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Enterprise Risk Management in Insurance Groups: Measuring Risk Concentration and Default Risk

Nadine Gatzert (corresponding author)
Hato Schmeiser
Stefan Schuckmann

Institute of Insurance Economics, University of St. Gallen
Kirchlistrasse 2, 9010 St. Gallen, Switzerland
Tel.: +41 (71) 243 4012
Fax: +41 (71) 243 4040
E-mail address: nadine.gatzert@unisg.ch

Abstract: In financial conglomerates and insurance groups, enterprise risk management is becoming increasingly important in controlling and managing the different independent legal entities in the group. The aim of this paper is to assess and relate risk concentration and joint default probabilities of the group's legal entities in order to achieve a more comprehensive picture of an insurance group's risk situation. Risk concentration is measured using the concept of economic capital. We further examine the impact of the type of dependence structure on results by comparing linear and nonlinear dependencies using different copula concepts under certain distributional assumptions. Our results show that even if financial groups with different dependence structures do have the same risk concentration factor, joint default probabilities of different sets of subsidiaries can vary tremendously.

Keywords: Economic Capital, Enterprise Risk Management, Diversification, Risk Concentration, Dependence Structures, Copulas/Multivariate Distributions, Financial Conglomerate

1. INTRODUCTION

During the last several years, there has been a trend toward consolidation (M&A activities) in the financial sector of many countries (see Amel, Barnes, Panetta, and Salleo, 2004). Regulatory authorities, as well as rating agencies, are concerned with new types of risk and risk concentrations arising in financial conglomerates and with how to properly assess them in group supervision (for the European Union, see, e.g., CEIOPS, 2006). In this context, enterprise risk management (ERM) has become increasingly important. ERM takes a comprehensive view of risk and helps manage risks in a holistic and consistent way (CAS, 2003). The aim of this paper is to provide a detailed and more comprehensive picture of an insurance group's risk situation by assessing and relating the risk concentration and joint default probabilities of its legal entities. We further examine the impact of the type of dependence structure on results by comparing linear and nonlinear dependencies using the concept of copulas.

Determination of the risk concentrations of an insurance group can be based on an analysis of diversification effects at a corporate level since diversification is the opposite of concentration. In particular, risk concentrations, like interdependencies or accumulation, reduce diversification effects. The diversification effect is measured with the economic capital of an aggregated risk portfolio, which implicitly relies on the assumption that different legal entities are merged into one. An essential aspect in aggregating risks is modeling the dependence structure using linear and nonlinear dependencies. Copula theory can be used to model nonlinear dependencies in extreme events and to test the financial stability of a conglomerate structure.

There has been steady growth in research on the application of copula theory to risk management. Embrechts, McNeil, and Straumann (2002) introduce copulas in finance theory and analyze the effect of dependence structures on value at risk. Li (2000) applies copulas to the valuation of credit derivatives. Frey and McNeil (2001) model dependent defaults in credit portfolios, with a special emphasis on tail dependencies. An introduction to the use of copulas in the actuarial context can be found in Frees and Valdez (1998) and Embrechts, Lindskog, and McNeil (2003).

A central aspect of ERM is the aggregation of different types of risk to calculate the economic capital necessary as a buffer against adverse outcomes. Wang (1998, 2002) gives an overview on the theoretical background of economic capital modeling, risk aggregation, and the use of copula theory in enterprise risk management. Kuritzkes, Schuermann, and Weiner (2003) aggregate risks at different levels of a financial holding company under the assumption of joint normality; in an empirical study, they compute the relative diversification effect for several conglomerates. Ward and Lee (2002) use a normal copula approach to aggregate the risks of a diversified insurer in a combined analytical and simulation model. Dimakos and Aas (2004) apply a similar method to model total economic capital, and combine risks by pairwise aggregation; they present a practical approach to estimate the joint loss distribution of a Norwegian bank and a Norwegian life insurance company.

Faivre (2003) models the overall loss distribution for a four-lines-of-business insurance company and examines the influence of different types of copulas on the value at risk and the company's default probability. Tang and Valdez (2006) simulate the economic capital requirements for a multiline insurer, taking into account different types of distributions and different types of copulas; the resulting values for economic capital are used to compute absolute diversification benefits. Rosenberg and Schuermann (2006) relax the joint normality assumption and use copula theory to aggregate risks with non-normal marginals; they analyze the influence of the business mix between credit, market, and operational risk on value at risk and calculate diversification benefits by comparing the value at risk of the diversified conglomerate with the stand-alone value at risk.

However, most of the literature does not take into account the special risk profile of financial conglomerates that arises from the group-holding structure. Financial conglomerates or insurance groups consist of several independent legal entities, each with limited liability. In the European Union, for example, combining banking and insurance activities in the same legal entity is prohibited (see Article 6(1b) of the Life Insurance Directive 2002/83/EC and Article 8(1b) of the Non-Life Insurance Directive 73/239/EEC). Article 18(1) of the Life Insurance Directive prohibits the combination of life and non-life business in the same legal entity. Hence, where such is even lawful, a transfer of funds between different legal entities in case of an insolvency of one entity occurs only if a transfer-of-losses contract has been signed or if the management of the corporate group decides in favor of cross-subsidization (e.g., for reputational reasons).

Since the contracting party is usually not the whole group, but merely a single subsidiary, in principle, the structure of an insurance group is not important to those buying insurance from the subsidiary in respect to the insurer's default risk. Thus, generally, for policyholders and other debt holders, only the default risk of individual legal entities and their ability to meet outstanding liabilities are of relevance when there is no transfer-of-losses contract between members of the group. However, for the executive board of the insurance group and for shareholders, information on diversification—and thus on risk concentration—and joint default probabilities is important when considering the risk profile of the conglomerate for ERM. Information on risk concentration may also be helpful in obtaining a certain rating level from a rating agency. From the perspective of regulatory agencies, risk concentration information can be valuable in analyzing systemic risk of insolvency. The default of a whole financial group will, in general, have a stronger impact on financial markets than the default of a single subsidiary company.

This paper extends previous literature by analyzing risk concentrations in an insurance group and by concurrently reporting joint default probabilities for sets of legal entities within a financial conglomerate. Noting that joint default probabilities only depend on individual default probabilities and the coupling dependence structure, we further study the influence of different dependence structures using the concept of copulas. In particular, we consider an insurance group with three legal entities and compare results for Gauss, Gumbel, and Clayton copulas for normal and non-normal marginal distributions.

Our results demonstrate that even if different dependence structures imply the same risk concentration factor for the financial group, joint default probabilities of different sets of subsidiaries can vary tremendously with the dependence structure. The analysis shows that the simultaneous consideration of risk concentration and default probabilities can provide information of substantial value.

The remainder of the paper is organized as follows. Section 2 introduces the concept of diversification on economic capital and risk concentration of a financial conglomerate. Dependence structures are presented in Section 3, including linear and nonlinear dependencies modeled with copulas. In the numerical analysis, Section 4, we compare results for Gauss, Gumbel, and Clayton copulas under different distributional assumptions. Section 5 summarizes the findings.

2. RISK CONCENTRATION AND DEFAULT RISK

This section describes, first, a framework for measuring risk concentrations by calculating the diversification effect on the economic capital of an insurance group, assuming that the different legal entities are merged. The economic capital is the amount necessary to buffer against unexpected losses from business activities so as to prevent default at a specific risk tolerance level for a fixed time horizon. Diversification is generally intended to reduce the overall risk level in an insurance group and thus acts to alleviate the dangers inherent in risk concentration. Calculating risk concentration in an insurance group can thus be based on an examination of diversification effects at the group level. In a second step, the joint default probabilities of legal entities within a conglomerate are introduced.

We focus on an insurance group with a holding structure and different companies (legal entities) with limited liability. Generally, such a conglomerate is subject to market risks, credit risks, underwriting risks, and operational risks. If each member of the group is faced with identical risks, one would expect the stochastic liabilities of the different entities to be highly correlated. This will usually be the case if several firms of the same type, for example, financial firms, form a group. However, if the corporate group is composed of companies from widely different industries, the liabilities between the different legal entities might be rather uncorrelated. In what follows, we will consider an insurance group consisting of a bank, a life insurer, and a non-life insurer.

In our framework, the equity value of each subsidiary legal entity is modeled at two points in time (t = 0, 1). The value of the assets (liabilities) at time t = 1 of company i is defined as A_i (L_i). Debt and equity capital in t = 0 is invested in riskless assets, leading to a deterministic cash flow for the assets in t = 1, whereas liabilities paid out in t = 1 are modeled stochastically.

Stand-alone economic capital

The amount of necessary economic capital depends on the specific risk tolerance level and on the measure chosen to evaluate corporate risk. In the following, we determine the necessary amount of capital using the default probability. The default probability α of each legal entity i can be written as

$$P(A_i < L_i) = \alpha_i.$$

In the next step, the invested assets A_i are divided into two parts—the expected value of the liabilities $E(L_i)$ and the economic capital EC_i:

$$P(E(L_i) + EC_i < L_i) = \alpha_i$$
$$\Leftrightarrow P(L_i - E(L_i) > EC_i) = \alpha_i.$$

Hence, given a probability distribution for the liabilities and a certain safety level α_i, the economic capital EC_i can be derived. The necessary economic capital EC_i for *N* different legal entities within an insurance group can be calculated by

$$EC_i = VaR_{1-\alpha}(L_i) - E(L_i) \quad i = 1, \ldots, N. \tag{1}$$

For consistency, all companies within the conglomerate should have the same safety level α. Therefore, value at risk (*VaR*) is defined by

$$VaR_\alpha(L_i) = F_{L_i}^{-1}(\alpha) = \inf\{x | F_{L_i}(x) \geq \alpha\},$$

where F_{L_i} stands for the distribution function of the liabilities for company *i*.

Aggregation

Assuming that the several companies in the insurance group are merged into one company (full liability between legal entities), the necessary economic capital for the safety level α on an aggregate level for $\sum_{i=1}^{N} L_i$ can be written as

$$EC_{aggr} = VaR_{1-\alpha}\left(\sum_{i=1}^{N} L_i\right) - E\left(\sum_{i=1}^{N} L_i\right). \tag{2}$$

To calculate the quantile in Equation (2), information about the cumulative distribution of the liabilities is needed. Closed-form solutions for $\sum_{i=1}^{N} L_i$ can be derived only for a limited number of distributions. In the case of a normal distribution, only the variance of the portfolio is needed to determine the aggregate economic capital EC_{aggr}. If no closed-form solution can be obtained, the quantile of the distribution of the aggregate liabilities $\sum_{i=1}^{N} L_i$ can be estimated using either numerical simulation techniques or analytical approximations (for an overview, see Daykin, Pentikainen, and Pesonen, 1994, pp. 119 ff.).

Diversification versus concentration

Given Equations (1) and (2), diversification can be measured with the ratio of aggregated economic capital to the sum of stand-alone economic capital (see, e.g., Kuritzkes, Schuermann, and Weiner, 2003),

$$d = \frac{EC_{aggr}}{\sum_{i=1}^{N} EC_i}. \tag{3}$$

In the case of linear dependencies, the factor *d* takes on values between zero and one and can be used to compare the level of risk concentration in conglomerates. A value of one corresponds to perfect correlation, which means that there would be no diversification benefits if the different legal entities merged into one company. When risk factors are less than perfectly correlated, some of the risk can be diversified. Absolute measures for diversification can be found in the literature (see, e.g., Tang and Valdez, 2006); however, absolute measures do not allow the comparison of companies and conglomerates that are of different sizes. Given a benchmark company, a higher value of *d* implies possible risk concentration, since lower values of *d* mean a higher diversification and thus a lower risk concentration. We henceforth refer to the coefficient in Equation (3) as the *risk concentration factor*. To keep the different quantities in Equation (3) comparable, it is important to use the same risk measure and the same time horizon for all legal entities when calculating the economic capital.

Generally, diversification of the group is of no relevance to debt holders of the group's individual companies (e.g., policyholders in the case of an insurance group) since the whole group is not the contracting party, that is, the contract is between the policyholder and the insurance subsidiary only (although there might be transfer-of-loss contracts). However, for management and shareholders of the corporate group, information about risk concentration in the different sectors is of high importance.

Determination of default probabilities

Even though calculation of the diversification factor may enable the detection of risk concentrations within the conglomerate, the factor is in most cases only a hypothetical number since individual legal entities generally do not (fully) cover the losses of the other entities. To obtain further insight about the conglomerate's risk situation, joint default probabilities are appropriate and can provide additional and valuable information.

In contrast to the determination of the risk concentration factor, which requires a convolution over different entities $\sum_{i=1}^{N} L_i$, default probabilities make use of only the joint distribution function. To determine the joint default probability of two or more legal entities, the joint cumulative distribution function is needed.

For the case of a conglomerate comprised of three legal entities, the joint default probabilities of exactly one, two, and three legal entities are given by

$$P_1 = 1 - P\left(L_1 \le A_1, L_2 \le A_2, L_3 \le A_3\right) - P_2 - P_3,$$

$$P_2 = P\left(L_i > A_i, L_j > A_j, L_k \le A_k, i \ne j \ne k\right),$$

$$P_3 = P\left(L_1 > A_1, L_2 > A_2, L_3 > A_3\right).$$

It is assumed that no transfer of losses between companies will occur.

3. MODELING THE DEPENDENCE STRUCTURE

In risk management, appropriate modeling of dependence structures is very important. One recommendation in the literature is to apply copulas in addition to linear correlation to ensure an adequate mapping of dependence (see Embrechts, McNeil, and Straumann, 2002). Copulas allow for the inclusion of features such as fat tails and skewness for nonelliptically distributed risks. In this section, first, the concept of copulas is presented in regard to modeling nonlinear dependencies. Second, linear correlations are discussed as a special case, which further allows for a closed-form solution for economic capital if liabilities are normally distributed.

3.1. Copulas

For continuous multivariate distribution functions, copulas serve to separate the univariate margins and the multivariate dependence structure. The copula C represents the dependence structure and couples the marginal distributions to a joint multivariate distribution. Let the random variables X_i (with $i = 1,...,N$) have (continuous) marginal distribution functions F_i. From Sklar's theorem it follows that (see Nelsen, 1999, p. 15)

$$\begin{aligned} P\left(X_1 < x_1,...,X_N < x_N\right) &= F_{X_1,...,X_N}\left(x_1,...,x_N\right) \\ &= C\left(F_{X_1}\left(x_1\right),...,F_{X_N}\left(x_N\right)\right). \end{aligned}$$

For joint default probabilities, this result implies that for fixed individual default probabilities

$$P\left(X_i < 0\right) = \alpha_i, i = 1,...,N,$$

one can obtain for the joint default probability of all N entities

$$\begin{aligned} P\left(X_1 < 0,...,X_N < 0\right) &= F_{X_1,...,X_N}\left(0,...,0\right) \\ &= C\left(F_{X_1}(0),...,F_{X_N}(0)\right) = C\left(\alpha_1,...,\alpha_N\right). \end{aligned}$$

Hence, the joint default probability depends on the dependence structure expressed by the copula C and on the marginal default probabilities α_i. In our case, these quantities are given and fixed since the economic capital for each entity is adjusted for each entity such that the marginal default probabilities remain constant. For example, in the case of the three entities considered here, the probability that legal entity 1 and 2 default, and entity 3 survives is

$$\begin{aligned} P\left(X_1 < 0, X_2 < 0, X_3 > 0\right) &= C\left(F_{X_1}(0), F_{X_2}(0), 1\right) - C\left(F_{X_1}(0), F_{X_2}(0), F_{X3}(0)\right) \\ &= C\left(\alpha_1, \alpha_2, 1\right) - C\left(\alpha_1, \alpha_2, \alpha_3\right). \end{aligned}$$

The probability of default for exactly two legal entities can thus be calculated by

$$\begin{aligned} P_2 &= P\left(X_i < 0, X_j < 0, X_k > 0, i \neq j \neq k\right) \\ &= C\left(1,\alpha_2,\alpha_3\right) + C\left(\alpha_1,1,\alpha_3\right) + C\left(\alpha_1,\alpha_2,1\right) - 3\cdot C\left(\alpha_1,\alpha_2,\alpha_3\right). \end{aligned} \tag{4}$$

Furthermore, the probability that exactly one company defaults is determined by

$$P_1 = 1 - P\left(X_i > 0 \ \forall i = 1,2,3\right) - P_2 - P_3 \tag{5}$$

where

$$P_3 = P\left(X_1 < 0, X_2 < 0, X_3 < 0\right) = C\left(\alpha_1,\alpha_2,\alpha_3\right). \tag{6}$$

In the analysis, we will compare several copulas. To obtain boundaries, we include the case of independence and perfect dependence (*comonotonicity*), represented by the copula (McNeil, Frey, and Embrechts, 2005, p. 189)

$$M\left(u_1,...,u_N\right) = \min\left\{u_1,...,u_N\right\}.$$

The *independence copula* is given by (McNeil, Frey, and Embrechts, 2005, p. 189)

$$\Pi\left(u_1,...,u_N\right) = \prod_{i=1}^{N} u_i .$$

The default probability for all three entities is thus the product of the marginal default probabilities. From Equations (4)–(6), it follows that in the case of independence the default probabilities of exactly one, two, and three companies are

$$P_1 = 1 - \left(1-\alpha_1\right)\cdot\left(1-\alpha_2\right)\cdot\left(1-\alpha_3\right) - P_2 - P_3$$

$$P_2 = \alpha_1\cdot\alpha_2 + \alpha_1\cdot\alpha_3 + \alpha_2\cdot\alpha_3 - 3\cdot\alpha_1\cdot\alpha_2\cdot\alpha_3$$

$$P_3 = \alpha_1\cdot\alpha_2\cdot\alpha_3 .$$

The two most common Archimedian copulas, Clayton and Gumbel, are used to model the dependence structure between the entities. These are explicit copulas that have closed-form solutions and are not derived from multivariate distribution functions as is the implicit Gaussian copula. In general, an *N*-dimensional Archimedian copula may be constructed by using the respective generator $\Phi(t)$

$$C(u_1,...,u_N) = \Phi^{-1}\left(\Phi(u_1)+...+\Phi(u_N)\right),$$

which must have certain properties (as described in McNeil, Frey, and Embrechts, 2005, p. 222). In the case of the *N*-dimensional *Clayton copula*, the generator and its inverse are given by (see Wu, Valdez, and Sherris, 2006, p. 7)

$$\Phi(t) = \frac{1}{\theta}\left(t^{-\theta}-1\right) \text{ and } \Phi^{-1}(t) = (\theta\cdot t+1)^{-1/\theta},$$

which leads to the following representation

$$C_{\theta,N}^{Cl}(u_1,...,u_N) = \left(\sum_{i=1}^{N} u_i^{-\theta} - N + 1\right)^{-1/\theta},$$

where $0 \le \theta < \infty$. For $\theta \to \infty$, one obtains perfect dependence; $\theta \to 0$ implies independence (McNeil, Frey, and Embrechts, 2005, p. 223). For the *N*-dimensional *Gumbel copula*, the generator and its inverse are given by (Wu, Valdez, and Sherris, 2006, p. 7)

$$\Phi(t) = \left(-\log(t)\right)^{\theta} \text{ and } \Phi^{-1}(t) = \exp\left(-t^{1/\theta}\right),$$

which implies the expression

$$C_{\theta,N}^{Gu}(u_1,...,u_N) = \exp\left[\left(-\sum_{i=1}^{N}\left(-\log u_i\right)^{\theta}\right)^{1/\theta}\right],$$

where $\theta \ge 1$. For $\theta \to \infty$, one obtains perfect dependence; $\theta \to 1$ implies independence (McNeil, Frey, and Embrechts, 2005, p. 220). Both Clayton and Gumbel copulas exhibit asymmetries in the dependence structure. The Clayton copula is lower tail dependent; the Gumbel copula is upper tail dependent.

3.2. The special case of linear dependence

Linear dependence is a special case of the copula concept. If X is a multivariate Gaussian random vector, then its copula is a so-called *Gauss copula* (McNeil, Frey, and Embrechts, 2005, p. 191)

$$C_R^{Ga}\left(u_1,...,u_N\right)=\Phi_N\left(\Phi^{-1}\left(u_1\right),...,\Phi^{-1}\left(u_N\right)\right),$$

where R is the correlation matrix, Φ denotes the standard univariate standard normal distribution function, and Φ_N denotes the joint distribution function of X. The Gauss copula measures the degree of monotonic dependence and has no closed-form solution, only an integral representation. The copula may be constructed by the inverse method, which maps linear dependence in the form of the linear correlation of ranks, as described by Iman and Conover (1982) and by Embrechts, McNeil, and Straumann (2002). The use of rank correlations ensures the existence of a multivariate distribution with the prescribed marginals. Given linear correlations $\rho\left(X_i,X_j\right)$, Spearman's rank correlation matrix for the Gauss copula can be derived from (McNeil, Frey, and Embrechts, 2005, p. 215)

$$\rho\left(X_i,X_j\right)=2\cdot\sin\left(\frac{\pi}{6}\rho_S\left(X_i,X_j\right)\right).$$

If the joint distribution is a multivariate normal with standard normal marginals, the economic capital EC_i for each entity can be calculated by (see, e.g., Hull, 2003, pp. 350 ff.)

$$EC_i=\sigma\left(L_i\right)\cdot z_\alpha, \tag{7}$$

where z_α denotes the α-quantile of the standard normal distribution and σ stands for the standard deviation. To aggregate the economic capital under the assumption that all sectors are carried in one company, correlations between the liabilities of the different entities are needed. The symmetric correlation matrix R with coefficients ρ_{ij} between the liabilities of entity i and entity j is given by

$$R = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1N} \\ \rho_{21} & 1 & \cdots & \rho_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{N1} & \rho_{N2} & \cdots & 1 \end{pmatrix}.$$

The corresponding entries in the covariance matrix Σ are given by $\Sigma_{ij} = \rho_{ij} \cdot \sigma(L_i) \cdot \sigma(L_j)$ and the standard deviation of the portfolio of the liabilities $L = \sum_{i=1}^{N} L_i$ can be calculated by

$$\sigma(L) = \sqrt{\sum_{i,j=1}^{N} \Sigma_{ij}} \;. \tag{8}$$

The aggregated economic capital, assuming that all sectors in the insurance group are merged, can be calculated from

$$EC_{aggr} = \sigma(L) \cdot z_\alpha \,. \tag{9}$$

Equation (9) can be reformulated using Equation (8) to obtain the following formula (see Kuritzkes, Schuermann, and Weiner, 2003; Groupe Consultatif, 2005):

$$\begin{aligned} EC_{aggr} &= \sigma(L) \cdot z_\alpha = \sqrt{\sum_{i,j=1}^{N} \Sigma_{ij}} \cdot z_\alpha = \sqrt{\sum_{i,j=1}^{N} z_\alpha \cdot \sigma(L_i) \cdot \rho_{ij} \cdot z_\alpha \cdot \sigma(L_j)} \\ &= \sqrt{\sum_{i,j=1}^{N} EC_i \cdot \rho_{ij} \cdot EC_j} \qquad (10) \\ &= \sqrt{\begin{pmatrix} EC_1 \\ EC_2 \\ \vdots \\ EC_N \end{pmatrix}^T \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1N} \\ \rho_{21} & 1 & \cdots & \rho_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{N1} & \rho_{N2} & \cdots & 1 \end{pmatrix} \begin{pmatrix} EC_1 \\ EC_2 \\ \vdots \\ EC_N \end{pmatrix}}. \end{aligned}$$

Equation (10) illustrates that the effect of diversification on the aggregated economic capital EC_{aggr} depends on the number N of legal entities, the relative portion of the economic capital of the individual companies EC_i, and the correlation between the liabilities of the different companies.

One way to calculate economic capital for liability distributions that are not normally distributed is to use analytical approximation methods such as the normal-power concept (see, e.g., Daykin, Pentikainen, and Pesonen, 1994, pp. 129 ff.).

4. NUMERICAL EXAMPLES

In this section we present numerical examples in order to examine the influence of the dependence structure (nonlinear vs. linear dependence) and the distributional assumptions (normal vs. non-normal) on risk concentration and default probabilities. First, the case of linear dependence is presented for normally and non-normally distributed liabilities with different sizes. Second, nonlinear dependencies are examined for normality and non-normality. Table 1 sets out the input parameters that are the basis for the numerical examples analyzed in this section. The values and distributions are chosen to illustrate central effects.

TABLE 1

Economic capital for individual entities in an insurance group for different distributional assumptions given a default probability α = 0.50% and $E(L_i)$ = 100, i = 1, 2, 3.

Legal entity	*Distribution type*	*Case (A)*		*Case (B)*	
		$\sigma(L_i)$	EC_i	$\sigma(L_i)$	EC_i
	“normal”				
Bank	Normal	15.00	38.64	35.00	90.15
Life insurer	Normal	15.00	38.64	5.00	12.88
Non-life insurer	Normal	15.00	38.64	5.00	12.88
Sum			115.91		115.91
	“non-normal”				
Bank	Normal	15.00	38.64	35.00	90.15
Life insurer	Lognormal	15.00	45.22	5.00	13.59
Non-life insurer	Gamma	15.00	42.84	5.00	13.35
Sum			126.70		117.09

Table 1 contains values for two different cases, (A) and (B), for normally and non-normally distributed liabilities. The given default probability of 0.50% is adapted to Solvency II regulatory requirements, which are currently being debated (European Commission, 2005). For normally distributed liabilities, economic capital can be calculated using Equation (7) with a standard normal quantile of $z_\alpha = 2.5758$. The conglomerate under consideration consists of a bank, a life insurance company, and a non-life insurer. In case (A), the liabilities of all three entities have the same standard deviation and thus require the same economic capital. In case (B), the bank has a substantially higher standard deviation than the insurance entities. Accordingly, the resulting economic capital differs.

We next change the distribution assumption to allow for non-normality. Now, only the liabilities of company 1 (Bank) are normally distributed, whereas the liabilities of company 2 (Life Insurer) and company 3 (Non-Life Insurer) follow, respectively, a lognormal and a gamma distribution. To keep the cases comparable, the expected value μ and standard deviation σ remain fixed. In the case of lognormal (a, b) distribution, the parameters can be calculated by $a = \ln(\mu) - b^2/2$ and $b^2 = \ln(1 + \sigma^2/\mu^2)$ (Casella and Berger, 2002, p. 109). For gamma distribution (α, β), the parameters are given by $\alpha = \mu^2/\sigma^2$ and $\beta = \sigma^2/\mu$ (Casella and Berger, 2002, pp. 63–64).

The assumption of non-normal distributions leads to different individual economic capital values in case (A) and case (B) compared to the values under the normality assumption. As a result, the sum of the individual economic capital (126.70 in case (A) and 117.09 in case (B)) differs also (115.91 for both cases under the normality assumption).

The numerical analysis proceeds as follows. First, we calculate the necessary aggregated economic capital based on the value at risk at the group level for a confidence level $\alpha = 0.50\%$ (Equation (2)). The concentration factor can then be derived using the stand-alone economic capital of the legal entities given in Table 1 by way of Equation (3). Subsequently, we calculate the corresponding default probabilities P_1, P_2, P_3.

4.1. Numerical results for linear dependence

To calculate the necessary economic capital at the group level, the correlation matrix for the liabilities is needed. Estimation of dependencies can be made on the basis of macroeconomic models. For instance, Estrella (2001) derives correlations from stock market returns to measure possible diversification effects between the bank and insurance sectors.

To obtain more comprehensive information on the risk situation of conglomerate under consideration (see Table 1), we compare the effect of distributional assumptions on the concentration factor and default probabilities. Figure 1 shows a plot of the default probabilities for different choices of the correlation matrix with increasing dependency and the corresponding concentration factors for different distributional assumptions. In particular, we compare the cases (A) and (B) given in Table 1 when liabilities follow a normal distribution (normal) and when they are partly non-normally distributed (non-normal). For ease of exposition we use the same coefficient of correlation ρ between the liabilities of all entities, i.e., $\rho\left(L_i, L_j\right) = \rho, i \neq j$.

Figure 1 shows how the concentration factor and information on default probabilities can complement each other. Part a) illustrates that the joint default probabilities depend on the dependence structure between the legal entities and individual default probabilities. Hence, for normal and non-normal distributions, the joint default probabilities remain unchanged, whereas the concentration factor can differ substantially. In the case of independence, joint default probabilities of two and three companies are (approximately) zero in the example considered and only individual default occurs within the group. With increasing dependence, the probability of a single default (P_1) decreases, while the probability of combined defaults (P_2, P_3) increases. For higher correlations, the probability of a combined default of two entities (P_2) decreases again. For perfectly correlated liabilities, all three entities default with probability 0.50%, while $P_1 = P_2 = 0$.

Part b) of Figure 1 illustrates that—given that the liabilities have the same standard deviations (case (A))—the distributional assumption has only marginal influence on the concentration factor, but that different correlation factors and firm size (case (B)) do matter. As an example, consider the case $\rho = 0.3$, implying a correlation matrix

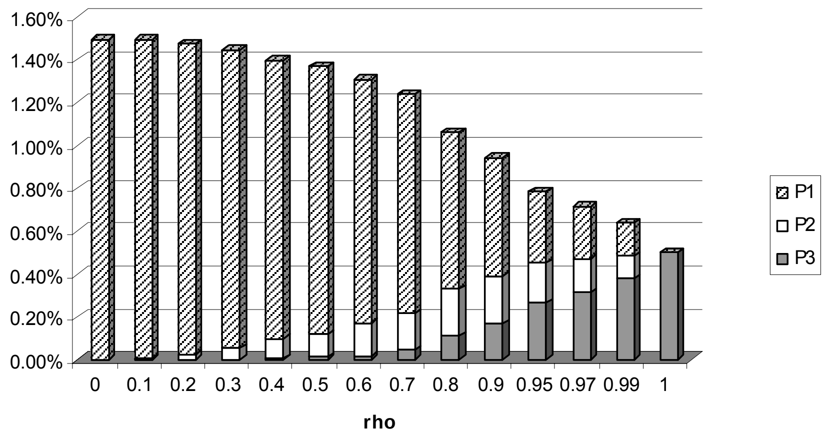
$$R = \begin{pmatrix} 1 & 0.3 & 0.3 \\ 0.3 & 1 & 0.3 \\ 0.3 & 0.3 & 1 \end{pmatrix}.$$

The corresponding concentration factor can be derived using Equation (10). For normally distributed liabilities in case (A), the aggregated economic capital is $EC_{aggr}^{A} = 84.65$. This is lower than the sum of stand-alone economic capital (115.91) due to diversification effects. Hence, the concentration factor is given by $d = 84.65/115.91 = 73.03\%$. Changing only the distributional assumption to a non-normal distribution leads to a lower value of $d = 90.78/126.70 = 71.65\%$. Hence, the concentration factor decreases in case of non-normal distribution even though the aggregated economic capital increases to 90.78. This illustrates that an absolute comparison of aggregated economic capital may be misleading. The concentration factor is very similar to the results for the normal case, with a difference of only 1.38 percentage points, which can be explained by the calibration of the distributions. We calculated the parameters of the lognormal and gamma distributions using the same values for expected value and standard deviation so as to achieve better comparability between different situations. Hence, we can say that in the example given the choice of a non-normal distribution has very little impact on concentration factors.

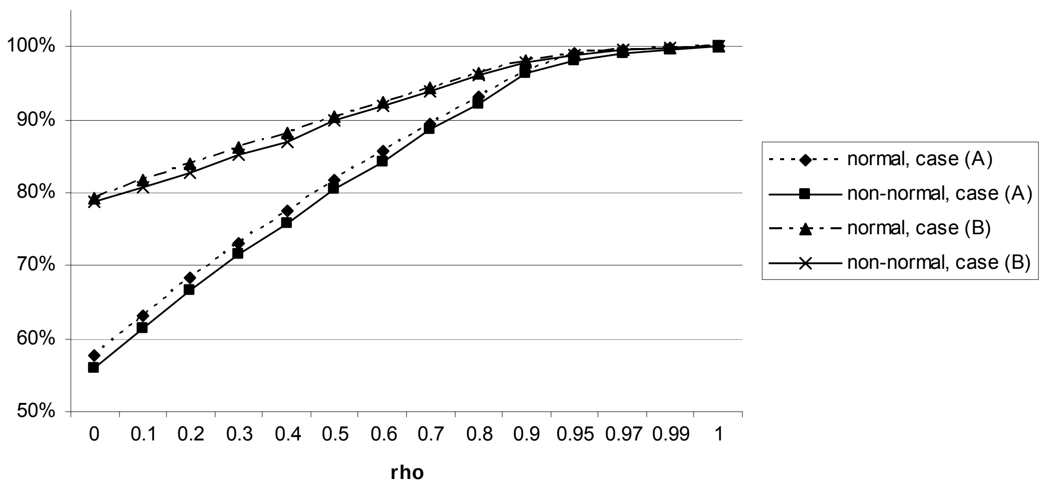
FIGURE 1

Default probabilities and risk concentration factor for linear dependence on the basis of Table 1.

a) Joint default probabilities for linear dependence



b) Risk concentration factor for linear dependence



Notes: P_1 = probability that exactly one entity defaults; P_2 = probability that exactly two entities default; P_3 = probability that all three entities default.

A much larger effect can be observed when comparing case (A) with case (B). For normal distribution, the concentration factor for case (B) is $d = 99.76/115.91 = 86.07\%$. Hence, this situation leads to a higher concentration factor than case (A) (d = 73.03%). Thus, the situation given in case (B) indicates a possible existence of risk concentration within the conglomerate, originating from the bank. The bank's relatively large risk contribution to total group risk causes a less effective diversification of risks. Losses resulting from banking activities in case (B) are less likely to be compensated by good results from insurance activities than in case (A). Thus the concentration factor d is useful for examining the existence of risk concentrations whenever a benchmark company is available.

Overall, Figure 1, part b) demonstrates that the difference between the concentration factors of cases (A) and (B) decreases with increasing correlation. In the case of perfect positive correlation, $\rho = 1$, the difference vanishes and the concentration factor takes on its maximum of 100%.

Even though all four curves imply the same joint default probabilities, they have different risk concentration factors. The differences in d result from changes in the amount of economic capital needed to retain a constant default probability.

4.2. Numerical results for nonlinear dependence

In this section, we alter the assumption for the dependence structure and examine the impact of nonlinear dependencies on risk concentration and joint default probabilities using Clayton and Gumbel copulas as described in Section 3.1. Both copulas are constructed using Monte Carlo simulation with the same 200,000 paths so as to increase comparability. The Clayton and Gumbel copulas are simulated using the algorithms in McNeil, Frey, and Embrechts (2005, p. 224). The algorithm for the Gumbel copula uses positive stable variates, which were generated with a method proposed in Nolan (2005).

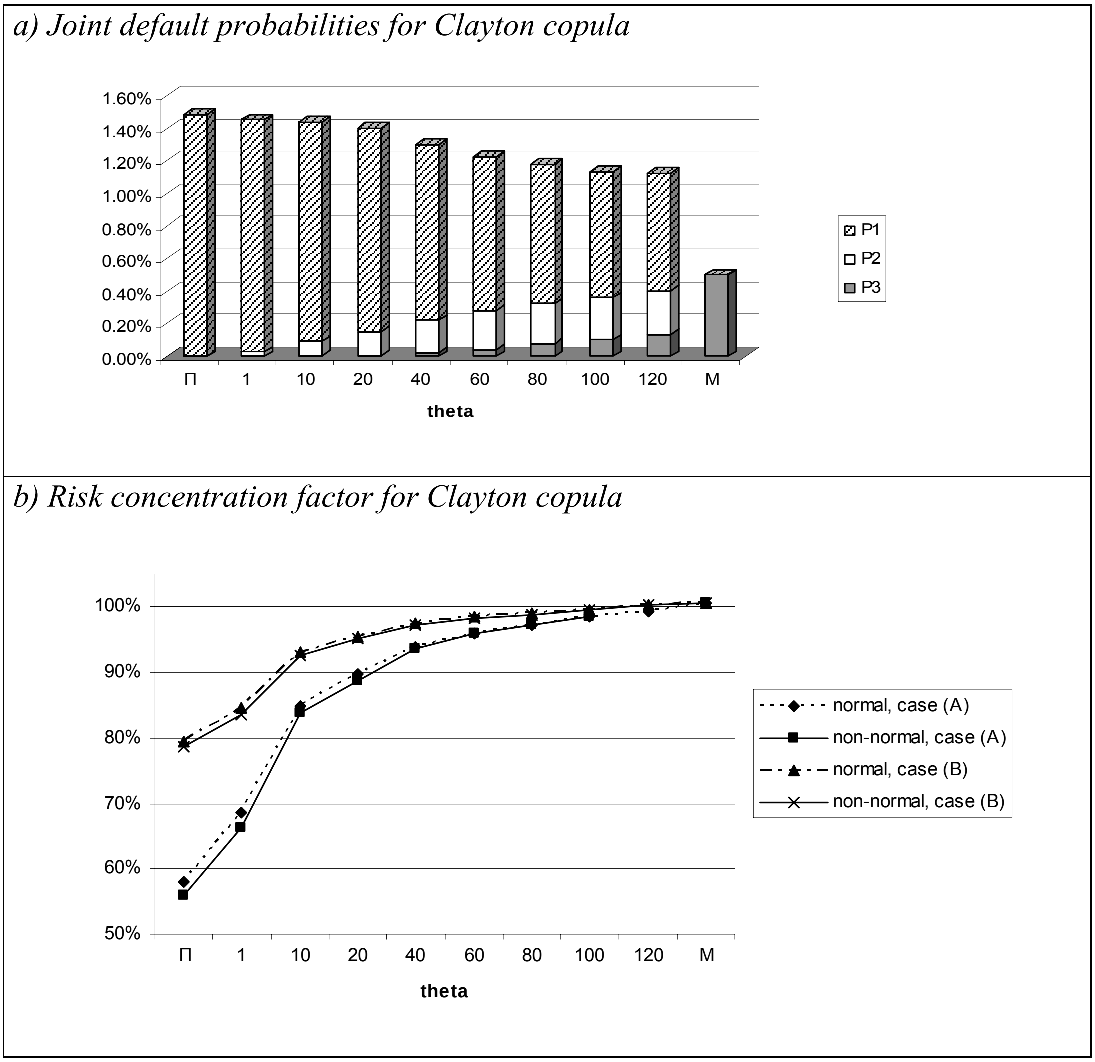
Numerical results for the Clayton and Gumbel copulas are illustrated in Figures 2 and 3, respectively. In both figures, part a) displays default probabilities as a function of the dependence parameter θ and part b) shows the corresponding concentration factors. The independence copula marked as Π in the figures serves as a lower boundary, while the case of comonotonicity (M) represents perfect dependence and is thus an upper bound.

At first glance, both dependence structures in Figures 2 and 3 appear to lead to similar results as in the linear case in Figure 1. Overall, the probability that any company defaults decreases with increasing θ. As before, under perfect comonotonicity, all three entities always become insolvent at the same time with probability 0.50%, while the probability for one or two defaulted companies is zero ($P_1 = P_2 = 0$). In fact, in this case, the concentration factor exceeds

100% since the value at risk is not a subadditive risk measure (for a discussion, see Embrechts, McNeil, and Straumann, 2002, p. 212).

FIGURE 2

Default probabilities and risk concentration factor for Clayton copula on the basis of Table 1.



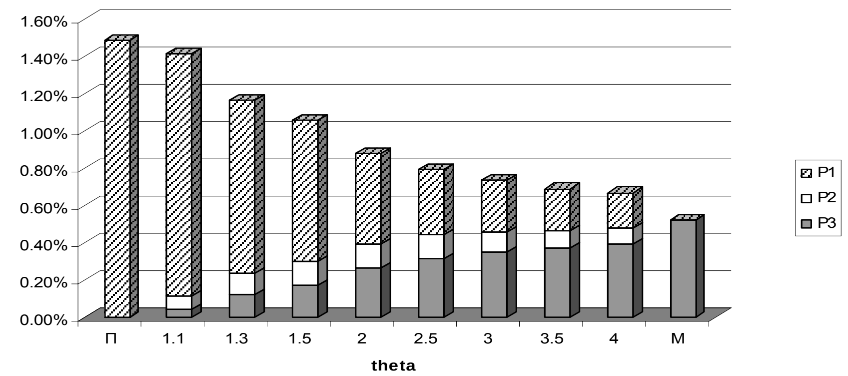
Notes: P_1 = probability that exactly one entity defaults; P_2 = probability that exactly two entities default; P_3 = probability that all three entities default.

A comparison of Figures 2 and 3 reveals that the type of tail dependence (upper vs. lower) has a significant impact on the particular characteristics of the joint default probabilities curves. In case of the upper tail dependent Gumbel copula, companies become insolvent far more often, and hence the joint default probability of all three entities quickly approaches 0.50% in the limit M. In contrast, the default probabilities of the lower tail dependent Clayton copula converge to 0.50% much more slowly. In fact, even for θ close to 120, the generation of random numbers from the Clayton copula becomes increasingly difficult, despite the fact that the joint default probability of all three entities is only 0.13%.

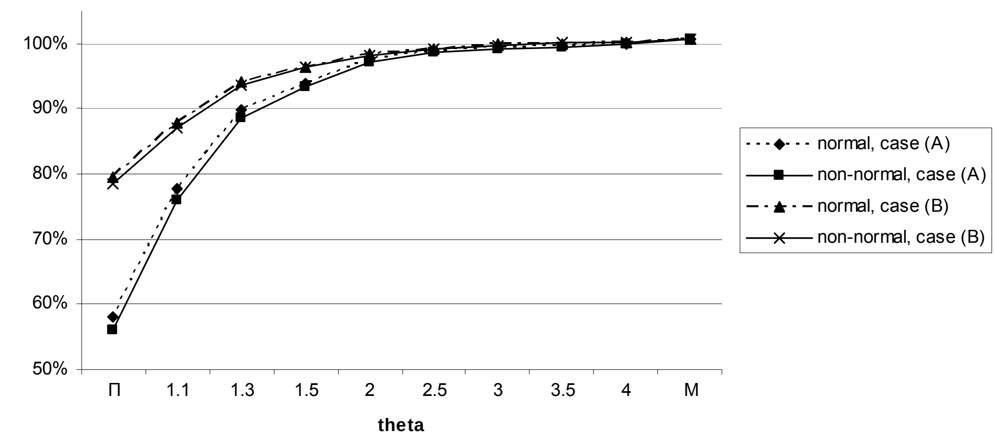
FIGURE 3

Default probabilities and risk concentration factor for Gumbel copula on the basis of Table 1.

a) Joint default probabilities for Gumbel copula



b) Risk concentration factor for Gumbel copula



Notes: P_1 = probability that exactly one entity defaults; P_2 = probability that exactly two entities default; P_3 = probability that all three entities default.

Even though results for default probabilities and concentration factors under the Gauss, Gumbel, and Clayton copulas look very similar at first glance, they can differ tremendously, which will be demonstrated in the next subsection.

4.3. Comparing the impact of nonlinear and linear dependencies

To compare and identify the considerable effects of the underlying dependence structures on default probabilities, we take examples from the Figures 1, 2, and 3 that have the same concentration factor, using case (A) with normally distributed marginals so as to make the results comparable.

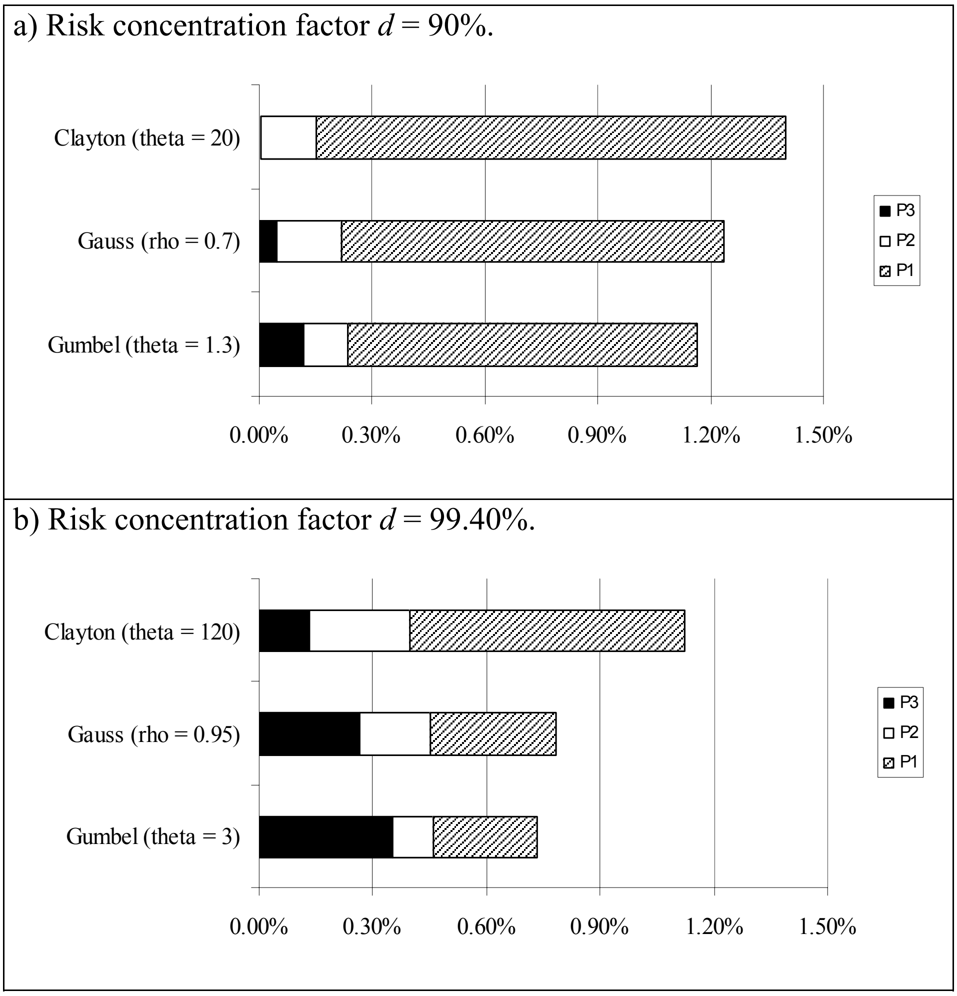
Two examples are presented in Figure 4 for fixed concentration factors of 90% in part a) and 99.40% in part b) from the Clayton, Gauss, and Gumbel copulas. The examples in each part of the figure have the same concentration factor and thus exhibit the same value at risk. Although the compared companies have the same risk, default probabilities differ substantially with the dependence structure.

A comparison of parts a) and b) of Figure 4 shows that the sum of default probabilities ($= P_1 + P_2 + P_3$)—i.e., the probability that one, two, or three companies default—is higher for the lower concentration factor d = 90%. Furthermore, for d = 99.40%, the partitioning between the three joint default probabilities (P_1, P_2, P_3) is shifted toward P_3, while P_1 decreases. Hence, a higher concentration factor is accompanied by a lower sum of default probabilities, but induces a significantly higher joint default probability of all three entities.

Figure 4 demonstrates the considerable influence that the choice between Clayton, Gauss, and Gumbel copulas has on joint default probabilities. The Clayton copula leads to the highest sum of default probabilities, but has the lowest probability of default for all three companies (P_3). The other extreme occurs under the Gumbel copula, where P_3 is highest and P_1 takes the lowest value, while the Gauss copula induces values between those of the Clayton and Gumbel copulas.

FIGURE 4

Comparison of joint default probabilities for one (P_1), two (P_2), and three (P_3) companies for different dependence structures; case (A), normal distributions.



Notes: P_1 *= probability that exactly one entity defaults;* P_2 *= probability that exactly two entities default;* P_3 *= probability that all three entities default.*

Our results show that even if different dependence structures imply the same value at risk and thus the same risk concentration factor, joint default probabilities can differ tremendously. Furthermore, our analysis demonstrates that the simultaneous reporting of risk concentration factors and default probabilities can be of substantial value, especially for the management of the corporate group. By comparing linear and nonlinear dependencies, we found that the effect of mismodeling dependencies may not only lead to significant differences in assessing risk concentration, but can also lead to misestimating joint default probabilities. Hence, there is a substantial model risk involved with respect to dependence structures.

5. SUMMARY

This paper assessed and related risk concentrations and joint default probabilities of legal entities in a conglomerate composed of three entities, a bank, a life insurance company, and a non-life insurance company. Our procedure provided valuable insight regarding the group's risk situation, which is highly relevant for enterprise risk management purposes. An insurance group (a conglomerate) typically consists of several legally independent entities, each with limited liability. However, diversification concepts assume that these entities are fully liable and all together meet all outstanding liabilities of each. Even if diversification is of no importance from a policyholder perspective, it is useful in determining risk concentration in an insurance group because greater diversification generally implies less risk.

To determine default probabilities, we focused on the case of limited liability without transfer of losses between the different legal entities within the group. Joint default probabilities only depend on individual default probabilities and the coupling dependence structure. Hence, we studied the effect of different dependence structures using the concept of copulas.

In the numerical analysis, we considered an insurance group comprised of three legal entities and compared results from the Gauss, Gumbel, and Clayton copulas for normal and non-normal marginal distributions. Economic capital was adjusted for each situation to satisfy a fixed individual default probability. In contrast to the risk concentration factor, joint default probabilities only depend on individual default probabilities and on the dependence structure, not on distributional assumptions.

For all models, we found that the risk concentration factor and the joint default probability of all three entities increase with increasing dependence between the entities, while the probability of a single default decreases. Overall, the sum of default probabilities of one, two, or three entities decreases with increasing dependence. Furthermore, one entity's large risk contribution, in terms of volatility, led to a much higher risk concentration factor for the group as a whole. Our findings further demonstrated that even if different dependence structures imply the same risk concentration factor for the group, joint default probabilities for different sets of subsidiaries can vary tremendously. In particular, the lower tail dependent Clayton copula led to the lowest probability of default for all three entities, while the upper tail dependent Gumbel copula exhibited the highest default probability.

The analysis showed that a simultaneous consideration of risk concentration factor and default probabilities can be of substantial value, especially for the management of the corporate group with respect to enterprise risk management.

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