) = P(X \le x)$.

In the present paper, the random variable X represents a financial loss such that for $\omega \in \Omega$ the real number $X(\omega)$ is the realization of a loss and profit function with $X(\omega) \ge 0$ for a loss and $X(\omega) < 0$ for a profit. A set of financial losses is denoted by χ. A *risk measure* is a functional from the set of losses to the extended non-negative real numbers described by a map $R:\chi \to [0,\infty]$. A *coherent risk measure* is a risk measure, which satisfies the following desirable properties (e.g. Arztner et al.(1997/99)) :

(M) (monotonicity) If $X, Y \in \chi$ are ordered in stochastic dominance of first order, that is $F_X(x) \ge F_Y(x)$ for all x, written $X \le_{st} Y$, then $R[X] \le R[Y]$

(P) (positive homogeneity) If $a > 0$ is a positive constant and $X \in \chi$ then $R[aX] = aR[X]$

(S) (subadditivity) If $X, Y, X+Y \in \chi$ then $R[X+Y] \le R[X] + R[Y]$

(T) (translation invariance) If c is a constant and $X \in \chi$ then $R[X+c] = R[X] + c$

Consider a continuous increasing function $g:[0,1] \to [0,1]$ such that $g(0) = 0$ and $g(1) = 1$, called a *distortion* function. The dual transform $\gamma(x) = 1 - g(1-x)$ of a distortion function is called a *dual distortion* function. For $X \in \chi$ with probability distribution $F_X(x)$, the transform $F_X^g(x) := g(F_X(x))$ defines a distribution function, which is called the *distorted distribution* function. The dual distortion function defines a transformed distribution function $F_X^\gamma(x) := \gamma(F_X(x))$, which is called the *dual distorted distribution* function.

The continuity assumption ensures the validity of the following representation. Taking the mean value with respect to the distorted distribution of a loss $X \in \chi$ with probability distribution $F_X(x)$, one obtains the *distortion (risk) measure*

$$R_g[X] = \int_0^\infty \left[1 - F_X^g(x))\right] dx - \int_{-\infty}^0 F_X^g(x) dx . \tag{3.1}$$

Similarly, the dual distorted distribution defines the *dual distortion (risk) measure*

$$R_\gamma[X] = \int_0^\infty \left[1 - F_X^\gamma(x))\right]dx - \int_{-\infty}^0 F_X^\gamma(x)dx \tag{3.2}$$

One notes that the dual transform $\gamma(x) = 1 - g(1-x)$ implies the following alternative dual representations of the distortion measures (3.1) and (3.2) in terms of the *distorted survival* function $\overline{F}_X^g(x) := g(\overline{F}_X(x)) = 1 - F_X^\gamma(x)$ and the *dual distorted survival* function $\overline{F}_X^\gamma(x) := \gamma(\overline{F}_X(x)) = 1 - F_X^g(x)$ associated to the survival function $\overline{F}_X(x) = 1 - F_X(x)$:

$$\overline{R}_g[X] = \int_0^\infty \overline{F}_X^g(x)dx - \int_{-\infty}^0 \left[1 - \overline{F}_X^g(x))\right]dx = R_\gamma[X] \tag{3.3}$$

$$\overline{R}_\gamma[X] = \int_0^\infty \overline{F}_X^\gamma(x)dx - \int_{-\infty}^0 \left[1 - \overline{F}_X^\gamma(x))\right]dx = R_g[X] \tag{3.4}$$

Wang et al.(1997), Theorem 3, implies that the risk measures (3.3) and (3.4) are coherent risk measures provided $g(x)$ ($\gamma(x)$) is a concave (convex) function. This implies that (3.1) and (3.2) are coherent provided $g(x)$ ($\gamma(x)$) is a convex (concave) function. The only disadvantage of the present approach is that risk measures such as value-at-risk (VaR) are not distortion risk measures because the distortion function is discontinuous in this case.

4. A minimax solvency approach to the parameter-free actuarial pricing principle.

Suppose an insurance risk business of a corporate company covers random claims of total amount S for an appropriate *actuarial premium* $P = P[S]$. The *insurance risk* at the beginning of some insurance period, which is associated to this risk business, is measured by the *risk process* random variable $X = S - P$. To protect the insurance business against random fluctuations in the risk process, one is interested in the evaluation of *economic capital* for the insurance risk, which is assumed to be a function of the risk process only and is denoted by $EC[X]$. To finance the economic capital, the corporate company has to consider the associated *cost of capital*, which also depends on the risk process and is denoted by $CoC[X]$. The total cost of risk, which consists of the sum of the actuarial premium and the cost of capital, is called *risk adjusted premium* and is denoted by $P^a = P^a[S] = P[S] + CoC[S - P[S]]$. To determine the economic risk capital and the cost of capital we apply a simple model of solvency introduced in Dhaene et al.(2003) as further developed in Hürlimann(2004a).

To avoid the technical insolvency risk, which happens when $X > 0$, an insurer borrows at the beginning of the period and at the interest rate i some economic capital $C = EC[X]$. The insurance company invests this capital at the risk-free interest rate $r < i$. The resulting (net) *interest on capital* $i_C \cdot C$, with $i_C = i - r$ the *cost of capital rate*, should be as small as possible. On the other hand, insolvency occurs if $X > C$, hence C should be as large as possible. Therefore, an "optimal" compromise solution must be found. Theoretically, to eliminate the insolvency risk, the insurer could buy on the insurance market (if available) a stop-loss contract with pay-off function $(X-C)_+$. If the price of such a contract is set using a risk measure $R[X]$, then the *cost of solvability* equals $R[(X-C)_+]$. The sum of the cost of solvability and interest on capital determines the *cost of capital function* $f(C)=CoC[X]=R[(X-C)_+]+i_C\cdot C$, which should be minimized. Assume insurance market prices are set using a coherent distortion measure such that $R[X]=\overline{R}_g[X]$ for all X. Using the distorted survival function $\overline{F}_X^g(x)=g(\overline{F}_X(x))$ associated to the survival function of X, the cost of capital function can be rewritten as

$$f(C)=E^g[(X-C)_+]+i_C\cdot C, \tag{4.1}$$

where $E^g[X]$ denotes expectation of X under the distorted survival function. Under the assumption of *continuous* distributions, the *optimal economic capital*, which minimizes (4.1), and the corresponding *minimum cost of capital* are determined as follows (formulas (5) and (7) in Dhaene and Goovaerts(2002)) :

$$EC[X]=\left(\overline{F}_X^g\right)^{-1}(i_C)=F_X^{-1}\left(1-g^{-1}(i_C)\right) \tag{4.2}$$

$$CoC[X]=f(EC[X])=i_C\cdot E^g\left[X\,|\,X>EC[X]\right] \tag{4.3}$$

The formula (4.2) identifies the value of the optimal economic capital, that is $EC[X]$, as the *value-at-risk* of the risk process at the confidence level $\alpha=1-g^{-1}(i_C)$, that is $EC[X]=VaR_\alpha[X]$ in the usual notation. Similarly, the value of the minimum cost of capital identifies with the interest at the cost of capital rate on the *distorted conditional value-at-risk* of the risk process at the same confidence level evaluated with respect to the distorted survival function, that is $i_C\cdot E^g\left[X\,|\,X>VaR_\alpha[X]\right]=i_C\cdot CVaR_\alpha^g[X]$, where the latter notation remembers the usual notation of conditional value-at-risk. In this setting, the considerable amount of research on related matters remains applicable (see for example Hürlimann(2003/04b), Furman and Landsman(2006)).

Using the obtained optimal economic capital and minimum cost of capital, one obtains immediately an *optimal risk adjusted premium* formula of the type

$$P^a=P^a[S]=(1-i_C)\cdot P[S]+i_C\cdot CVaR_\alpha^g[S]. \tag{4.4}$$

If we use as coherent distortion risk measure the Wang right-tail measure defined by $g(t)=\sqrt{t}$, which has been justified through mathematical characterization in Hürlimann(2004a), the following relationship between confidence level and cost of capital rate must hold:

$$\alpha=1-i_C^2. \tag{4.5}$$

Of course, the straightforward choice $g(t)=t$ associated to the risk-neutral expected value measure $R[X]=E[X]$ also fulfills the required axioms for a sound risk measure and leads to the less conservative confidence level

$$\alpha = 1 - i_C . \tag{4.6}$$

A recent generalization of the results in Hürlimann(2004a) by Bellini and Caperdoni(2006) shows that the choice $g(t)=t$ is the unique possible choice provided the space of risks contains some specific 4-atomic discrete type distributions. To the knowledge of the author it is still unknown whether these generalized results apply to continuous type distributions. Nevertheless, a regulatory authority, whose first concern is to settle minimum requirements a risk adjusted premium should fulfill, will logically opt for the minimum level of "fair" actuarial premium, that is choose $P[S]=E[S]$ according to axiom (P1) in Section 2, and also measure risk using the risk-neutral expected value measure $R[X]=E[X]$ with $g(t)=t$. Inserting these minimum requirement choices into (4.4) yields the *minimum risk adjusted premium* formula

$$P_{\min}^{a}(\alpha) = P_{\min}^{a}(\alpha)[S] = \alpha \cdot E[S] + (1-\alpha)\cdot CVaR_{\alpha}[S]. \tag{4.7}$$

Recall that by known mean and standard deviation, the conditional value-at-risk functional satisfies the inequality (e.g. Hürlimann(2002) and Schmitter(2005)):

$$CVaR_{\alpha}[S] \le \mu + \sqrt{\frac{\alpha}{1-\alpha}}\cdot\sigma . \tag{4.8}$$

Inserting this upper bound into (4.7) and maximizing the obtained quantity with respect to the confidence level, which is attained when $\alpha = \frac{1}{2}$, shows that the robust parameter-free pricing principle (2.3) identifies with the following *minimax risk adjusted premium*

$$\max_{S,\alpha}\left\{P_{\min}^{a}(\alpha)[S]\right\} = \mu + \tfrac{1}{2}\sigma . \tag{4.9}$$

The present alternative derivation of the robust parameter-free pricing principle justifies its use from a solvency point of view.

5. Dynamic Monte Carlo simulation of the profit of a life insurance portfolio.

Let us briefly illustrate our proposal for a robust pricing principle at the typical example of a life insurance portfolio of endowment policies. Our goal is to show that the introduced profit loading absorbs in average the random claims fluctuations.

Consider a portfolio of endowment policies with the following characteristics:

$N_0 = 1'000$	: number of contracts at entry
$x = 40$	: age at entry
$n = 20$	: term of insurance
$S = 100$	: sum insured
$i = 2\%$	: technical interest rate

For the sake of analytical verification assume that the Life Table is modeled by Gompertz' survival probability law such that

$$ {}_t p_x = \exp\left\{ e^{-\left(\frac{m-x}{b}\right)} \cdot \left(1 - e^{\frac{t}{b}}\right) \right\}, \quad m = 85, b = 10, x = 40. \tag{5.1}$$

The usual standard actuarial calculations yield the net level premium

$$P^N = \frac{A_{x:n]}}{\ddot{a}_{x:n]}} = 4.15899 \tag{5.2}$$

and the standard deviation of the portfolio loss (e.g. Gerber(1986), p.65)

$$\sigma = \sqrt{\frac{1}{N_0} \sum_{k=0}^{n-2} v^{2(k+1)} \left(S - {}_{k+1}V\right)^2 {}_{k+1}p_x q_{x+k}} = 0.32257 \tag{5.3}$$

Our robust pricing principle yields the following profit loaded actuarial premium

$$P^L = P^N + \frac{1}{2} \frac{\sigma}{\ddot{a}_{x:n]}} = 4.16886\,. \tag{5.4}$$

To study the effect of the loading on the balance sheet of the life portfolio, let us apply dynamic Monte Carlo simulation to compare simulated values of the profit loading and the equity (difference between assets and liabilities) at the expiration date of the life portfolio. For this we need the following quantities:

E_t	: equity at time t
G_t^L	: profit loading income in period $(t-1, t]$ at time t
$AG_t^L = AG_{t-1}^L(1+i) + G_t^L$	: accumulated aggregate profit loading income at time t
Q_t	: random number of deaths in period $(t-1, t]$
$N_t = N_{t-1} - Q_t$	: random number of contracts in portfolio at time t
π_t^R	: actuarial risk premium in period $(t-1, t]$ due at time $t-1$
${}_tV$	: actuarial reserve at time t

For the purpose of a simplified illustration, we assume that the market interest rate in each period is equal to the technical interest rate. Under this assumption the equity develops dynamically as follows:

$$E_t = E_{t-1} \cdot (1+i) + N_{t-1} \cdot \left(\pi_t^R \cdot (1+i) - Q_t \cdot (S - {}_tV)\right) + G_t^L, \qquad (5.5)$$
$$G_t^L = N_{t-1} \cdot \left(P^L - P^N\right)(1+i).$$

In theory, and in the special case $E_0 = 0$, the expected values of the equity and accumulated aggregate profit loading income at expiration time coincide:

$$E_Q[E_n] = E_Q[AG_n^L] = 239.7\,. \qquad (5.6)$$

However, Chief Risk Officers of today's insurance companies would like to see in simulation runs that the profit loading effectively absorbs in average claims fluctuations. A potential efficient software tool to get at once all useful entries of a simulated life insurance portfolio will be a dynamic Monte Carlo simulation module planned in future *riskpro*™ versions for life insurance by IRIS integrated risk management AG. To run Monte Carlo simulations we assume that the number of deaths given the number of contracts in the portfolio is conditional Poisson distributed, that is $Q_t | N_{t-1} \sim Po(N_{t-1} q_{x+t-1})$. The figures 5.1 and 5.2 show the result of 50 simulation runs of the balance sheet. These results show that the difference between equity and aggregate profit loading at expiration is slightly positive, which shows that the "robust" profit loading absorbs in average the random claims fluctuations. In fact, for our 50 simulation runs we have $Average(E_n) = 249.6 > Average(AG_n^L) = 239.7$.

<u>Figure 5.1</u>: Simulated values of the accumulated aggregate profit loading

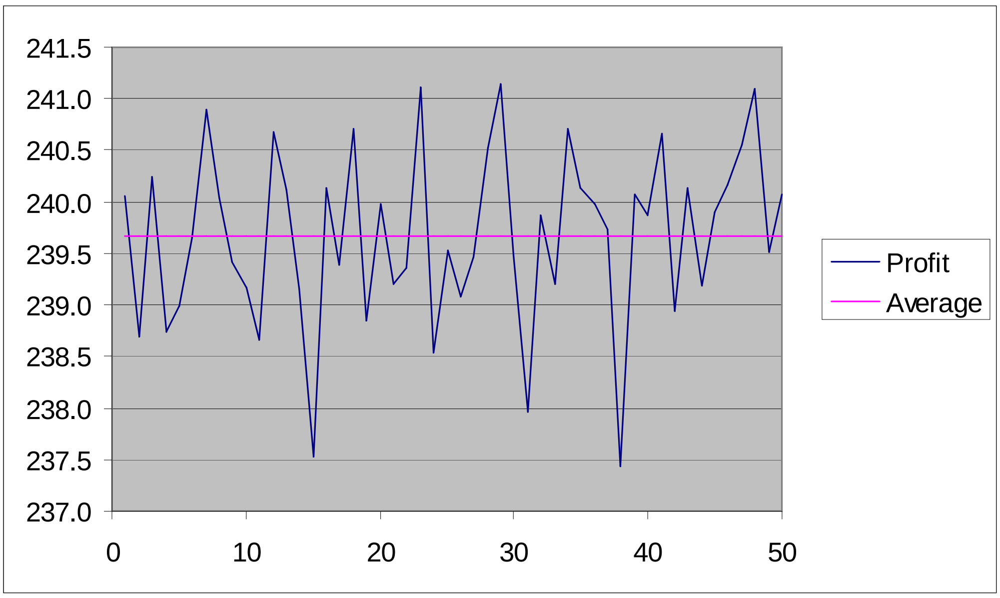
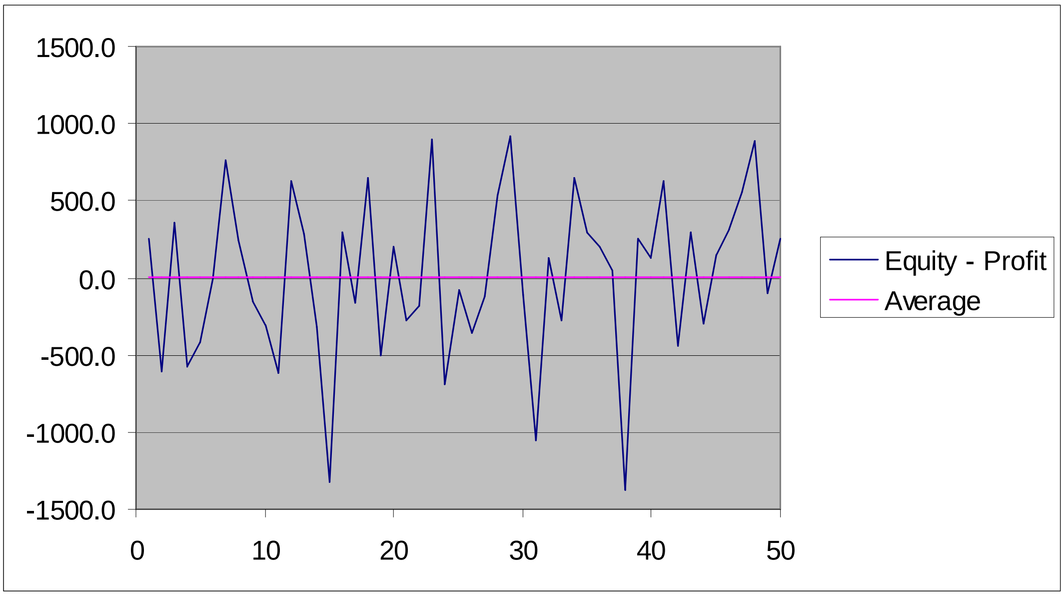


Figure 5.2: Simulated values of the difference between equity and aggregate profit loading



It is now justified to ask the following question. What will be the effect of choosing a smaller profit loading? In general, the profit loading does not absorb with certainty the random claims fluctuations in simulation runs. It is intuitively clear that a smaller profit loading will reduce the margin between average equity and average aggregate profit loading at expiration. To illustrate, if we use 20% of the standard deviation instead of 50% as profit loading in (5.4), then a run of 50 simulations of the balance sheet shows that $Average(E_n) = 116.9 < Average(AG_n^L) = 191.6$, which represents an average loss in profit of amount 74.7 or about 40% of the expected profit loading.

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