Regret Aversion and Annuity Risk in Defined Contribution Pension Plans

Don't Look Back in Anger

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Abstract

The high value of the implicit option to time the retirement date leads to regret aversion in the retirement investment decision of Defined Contribution plan participants. As a solution, this paper develops and prices a lookback option on a life annuity contract. We determine a closed-form option value, price the option via Monte Carlo simulations in an augmented economic environment and determine the price sensitivities via the pricing of alternative options. We find that the price of a lookback option, with a maturity of three years, is 8-9% of the wealth at the option issuance date. The option price is highly sensitive to the exercise price of the option, i.e. pricing alternative options (e.g. Asian) substantially lowers the price. Time to maturity is another important price driver. Asset allocation decisions and initial interest rates hardly affect the option price.

1 Introduction

In recent years an increasing number of pension funds shifted from defined benefit (DB) to defined contribution (DC) plans. Particularly in anglo-saxon countries DC plans have gained popularity. Boulier, Huang, and Taillard (2001) attribute this shift to two major advantages of DC plans over traditional DB plans. Participants can observe their pension claim at any moment in time and can transfer this claim more easily among employers. Moreover, plan sponsors are unburdened from risk associated with the pension plan. This risk is passed on to participants. As a direct result of the change in risk bearer, large numbers of uninformed participants face a difficult decision at retirement: If and when should they invest their retirement wealth irreversibly in a life annuity? The high price sensitivity of the life annuity with respect to the long-term interest rate complicates the retirement investment decision. This high price sensitivity makes the implicit option to time the retirement date valuable (see Milevsky and Young, 2002). The large value of the timing option creates fertile ground for regret aversion. For instance, the pension income of a participant who retires at the end of our data set (December 2003) is, ceteris paribus, 6.1% higher than a colleague's income who retired half a year earlier. The 6.1% difference is even more remarkable if we take into account that long-term interest rate differences at the sample end are small relative to differences earlier in the sample (see Figure 1).

[Figure 1 about here.]

Numerous experiments document that regret aversion plays a role in human decision making (e.g. Kahneman and Tversky, 1979). Bell (1982) and Loomes and Sugden (1982) are the first to extend standard utility functions by incorporating regret aversion components. Since this development, the effect of regret aversion has been explored in many different areas. Braun and Muermann (2004) document that regret aversion has a mitigating effect on extreme demands for insurance. Michenaud and Solnik (2006) show that even if the currency risk premium is zero, regret averse investors hold positive currency positions. Gollie and Salanié (2006) document that the introduction of regret aversion in a complete market with Arrow-Debreu securities, shifts the optimal asset allocation more to states with low probabilities. Muermann, Mitchell, and Volkman (2006) are the first to analyze the role of regret aversion in pension investment decisions. They evaluate the effect of regret aversion on the willingness to pay for a pension guarantee. One of their main findings is that the willingness to pay for a pension guarantee, of an investor with a risky portfolio, increases with the introduction of regret aversion.

This paper prices an option that protects regret averse participants against annuity risk at retirement. At retirement, many retirees convert their pension wealth into a life annuity and regret aversion plays an important role in the timing of the life annuity purchase. Merton and Bodie (2005) argue that lookback options can be used as a remedy for regret aversion. We therefore address regret aversion and annuity risk at retirement by developing and pricing a lookback option on a life annuity. The option holder effectively buys an insurance against regret aversion shortly before the retirement date. Using a replicating portfolio strategy, we derive an exact closed-form price for the lookback life annuity option under some simplifying assumptions. However, the lookback life annuity option does not eliminate annuity risk completely. Extremely regret averse participants who want to convert the exact amount of terminal wealth into a life annuity confront difficulties. Uncertainty about the future minimum life annuity price and future investment returns leaves DC plan participants doubtful about the number of lookback options to buy. To eliminate annuity risk and relax some simplifying assumptions, we also price the lookback life annuity option in an augmented economic environment. In this alternative setting, we price the option under the restriction that the complete retirement wealth is converted into a life annuity. Monte Carlo simulations and a stochastic discount factor are used to price the option. Finally, we determine the option Greeks by changing different parameters in the pricing environment.

We find that the price of the lookback life annuity option, expressed in percentages of the wealth at the option issuance date, is approximately 8% for men and 9% for women. Furthermore, we document that the lookback feature and time to maturity are important drivers of the option price. If the participant is offered the right to buy the life annuity for the average life annuity price (Asian option), the option price decreases by 67%. Extending the time to maturity of the option by one year increases the option price by 21%. We also find that the option price is insensitive to the initial interest rate and hardly affected by the asset allocation during the time to maturity.

The presence of regret aversion in retirement investment decisions contrasts our paper sharply with most earlier literature on the topic. Since we are the first to consider regret aversion in pension retirement investment decisions, the lookback life annuity option, which offers protection against regret aversion, is not available in financial markets yet. Earlier literature addresses the problem of annuity risk at retirement effectively using standard utility functions and asset allocation approaches. Assuming power utility for participants, Campbell and Viceira (2002) derive the optimal pre-retirement asset allocation. Furthermore, Yaari (1965) points out that it is optimal for participants without bequest motives to convert retirement wealth into a life annuity. Koijen, Nijman, and Werker (2006) determine the optimal allocation to nominal, real and equity-linked annuities at retirement. Subsequently, they determine the optimal pre-retirement hedging strategy in stocks, nominal and inflation-linked bonds and cash, for the risk created by the optimal annuity mix. Boulier, Huang, and Taillard (2001) optimize the asset allocation in a DC pension plan. They impose a minimum guarantee on the benefits in the form of a life annuity. Deelstra, Grasselli, and Koehl (2003) generalize the problem to an allocation optimization in the presence of a lower bound on the retirement benefits (not necessarily converted into a life annuity). These solutions often impose long-term restrictions on the asset allocation and therefore imply a serious reduction in the freedom of choice of the DC plans. A solution that suffers less from the loss in freedom of choice is the possibility to buy an option that protects participants against annuity risk. Lachance, Mitchell, and Smetters (2003) develop an option to buy back a defined benefit claim. However, this option turns out to be extremely expensive and also fails to recognize possible regret aversion in participant investment decisions. The lookback life annuity option creates only a minor loss in freedom of choice and provides a possibility to insure participants against regret aversion.

In section 2 we develop the lookback life annuity option and determine a closed-form price. Section 3 describes the Monte Carlo valuation setting. Our empirical results are discussed in section 4.

2 Defined Contribution and the Lookback Option

In this section we develop and price the lookback option on a life annuity contract in a continuous time framework. Gerber and Shiu (2003) simplify the pricing of path-dependant derivatives significantly, by deriving a closed-form expression for the Laplace-Stieltjes transform of the expected value of the minimum (maximum) of a Wiener process. Nevertheless, we follow the approach by Goldman, Sosin, and Gatto (1979) and determine a closed-form option price by constructing a hedging portfolio.

2.1 The Lookback Option on a Life Annuity

To characterize the lookback option on a life annuity we specify the option payoff and time to maturity. Since we price the option via a replicating portfolio technique this characterization is sufficient to determine a closed-form option price. The option considered provides the right to lookback at retirement and buy a life annuity for the minimum forward price. Hence, the option we consider is a lookback call option on a life annuity with a floating strike price (the minimum forward price of a life annuity). The option payoff U_T can consequently be characterized as

$$U_T = P^L(r,T) - F^L(\overline{f},T), \tag{1}$$

with $P^L(r,T)$ the time T price of a life annuity and $F^L(\overline{f},T)$ the minimum forward life annuity price in the time span $[t_a, T]$, i.e. the time the option was issued until retirement date T. The time T life annuity price P^L is a function of the time T interest rate. Moreover, the minimum forward life annuity price F^L is a function of the maximum time T forward rate $\overline{f}$ during the time to maturity of the option.

The optimal time to maturity of the lookback life annuity option can be determined in a trade off between effectiveness and price. The time to maturity of the option determines to a large extent the expected value of the difference between the time T life annuity price and the maximum time T forward life annuity price during the time to maturity of the option. For very short horizons, the expected difference is small and the option therefore in many cases ineffective. For very long horizons, the expected difference grows very large and consequently the option price is high. Furthermore, while determining the optimal time to maturity attention should be paid to the pension setting within which the option is priced. As optimal trade off between price and effectiveness, taking into account the pension setting, we selected a time to maturity of thee years.

2.2 Closed-form Lookback Life Annuity Option Pricing

In this section we determine a closed-form price for the lookback option on a life annuity. We assume that mortality rates are exogenous and constant and that the life annuity is fairly priced. As a result, a life annuity is nothing more than a portfolio of zero-coupon bonds. The pricing can therefore be simplified to pricing a lookback option on a pure discount bond. We therefore restrict the analysis to pricing a lookback option on a zero coupon bond with a fixed maturity date s. We apply bond option formulas developed by Jamshidian (1989) and use Goldman, Sosin, and Gatto (1979) as a guideline to price a lookback option on a zero coupon bond. The guideline prescribes a replicating portfolio approach, and therefore absence of arbitrage is needed to develop an option price. We furthermore assume that markets are frictionless, unlimited borrowing and lending is possible and an Ornstein-Uhlenbeck interest rate process characterized as

$$dr = \alpha \left(\gamma - r\right) dt + \sigma_r dW, \tag{2}$$

with r the instantaneous spot rate, γ the long-term average interest rate, σ_r the interest rate volatility and dW a standard Brownian motion.

We price the lookback option by forming a hedging portfolio. This replicating portfolio has to satisfy two criteria, its payoff has to match the option payoff and it should be self-financing. Self-financing means that any change in portfolio composition can take place without extra financing. If the hedging portfolio satisfies the two criteria and we use the absence of arbitrage, the portfolio price should equal the option price. The hedging portfolio consists of a straddle on a zero coupon bond with maturity s. The expiration dates of the straddle and lookback option match exactly. At the time the option is issued (t_a), the exercise price equals the forward price of a pure discount bond bought at time T. If the forward bond price attains a new minimum, the straddle is sold and a new straddle with the new minimum as exercise price is bought. The replicating portfolio value can consequently be represented as

$$H(P(r,t,s),Q(\overline{f},t,s),T-t,s) = C^B(P(r,t,s),Q(\overline{f},t,s),T-t,s) + P^B(P(r,t,s),Q(\overline{f},t,s),T-t,s), \tag{3}$$

where $Q(\overline{f},t,s)$ is the minimum forward price of a zero coupon bond with maturity date s during the interval $[t_a,t]$. The minimum forward price is a function of the maximum forward rate $\overline{f}$, time t and bond maturity date s. T is the lookback option expiration date and $T-t$ is the time to maturity of the lookback option. C^B and P^B are the prices of respectively (regular) call and put options with expiration date T and exercise price $Q(\overline{f},t,s)$ on the pure discount bond with maturity s. An application of the put-call-parity transforms the hedging portfolio into

$$H(P(r,t,s),Q(\overline{f},t,s),T-t,s) = 2C^B(P(r,t,s),Q(\overline{f},t,s),T-t,s) + P(r,t,T)Q(\overline{f},t,s) - P(r,t,s). \tag{4}$$

Equation (4) shows that if the hedging portfolio satisfies the matching payoff and self-financing criteria, we reduce our problem to pricing a call option on the zero coupon bond, since all other terms in equation (4) are directly observable. This remaining call option is priced by Jamshidian (1989).

Under the above-mentioned assumptions, the price of a call option on a pure discount bond can be represented as

$$C^B(P(r,t,s),Q(\overline{f},t,s),T-t,s) = P(r,t,s)N(h)-Q(\overline{f},t,s)P(r,t,T)N(h-\sigma_p), \tag{5}$$

with

$$h = \frac{ln\left(\frac{P(r,t,s)}{P(r,t,T)Q(\overline{f},t,s)}\right)}{\sigma_p} + \frac{1}{2}\sigma_p, \tag{6}$$

$$\sigma_p = \frac{v(t,T)(1-e^{-\alpha(s-T)})}{\alpha}, \tag{7}$$

$$v^2(t,T) = \sigma_r^2 \frac{1-e^{-2\alpha(T-t)}}{2\alpha}, \tag{8}$$

where N represents the normal cumulative distribution function.

Given the price of the call option, it suffices to show that the hedging portfolio matches the option payoff and is self-financing. When the maturity date of the option is approaching, equations (4) to (8) show that the portfolio has limit value

$$\lim_{t \to T} H(P(r,t,s), Q(\overline{f},t,s), T-t, s) = P(r,T,s) - Q(\overline{f},T,s). \tag{9}$$

In particular, the option pays out exactly the time T difference between the bond price and its minimum forward price. The option holder is thus able to buy the bond for the minimum forward price and the first criterion is satisfied. In addition to the payoff criterion fulfilment, Appendix A shows that an additional investment is needed to assure the self-financing property. An investment of

$$\frac{\alpha H_r}{1 - e^{-\alpha(s-t)}}, \tag{10}$$

with H_r derived in equation (50) in Appendix C, in a pure discount bond with maturity date s is necessary to hedge the interest rate risk of the earlier specified straddle. The price of a lookback option on a zero coupon bond maturing at time s can therefore be characterized as

$$\begin{aligned} C_L^B(P(r,t,s), Q(\overline{f},t,s), T-t, s) =& 2P(r,t,s)N(h) \\ &- 2Q(\overline{f},t,s)P(r,t,T)N(h-\sigma_p) \\ &+ P(r,t,T)Q(\overline{f},t,s) - P(r,t,s) \\ &+ \frac{\alpha H_r}{1 - e^{-\alpha(s-t)}} P(r,t,s). \end{aligned} \tag{11}$$

Jamshidian (1989) proves that the value of a portfolio of bond options equals the value of an option on a bond portfolio. Consequently, the price of a lookback call option on a life annuity is characterized as

$$C_L^L(r, \overline{f}, t, T) = \int_T^\infty C_L^B(P(r,t,s), Q(\overline{f},t,s), T-t, s)\pi_s ds, \tag{12}$$

with π_s the time t probability that the option holder is still alive at time s.

The closed-form solution presented in equation (12) is exact and the hedging strategy easily applicable in practice. The replicating portfolio strategy enables participants to buy a predetermined level of pension income for the minimum forward price over the lookback period. To achieve this, investments in straddles and bonds of the same maturity are necessary. However, at the investment date (t_a) the option holder's actual level of pension income is unresolved. The actual level of pension income is a function of three factors: mortality rates, the minimum forward life annuity price (F^L) and retirement wealth (W_T). Since it is impossible to determine the last two factors at the option issuance date (t_a), it is impossible to ensure that the exact amount of retirement wealth is converted into a life annuity for the minimum price. However, extremely regret averse participants may wish to eliminate all annuity risk and convert the exact amount of retirement wealth. Including the guarantee that participants convert their exact amount of retirement wealth into a life annuity, introduces investment risk drivers in the closed-form valuation procedure. The introduction of additional risk drivers complicates the valuation process tremendously. In the next section ,we therefore define a Monte Carlo valuation setting in which we assure that the exact amount of retirement wealth is converted into a life annuity.

3 Augmented Environment and Price Drivers

In this section we describe the pricing environment of the lookback life annuity option. As pointed out in the introduction, the lookback life annuity option reduces annuity risk for pension plan participants substantially, but can not eliminate it completely. The number of options held by the participant determines to what extent he is exposed to interest rate risk at retirement. Investment risk and the path-dependency of the lookback option make it impossible to determine the optimal number of options at the option issuance date. We therefore extend the pricing environment by pricing the lookback life annuity option conditional on the guarantee that the exact amount of retirement wealth is converted. In a Monte Carlo analysis, the DC pension setting is modeled using a stochastic discount factor that is consistent with market prices for stocks and bonds. Finally, we determine the drivers of the option price by pricing alternative options in the augmented DC setting.

3.1 Stylized Pricing Environment

This subsection describes the economic environment and pricing techniques applied to determine the lookback life annuity price. We price the option in a Monte Carlo framework, again assuming that mortality rates are exogenous and constant and that life annuities are fairly priced. We neglect the effect of the lookback option purchase on the pre-retirement wealth. Despite its implausible character, this assumption is necessary. Otherwise, the retirement wealth and thus the option price would be a function of the option price. This self-dependency is typically hard to model. We first determine the option payoff and specify the investment universe with the corresponding stochastic discount factor. Subsequently, 100,000 scenarios with discounted option payoffs are generated on a monthly frequency.[1] The option price is then approximated by the average discounted option payoff.

To ultimately determine the option price, the payoff, investment universe and discount factor need to be specified. In this setting option holders have the right to buy a life annuity for the minimum price attained during the time to maturity of the option. Furthermore, we require that both the option holder and non-option holder convert the exact amount of retirement wealth W_T into a life annuity. As a result, the right to buy the life annuity for minimum price is equivalent to the right to convert retirement wealth against the maximum interest rate during time to maturity of the option. We specify the guarantee of exact conversion of retirement wealth for non-option holders as

$$W_T = L^N \sum_{s=T+1}^{\infty} \frac{\pi_s}{(1+r_T)^s}, \tag{13}$$

where π_s is the time T probability that the participant is still alive at time s and L^N the after retirement income of the non-option holder. The option holder has the right to convert against the maximum rate. The guarantee for the option holder is therefore expressed as

$$W_T = L^H \sum_{s=T+1}^{\infty} \frac{\pi_s}{(1+r_{max})^s}, \tag{14}$$

where L^H is the after retirement income of the option holder and r_{max} the maximum interest rate during the time to maturity of the option. The option

[1] Option prices converge at 100,000 scenarios.

payoff is the difference between the present value of the option holder's and non-option holder's benefits

$$U_T = \sum_{s=T+1}^{\infty} \frac{\pi_s \left[L^H - L^N\right]}{(1+r_T)^s}$$
$$= \sum_{s=T+1}^{\infty} \frac{\pi_s L^H}{(1+r_T)^s} - W_T. \qquad (15)$$

Equations (14) and (15) show that the option payoff is a function of the retirement conversion rate r_T, maximum conversion rate r_{max} and terminal wealth W_T and mortality rates π_s.

In a DC pension plan, participants have more freedom with regard to the asset allocation than in a DB plan. DC plan participants are often offered a limited number of investment opportunities. They can either choose one of the alternatives offered or specify weights for each alternative. We assume that the investment universe consists of a portfolio of stocks, a two year bond and a ten year bond and that participants determine investment weights for each of the three alternatives. The investment universe with stocks and different bonds implies that investment risk during the time to maturity of the option is contained in the option price, in contrast to the closed-form analysis. Analogous to Ang and Piazzesi (2003) and Cochrane and Piazzesi (2005) we model the log returns of the different investment alternatives as a VAR(1)

$$X_{t+1} = \mu + \phi X_t + \eta_{t+1} \qquad \eta_{t+1} \sim N(0, \Sigma), \qquad (16)$$

where X_{t+1} contains log stock, 2 year and 10 year bond returns at time $t+1$. As Campbell, Lo, and MacKinlay (1997), we approximate n-maturity log bond returns by

$$r_{n,t+1} = D_{n,t} y_{n,t} - (D_{n,t} - 1) y_{n-1,t+1}, \qquad (17)$$

with $D_{n,t}$ and $y_{n,t}$ respectively the time t duration and yield of an n-maturity bond. We approximate $y_{n-1,t+1}$ by $y_{n,t+1}$ and consider zero coupon bonds, for which maturity equals duration.

The VAR(1) model is estimated using monthly U.S. data on bond yields and stock returns for the period ranging from January 1952 to December 2003. Value weighted stock returns are obtained from the CRSP database.

Bond yields are provided by McCullogh and Kwon and Bliss (1997). Summary statistics and parameter estimates are provided in Appendix E.

To accustom the simulation setting even more to DC pension plans and their specific risks we specify a stochastic rather than non-stochastic discount factor. The stochastic nature of the discount factor allows it to discount payoffs in relatively "good" scenarios stronger than payoffs in relatively "bad" scenarios. In line with Cochrane and Piazzesi (2005) we assume for the discount factor the following process

$$M_{t+1} = \exp\Big(-\delta_0 - \delta_1' X_t - \frac{1}{2}\lambda_t' \Sigma \lambda_t - \lambda_t' \eta_{t+1} \Big), \tag{18}$$

$$\lambda_t = \lambda_0 + \lambda_1 X_t, \tag{19}$$

where M_{t+1} denotes the time $t+1$ discount factor and $\lambda_t > 0$. By definition, the expected deflator value should equal the price of a zero coupon bond maturing in the same period. Considering a one period horizon, this yields

$$p_t^{(1)} = \log(E_t[M_{t+1}]), \tag{20}$$

with $p_t^{(1)}$ the log time t price of a zero coupon bond with maturity date $t+1$. Substitution of equations (18) and (19) in equation (20) leads to

$$-\delta_0 - \delta_1' X_t = -y_{1m_t}^{(1)}. \tag{21}$$

To determine values for λ_0 and λ_1, we note that the expected value of the discounted return of any asset in the economic environment should be equal to 1, represented as

$$\iota = E_t[M_{t+1} R_{t+1}], \tag{22}$$

with ι a 3x1 vector of ones and R_{t+1} a 3x1 vector containing the returns of the different products in the investment universe. Simple rewriting leads to

$$\begin{aligned} \iota &= E_t[M_{t+1} R_{t+1}], \\ &= E_t[\exp(\iota m_{t+1} + r_{t+1})], \end{aligned} \tag{23}$$

with m_{t+1} and r_{t+1} log values for respectively M_{t+1} and R_{t+1}. Taking logs allows us to rewrite equation (23) as

$$0 = \iota E_t[m_{t+1}] + E_t[r_{t+1}] + \frac{1}{2}\iota\sigma_m^2 + \frac{1}{2}\sigma_r^2 + \sigma_{mr}, \tag{24}$$

where σ_m^2 and σ_r^2 are respectively the variances of the log discount factor m_{t+1} and log return vector r_{t+1}. The covariances between log discount factor m_{t+1} and log return are denoted σ_{mr}. Substitution and some more rewriting then gives

$$\lambda_t = \Sigma^{-1}(\mu + \phi X_t + \iota(-\delta_0 - \delta_1' X_t) + \frac{1}{2}\sigma_r^2, \tag{25}$$

λ_0 and λ_1 can consequently be defined as

$$\lambda_0 = \Sigma^{-1}(\mu - \delta_0\iota + \frac{1}{2}\sigma_r^2), \tag{26}$$

$$\lambda_1 = \Sigma^{-1}(\phi - \iota\delta_1'), \tag{27}$$

where values for δ_0 and δ_1 can be obtained by simple regression and μ and ϕ and Σ from the VAR-regression.

We have specified the option payoff in equation (15) and discount factor in equation (18) in VAR parameters, mortality rates and terminal wealth. VAR parameters can be obtained from standard regressions and mortality rates are readily available. However terminal wealth depends on the investment weights and corresponding returns in each investment category for the investor. Hence, by specifying weights (w_s, w_{2b} and w_{10b}) for each investment category and repetitively, randomly drawing values for η from a normal distribution, scenarios-specific values for the terminal wealth and thus for the option payoff and discount factor can be determined. The option price can subsequently be approximated by averaging the discounted option payoffs across all scenarios.

Any n-maturity bond can be priced consistent with the VAR specified before. This allows us to determine the exact life annuity price and easily extend the investment universe. The exact price is obtained by discounting each pension payoff with the corresponding maturity bond rate. The investment universe as specified contains two bonds, a two year and ten year maturity zero coupon bond. This may seem a restrictive universe. However, Cochrane and Piazzesi (2005) document that a complete term structure of bonds can be defined recursively. As a result, it is straightforward to extend the universe with bonds of any maturity. However, we have to note that the term structure in our investment universe is based on only two maturities. As a result, the empirical term structure could differ (even in shape) from ours. If we assume that the n-maturity bond yield is a linear function of the VAR-variables

$$y_t^{(n)} = a_n + b_n' X_t, \tag{28}$$

a_n and b_n' can be specified recursively. In line with Cochrane and Piazzesi (2005) we characterize a_n and b_n' as

$$a_{n+1} = -\frac{1}{n+1}\Big(-\delta_0 + A_n + B_n'\mu - B_n'\Sigma\lambda_0 + \frac{1}{2}B_n'\Sigma B_n\Big), \tag{29}$$

$$b_{n+1}' = -\frac{1}{n+1}\Big(-\delta_1' + B_n'\phi - B_n'\Sigma\lambda_1\Big), \tag{30}$$

with

$$A_{n+1} = -\delta_0 + A_n + B_n'\mu - B_n'\Sigma\lambda_0 + \frac{1}{2}B_n'\Sigma B_n, \tag{31}$$

$$B_{n+1}' = -\delta_1' + B_n'\phi - B_n'\Sigma\lambda_1. \tag{32}$$

Since A_0=0 and B_0=0, the price of any n-maturity bond is specified by the VAR-parameters.

3.2 Measuring Price Drivers with Alternative Options

This subsection describes the different alternative options priced to quantify the Greeks of the lookback life annuity option. We start with a base option and first modify the exercise price (i.e. the option version) to determine the effect of the lookback characteristic. Subsequently the parameters time to maturity, initial interest rate and investment weights are changed to resolve the price sensitivities of the option price.

3.2.1 Base Option

The base option is a lookback life annuity option with a time to maturity of three years. This option provides plan participants the right to buy a life annuity at retirement for the minimum price during the last three years before retirement. In addition to the time horizon, the option price is a function of the retirement wealth W_T. W_T is determined by the investment weights (w_s, w_{2b} and w_{10b}) and their corresponding returns. We assume that contributions do not take place in the last years before retirement. For the base option we assume that investment weights are equal (w_s=w_{2b}=w_{10b}) and that the portfolio is monthly rebalanced. As a final characterization, we set the initial value of the conversion rate equal to its last value in the dataset (4.42%).

3.2.2 Option Greeks

As a first sensitivity test, we change the exercise price to determine the impact of the lookback feature on the price. We price an Asian option with the same characteristics as the lookback option. The Asian option provides the right to buy the life annuity for the average price during the time to maturity. This option only has a positive payoff if the time T life annuity price exceeds the average life annuity price. The second alternative option we price has a predetermined and fixed exercise price. This option offers holders the right to convert their retirement wealth against a predetermined rate (e.g. 6%). The option holder exercises the option if the retirement rate does not exceed the guaranteed rate (6%). For both alternatives the probability of having a payoff of zero is substantial, whereas the lookback option has a negligible chance of ending at-the-money (and generating a zero payoff). Since the option price is the expected value of the discounted payoff, the effect of changing the option version is large.

Furthermore, we measure the effect of a change in time to maturity. We price the lookback life annuity option on a horizon of two, four and five years. By providing the right to buy for the minimum price, the time to maturity is an important driver of the option price. An extension in time horizon with one year, gives the life annuity price 12 additional possibilities of attaining a new minimum. Hence, the longer (shorter) the time to maturity, the larger (smaller) the expected option payoff and thus the option price.

As a further possible price driver we select the interest rate at the time the option was issued. Particularly if the interest rate three years before retirement is high, a large demand for the lookback option is likely to arise. Participants then want to protect themselves against a fall in interest rate shortly before the retirement date. We quantify the impact of a change in initial rate. However, we pointed out in the closed-form valuation (Appendix

B) that changes in the lookback option price are not affected by changes in minimum price of the life annuity. On the issuance date t_a, the life annuity price is at a minimum by construction. Hence, the option price is unlikely to be affected by changes in the initial rate. The guarantee that the exact amount of retirement wealth is converted does not alter this result.

Finally, we measure the impact of the investment weights on the option price. In addition to the equally weighted portfolio of the base option, we construct three pure asset portfolios. Investments are restricted to one of the three assets (stock portfolio, two year and ten year bond) in the pure portfolios. The portfolios are re-balanced on a monthly basis. Since we consider the ten year bond yield as the conversion rate of the life annuity, especially the pure ten year bond portfolio is an interesting investment strategy. This portfolio is particularly risky. When the interest rate increases during the time to maturity of the option, the difference in interest rates $(r_T - r_{max})$ is likely to be small and the option payoff consequently low. Additionally, interest rate increases lead to a relatively low retirement wealth W_T, via low bond returns. On the other hand, interest rate decreases lead to large differences between r_T and r_{max} and thus high option payoffs. The high payoff is amplified by a relatively large retirement wealth. The large retirement wealth is a result of high bond returns. The higher risk of the pure ten year bond investment strategy is partially offset by the stochastic discount factor. Nevertheless,investing in the pure ten year bond portfolio is a risky strategy.

4 Empirical Results

After a description of the economic environment, pricing techniques and alternative options, this section reports the prices of the previously described options. We start with pricing a base option and then provide prices for different alternatives, to determine the option Greeks.

4.1 Option Prices

The base option as described in the previous section is a lookback life annuity option with a time to maturity of three years. Since men and women have different mortality rates, we determine option prices for men and women separately.

[Table 1 about here.]

Table 1 reveals that the price of the base option is 8% for male participants and slightly higher than 9% for female participants, due to better life expectancies for women. Furthermore, the simulation results show that the standard deviation across all scenarios of the discounted option payoff is high, approximately 55% for men and 63% for women. This is an indication that the annuity risk that participants encounter at the conversion date is considerable. Table 1 also shows convergence in option prices at 100,000 simulations. Running 10,000 simulations more or less does not alter results substantially. We therefore hereafter generate prices based on 100,000 simulations. Finally, the table with base option prices displays the average across all scenarios of the deflated cumulative stock returns. As expressed by equation (22) the expected value of discounted cumulative stock returns should be equal to one. We approximate the expected value by the average across all scenarios and the results shows that the theoretical condition is satisfied.

4.2 Alternative Option Prices

We first determine the sensitivity of the option price with respect to the exercise price. First an Asian option with the same time to maturity is priced. The Asian option holder has the right to convert his retirement wealth into a life annuity for the average interest rate during time to maturity.

[Figure 2 about here.]

Figure 2 shows that transforming the option into an Asian one, considerably lowers the price. For both men and women the price difference between the Asian and lookback option is approximately 6 percentage points. The Asian option has fewer scenarios with extremely high payoffs and many scenarios with a zero payoff. Only if the terminal rate r_T differs strongly from the average rate, high payoffs are realized.

As a second exercise price sensitivity test, we price an option that guarantees participants a certain conversion rate. Option prices are determined for guaranteed rates of 5% and 6%. Figure 2 again shows that the impact of changing the option version is large. Price differences between the guaranteed rate option and the base option range from 2.5 to 6 percentage points.

Beside the exercise price, we consider the sensitivity of the option price with respect to the time to maturity. The base option has a time to maturity of three years. We determine the effect of a change in time to maturity by pricing lookback life annuity option with times to maturity of 2, 4 and 5 years.

[Figure 3 about here.]

Figure 3 displays that the price effect of a change in time to maturity is large. Decreasing the lookback period with one year leads to a price decrease of 2 percentage points, whereas an increase of one year causes a price increase of 2 percentage points. A lookback life annuity with a time to maturity of five years would cost a participant approximately 12% of the wealth at the option issuance date. Compared to the base option this implies a price increase of 4 percentage points.

Furthermore, we document the effect of a change in initial interest rate. The base option has an initial rate equal to the last data point in our sample (4.42%).

[Figure 4 about here.]

Figure 4 shows that the initial rate does not affect the option price. These findings support the proof in Appendix B that the option payoff is not affected by changes in the minimum price.

As a final sensitivity test, we price the lookback life annuity option with different asset allocations. We restrict the investments during the lifetime of the option to one of the three available products: stock portfolio, two year bond and ten year bond.

[Figure 5 about here.]

A pure stock investment lowers the price compared to the base option by 1 percentage point. Restricting the investments to the two year bond reduces the price by 4 percentage points. Since stocks have a higher risk premium than two year bonds, the "stock-only" strategy is more expensive than the two year bond strategy. However, when investments are restricted to the ten year bond, we observe an increase in price. The price increase is a result of the risky character of the ten year bond. Since we consider the ten year bond rate as the conversion rate, restricting the investments to this category creates a multiplier effect. In relatively good scenarios this strategy leads to very high option payoffs via the difference in terminal and maximum interest rate and via high bond returns. In relatively bad scenarios this strategy leads to extremely poor payoffs vice versa. Nevertheless, we conclude that the asset mix drives the price of the option, but only to a minor extent.

Conclusion

Large numbers of uninformed plan participants face difficult decisions at retirement. Should they make an irreversible and risky investment in a life annuity or risk the possibility of outliving their money? If they buy a life annuity, when should they do so? These decisions affect their income for the rest of their lives. Especially the high value of timing the investment decision provokes regret aversion.

As a remedy for the regret aversion in that decision making process, we develop and price a lookback option on a life annuity. Participants buy this option in the last years before retirement. It provides them the right to lookback at retirement and buy a life annuity for the minimum price in the lookback period. First, we determine a closed-form lookback life annuity option price without the guarantee that the exact amount of retirement wealth is converted into a life annuity. Then we price the option in an augmented pension environment conditional on the guarantee that participants convert the exact amount of retirement wealth into a life annuity. The price of the lookback option is expressed as a percentage of the wealth at the option issuance date. For men the price is approximately 8% and for women 9%, due to the difference in life expectancies.

We determine the option Greeks by pricing alternative options. Important drivers of the option price are the lookback feature (i.e. the exercise price) and the time to maturity. If participants are offered the right to buy the life annuity for the average price (Asian option) or for a predetermined price, the option price decreases substantially. We find that the option price is neither sensitive to the initial interest rate, nor to the asset allocation during the time to maturity.

Future research could improve our analysis in multiple ways. Participants could be offered real instead of nominal pension income. Furthermore, mortality risk and life annuity risk premia could be added in the option pricing environment.

Appendix A

This appendix shows that the hedging portfolio is self-financing. To verify the self-financing property, we make sure that new positions in the hedging portfolio can be attained without extra financing and that the investment in the replicating portfolio is a riskless one. The replicating portfolio value is a function of the bond price $P(r, t, s)$, the exercise price $Q(\overline{f}, t, s)$, option time to maturity $T - t$ and the bond expiration date s. However, the bond price $P(r, t, s)$ is a contingent of the interest rate r, time t and expiration date s. Since T and s are fixed values, the change in hedging portfolio value is a function of r, $Q(\overline{f}, t, s)$ and t. An application of Itô's lemma shows that changes in hedging portfolio value can be characterized as

$$dH(r, Q(\overline{f}, t, s), t) = H_r dr + H_Q dQ(\overline{f}, t, s) + H_t d(t) + \frac{1}{2} H_{rr} (dr)^2, \qquad (33)$$

with

$$H_r = \frac{\partial H(r, Q(\overline{f}, t, s), T - t)}{\partial r}, \qquad (34)$$

$$H_Q = \frac{\partial H(r, Q(\overline{f}, t, s), T - t)}{\partial Q(\overline{f}, t, s)}, \qquad (35)$$

$$H_t = \frac{\partial H(r, Q(\overline{f}, t, s), T - t)}{\partial t}, \qquad (36)$$

$$H_{rr} = \frac{\partial^2 H(r, Q(\overline{f}, t, s), T - t)}{\partial r^2}. \qquad (37)$$

In Appendix B, we prove that $H_Q = 0$, i.e. if the forward bond price is at a minimum, the option value is not affected by a change in minimum. If $H_Q \neq 0$ and the forward bond price is at a minimum, the hedging portfolio has two different hedge ratios: one if the price attains a new minimum and one if it does not. Hedging would then be impossible. Proving that $H_Q = 0$ and noting that we have specified dr as a Vasicek process, the change in hedging portfolio value is characterized as

$$dH = H_r \sigma_r dW + (H_r \alpha(\gamma - r) + \frac{1}{2} H_{rr} \sigma_r^2 + H_t)dt. \tag{38}$$

Equation (38) shows that the hedging portfolio still bears interest rate risk. However, the self-financing property requires that the hedging portfolio is riskless. To hedge the risk in the hedging portfolio, an investment in a bond with maturity s is required. If an amount of

$$\frac{\alpha H_r}{1 - e^{-\alpha(s-t)}} P(r, t, s), \tag{39}$$

where H_r is determined in Appendix C, is invested in a bond with maturity s, the hedging portfolio is free of asset risk. The hedging portfolio now consists of a combined bond straddle and bond investment. Substitution of the bond price difference equation derived in Appendix D, shows that the change in hedging portfolio value can be represented as

$$dH + \frac{\alpha H_r}{1 - e^{-\alpha(s-t)}} \frac{dP(r,t,s)}{P(r,t,s)} = (H_r \alpha(\gamma - r) + \frac{1}{2} H_{rr} \sigma_r^2 + H_t + \frac{\alpha H_r}{1 - e^{-\alpha(s-t)}} r)dt. \tag{40}$$

Collecting terms from Appendix C, in which we determine expressions for H_r, H_t and H_{rr}, we can show that

$$dH + \frac{\alpha H_r}{1 - e^{-\alpha(s-t)}} \frac{dP(r,t,s)}{P(r,t,s)} = \Big(H + \frac{\alpha H_r}{1 - e^{-\alpha(s-t)}}\Big) r dt. \tag{41}$$

Hence, we conclude that all new positions can be financed from the proceeds of the old and that the investment in the hedging portfolio is a riskless one.

Appendix B

In this appendix we prove that $H_Q = 0$. The methodology applied is introduced by Goldman, Sosin, and Gatto (1979). However, the proofs are not identical.

This appendix proves that the raw moments of

$$\{Q(\overline{f}, T, s) | P(r, t, s), t, Q(\overline{f}, t, s)\} \tag{42}$$

are independent of $Q(\overline{f}, t, s)$ for any $t_a < t < T$. Since the hedging portfolio payoff and therefore also its value is a function of $Q(\overline{f},T,s)$, it suffices to show that the distribution of $Q(\overline{f},T,s)$ is unaffected by changes in $Q(\overline{f},t,s)$. To see this we note that

$$C_L^B(P(r,t,s),Q(\overline{f},t,s),T-t,s) = e^{-rt}E[P(r,T,s)-Q(\overline{f},T,s)]. \tag{43}$$

To prove the independence we can restrict ourselves to the cases where the forward bond price attains a minimum

$$F^B(r,t,s) \rightarrow Q(\overline{f},t,s).$$

If the forward bond price is not at a minimum $dQ(\overline{f},t,s) = 0$. Hence, $Q(\overline{f},T,s)$ is independent of $Q(\overline{f},t,s)$ in case $F^B(r,t,s) \nrightarrow Q(\overline{f},t,s)$.

We start our proof by splitting up the time to maturity of the option and defining ψ as

$$\psi = \begin{cases} \frac{Q(\overline{f},t,s)}{F^B(r,t,s)} \; if \; Q(\overline{f},t,s) < Q_T(\overline{f},t,s) \\ \frac{Q_T(\overline{f},t,s)}{F^B(r,t,s)} \equiv Z \; otherwise, \end{cases} \tag{44}$$

with $Q_T(\overline{f},t,s)$ the minimum bond price in the period $[t,T]$ and by construction $Q(\overline{f},T,s) = F^B(r,t,s)\psi$. The n^{th} raw moment of $Q(\overline{f},T,s)$ can be depicted as

$$G(n) = \int_{-\infty}^{F^B(r,t,s)} \left[Q(\overline{f},T,s)\right]^n d\Phi_Q\left[Q(\overline{f},T,s)\right], \tag{45}$$

with Φ_Q the CDF of $Q(\overline{f},T,s)$. Simple substitution transforms the n^{th} raw moment into

$$G(n) = \int_{-\infty}^{1} [F^B(r,t,s)\psi]^n d\Phi_Q[\psi]. \tag{46}$$

Decomposing ψ as in the definition and noting that we condition on the information set at time t leads to

$$G(n) = \left[Q(\overline{f},t,s)^n \int_{\frac{Q(\overline{f},t,s)}{F^B(r,t,s)}}^{1} d\Phi_Z(Z)\right] + \left[Q(\overline{f},t,s)^n \int_{-\infty}^{\frac{Q(\overline{f},t,s)}{F^B(r,t,s)}} Z^n d\Phi_Z(Z)\right], \tag{47}$$

with Φ_Z the distribution function of Z. Z is like a return and $F^B(r,t,s)$ follows a random walk (see Jamshidian, 1989), therefore the derivative of the second part towards $Q(\overline{f},t,s)$ is zero. Differentiation of the first part towards $Q(\overline{f},t,s)$ yields

$$\frac{\partial G(n)}{\partial Q(\overline{f},t,s)} = nQ(\overline{f},t,s)^{n-1} \int_{1}^{\frac{Q(\overline{f},t,s)}{F^B(r,t,s)}} d\Phi_Z(Z). \tag{48}$$

Since we only consider the case where $F^B(r,t,s) \to Q(\overline{f},t,s)$ and we assumed a Wiener process for the underlying interest rate, the probability mass at the point $\frac{Q(\overline{f},t,s)}{F^B(r,t,s)}$ is zero. Hence, the n-th raw moment of $Q(\overline{f},T,s)$ is independent of $Q(\overline{f},t,s)$ for any n.

Appendix C

This Appendix derives expressions for H_r, H_{rr} and H_t, based on bond option formulas provided by Jamshidian (1989). Since all three desired expressions are derivatives of the hedge portfolio we start with expressing the hedging portfolio value as

$$\begin{aligned} H(r,Q(\overline{f},t,s),t) =& \big[2N(h)-1\big]P(r,t,s) \\ &+ \big[1-2N(h-\sigma_p)\big]P(r,t,T)Q(\overline{f},t,s). \end{aligned} \tag{49}$$

The derivatives can be specified as

$$H_r = \frac{\partial H(r,Q(\overline{f},t,s),t)}{\partial r}, \tag{50}$$

$$H_t = \frac{\partial H(r,Q(\overline{f},t,s),t)}{\partial t}, \tag{51}$$

$$H_{rr} = \frac{\partial^2 H(r,Q(\overline{f},t,s),t)}{\partial r^2}. \tag{52}$$

The derivative of the hedging portfolio with respect to the interest rate r, H_r can be further specified as

$$\begin{aligned}H_r =&2N(h)\frac{\partial P(r,t,s)}{\partial r}+2n(h)P(r,t,s)\frac{\partial h}{\partial r}-2N(h-\sigma_p)Q(\overline{f},t,s)\frac{\partial P(r,t,T)}{\partial r}\\&-\frac{\partial P(r,t,s)}{\partial r}+Q(\overline{f},t,s)\frac{\partial P(r,t,T)}{\partial r}\\&-2n(h-\sigma_p)P(r,t,T)Q(\overline{f},t,s)\frac{\partial h-\sigma_p}{\partial r}.\end{aligned} \tag{53}$$

Furthermore, we note that

$$n(h)=n(h-\sigma_p)\frac{P(r,t,T)Q(\overline{f},t,s)}{P(r,t,s)}. \tag{54}$$

Jamshidian (1989) expresses the bond price as

$$P(r,t,s)=e^{\frac{1}{2}k^2(t,s)-n(r,t,s)}, \tag{55}$$

with

$$k^2(t,s)=\frac{\sigma_r^2(4e^{-\alpha(s-t)}-e^{-2\alpha(s-t)}+2\alpha(s-t)-3)}{2\alpha^3}, \tag{56}$$

$$n(r,t,s)=(s-t)\gamma+(r-\gamma)\frac{1-e^{-\alpha(s-t)}}{\alpha}. \tag{57}$$

The derivative of $P(r,t,s)$ towards r is characterized as

$$\begin{aligned}\frac{\partial P(r,t,s)}{\partial r}&=-\frac{\partial n(r,t,s)}{\partial r}P(r,t,s)\\&=-\frac{1-e^{-\alpha(s-t)}}{\alpha}P(r,t,s).\end{aligned} \tag{58}$$

We characterize the derivative of the hedge portfolio towards r as

$$\begin{aligned}H_r =&\big[1-2N(h)\big]\frac{1-e^{-\alpha(s-t)}}{\alpha}P(r,t,s)\\&+\big[2N(h-\sigma_p)-1\big]\frac{1-e^{-\alpha(T-t)}}{\alpha}P(r,t,T)Q(\overline{f},t,s).\end{aligned} \tag{59}$$

Having determined the first derivative towards r, we continue with the second derivative towards r, H_{rr}. This second derivative can be characterized as

$$
\begin{aligned}
H_{rr} =& -2n(h)\frac{1-e^{-\alpha(s-t)}}{\alpha}P(r,t,s)\frac{\partial h}{\partial r} - \big[2N(h)-1\big]\frac{1-e^{-\alpha(s-t)}}{\alpha}\frac{\partial P(r,t,s)}{\partial r}\\
&+2n(h-\sigma_p)Q(\overline{f},t,s)\frac{1-e^{-\alpha(T-t)}}{\alpha}P(r,t,T)\frac{\partial h}{\partial r}\\
&-\big[1-2N(h-\sigma_p)\big]Q(\overline{f},t,s)\frac{1-e^{-\alpha(T-t)}}{\alpha}\frac{\partial P(r,t,T)}{\partial r}\\
=&\big[2N(h)-1\big]\frac{\big(1-e^{-\alpha(s-t)}\big)^2}{\alpha^2}P(r,t,s)\\
&+\big[1-2N(h-\sigma_p)\big]\frac{\big(1-e^{-\alpha(T-t)}\big)^2}{\alpha^2}P(r,t,T)Q(\overline{f},t,s)\\
&-2n(h)P(r,t,s)e^{-\alpha(T-t)}\frac{\big(1+e^{-\alpha(s-T)}\big)}{\alpha}\frac{\partial h}{\partial r}\\
=&\big[2N(h)-1\big]\frac{\big(1-e^{-\alpha(s-t)}\big)^2}{\alpha^2}P(r,t,s)\\
&+\big[1-2N(h-\sigma_p)\big]\frac{\big(1-e^{-\alpha(T-t)}\big)^2}{\alpha^2}P(r,t,T)Q(\overline{f},t,s)\\
&-2n(h)P(r,t,s)e^{-\alpha(T-t)}\frac{\big(1+e^{-\alpha(s-T)}\big)}{\alpha}\frac{1}{\sigma_p}\Big[\frac{\partial ln(\frac{P(r,t,s)}{P(r,t,T)})}{\partial r}\Big]\\
=&\big[2N(h)-1\big]\frac{\big(1-e^{-\alpha(s-t)}\big)^2}{\alpha^2}P(r,t,s)\\
&+\big[1-2N(h-\sigma_p)\big]\frac{\big(1-e^{-\alpha(T-t)}\big)^2}{\alpha^2}P(r,t,T)Q(\overline{f},t,s)\\
&-2n(h)P(r,t,s)\frac{1}{\sigma_p\alpha^2}\Big[e^{-2\alpha(T-t)}\big(1-2e^{-\alpha(s-T)}+e^{-2\alpha(s-T)}\big)\Big].
\end{aligned}
\tag{60}
$$

Differentiating the hedging portfolio value towards t leads to

$$
\begin{aligned}
H_t =&\big[2N(h)-1\big]P(r,t,s)f(r,t,s)+\big[1-2N(h-\sigma_p)\big]f(r,t,T)\\
&+2n(h)P(r,t,s)\frac{\partial \sigma_p}{\partial t}\\
=&\big[2N(h)-1\big]P(r,t,s)\big(m(r,t,s)-q(t,s)\big)\\
&+\big[1-2N(h-\sigma_p)\big]\big(m(r,t,T)-q(t,T)\big)\\
&+2n(h)P(r,t,s)\frac{\partial \sigma_p}{\partial t}.
\end{aligned}
\tag{61}
$$

with

$$f(r,t,s) = m(r,t,s) - q(t,s), \tag{62}$$
$$m(r,t,s) = e^{-\alpha(s-t)}r + (1 - e^{-\alpha(s-t)})\gamma, \tag{63}$$
$$q(t,s) = \sigma_r^2 \frac{(1 - e^{-\alpha(s-t)})^2}{2\alpha^2}. \tag{64}$$

H_t can thus be expressed as

$$\begin{aligned}
H_t =& \big[2N(h) - 1\big] P(r,t,s)\Big(e^{-\alpha(s-t)}r + (1 - e^{-\alpha(s-t)})\gamma - \sigma_r^2 \frac{(1 - e^{-\alpha(s-t)})^2}{2\alpha^2}\Big) \\
&+ \big[1 - 2N(h - \sigma_p)\big] Q(\overline{f},t,s) P(r,t,T)\Big(e^{-\alpha(T-t)}r + (1 - e^{-\alpha(T-t)})\gamma\Big) \\
&- \big[1 - 2N(h - \sigma_p)\big] Q(\overline{f},t,s) P(r,t,T)\sigma_r^2 \frac{(1 - e^{-\alpha(T-t)})^2}{2\alpha^2} \\
&+ 2n(h)P(r,t,s)\frac{\partial \sigma_p}{\partial t} \\
=& \big[2N(h) - 1\big] P(r,t,s)\Big(e^{-\alpha(s-t)}r + (1 - e^{-\alpha(s-t)})\gamma - \sigma_r^2 \frac{(1 - e^{-\alpha(s-t)})^2}{2\alpha^2}\Big) \\
&+ \big[1 - 2N(h - \sigma_p)\big] Q(\overline{f},t,s) P(r,t,T)\Big(e^{-\alpha(T-t)}r + (1 - e^{-\alpha(T-t)})\gamma\Big) \\
&- \big[1 - 2N(h - \sigma_p)\big] Q(\overline{f},t,s) P(r,t,T)\sigma_r^2 \frac{(1 - e^{-\alpha(T-t)})^2}{2\alpha^2} \\
&- n(h)P(r,t,s)\frac{\sigma_r^2}{\sigma_p \alpha^2} e^{-2\alpha(T-t)}\big[1 - 2e^{-\alpha(s-T)} + e^{-2\alpha(s-T)}\big].
\end{aligned} \tag{65}$$

Appendix D

This Appendix determines the characteristics of the bond investment that is needed to hedge the interest risk of the bond straddle. The change in value of a zero coupon bond with maturity date s is represented by

$$dP(r,t,s) = P_r dr + P_t dt + \frac{1}{2} P_{rr} (dr)^2, \tag{66}$$

with

$$P_r = \frac{\partial P(r,t,s)}{\partial r}, \tag{67}$$
$$P_t = \frac{\partial P(r,t,s)}{\partial t}, \tag{68}$$
$$P_{rr} = \frac{\partial^2 P(r,t,s)}{\partial r^2}. \tag{69}$$

Consequently, the bond return can be represented as

$$\frac{dP(r,t,s)}{P(r,t,s)} = -\frac{1-e^{-\alpha(s-t)}}{\alpha}dr + \big(f(r,t,s) + q(t,s)\big)dt. \tag{70}$$

Premultiplying this bond investment with the derivative of $n(r,t,s)$ towards r leads to

$$\frac{\alpha}{1-e^{-\alpha(s-t)}}\frac{dP(r,t,s)}{P(r,t,s)} = -dr + \frac{\alpha m(r,t,s)}{1-e^{\alpha(s-t)}}dt. \tag{71}$$

Premultiplication with H_r as specified in Appendix C gives the bond return specification

$$\frac{\alpha H_r}{1-e^{-\alpha(s-t)}}\frac{dP(r,t,s)}{P(r,t,s)} = -\sigma_r H_r dW + \frac{\alpha H_r}{1-e^{-\alpha(s-t)}}rdt. \tag{72}$$

Hence, an investment of

$$\frac{\alpha H_r}{1-e^{-\alpha(s-t)}}P(r,t,s), \tag{73}$$

in a zero coupon bond with maturity date s is needed to hedge make the hedging portfolio riskless.

Appendix E

This Appendix provides data summary statistics and estimation results. Table 2 presents information on the mean, standard deviation, skewness and kurtosis of the return series. The figures show that the market risk premium and risk of stocks is highest. Furthermore, the data show higher average returns on longer maturity bonds. Finally, we note that the two year bond return series has skewness and kurtosis figures that deviate slightly from the third and fourth moment of the normal distribution.

[Table 2 about here.]

Finally, table 3 reports parameter estimates and T-statistics for the VAR(1)-model described in section 3 and for the parameters δ_0 and δ_1. These parameter estimates are used in the Monte Carlo procedure to generate scenarios. By randomly drawing error terms for the VAR(1)-model we generate discounted option payoffs for each scenario.

[Table 3 about here.]

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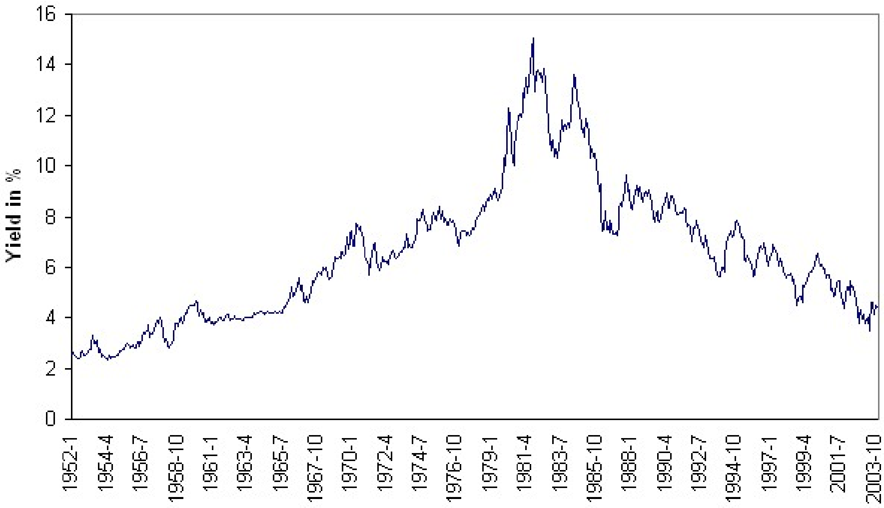
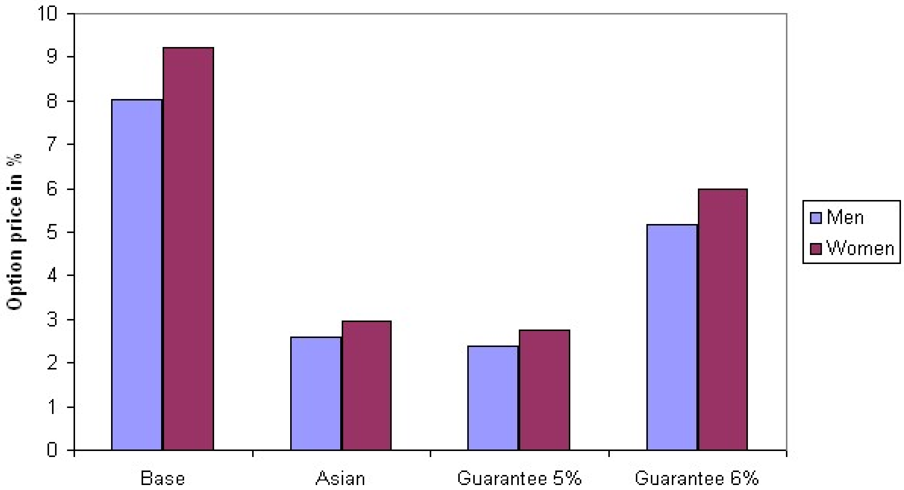
Figure 1: **Conversion Rate**



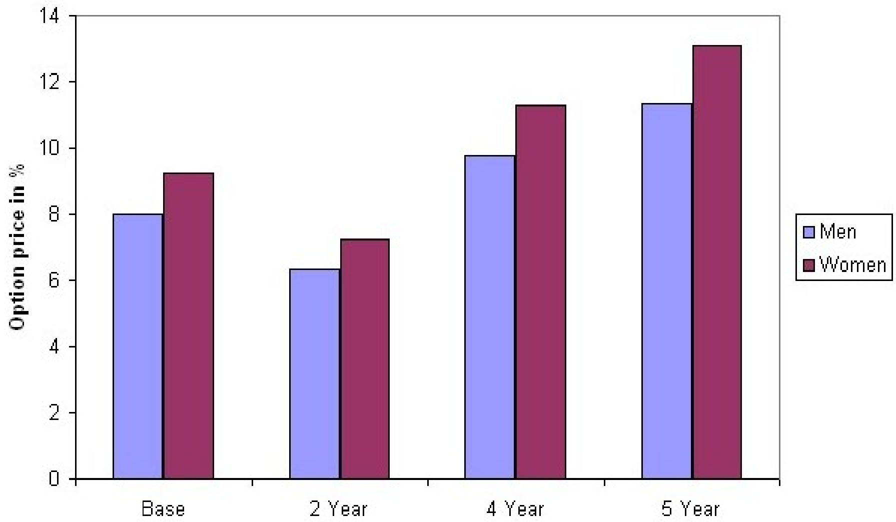
This figure displays the time series evolution of the 10 year bond yield from January 1952 to January 2004. The yields are annualized.

Figure 2: **Variation in Option Design**



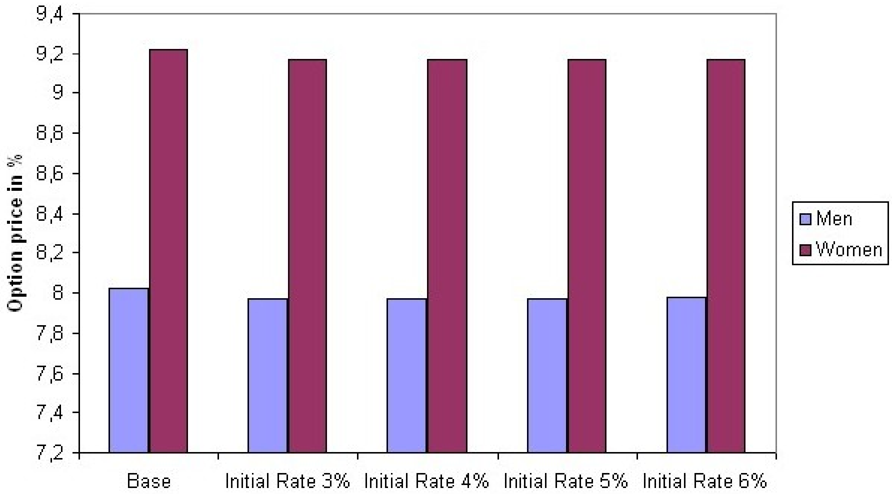
This figure displays the effect of a change in option design. Option prices are expressed in a percentage of wealth at the option issuance date, for both men and women. Prices are provided for the base option (3 year lookback life annuity) and an Asian option and Guaranteed Rate option with equal characteristics. Conversion rates of 5% and 6% are guaranteed in the Guaranteed Rate option.

Figure 3: **Variation in Time to Maturity**



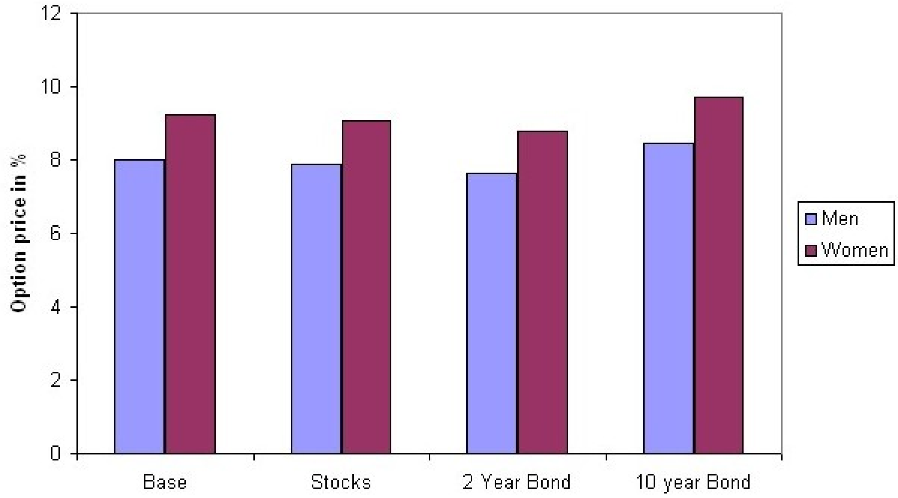
This figure displays the effect of a change in time to maturity. Option prices are expressed in a percentage of wealth at the option issuance date, for both men and women. Prices are provided for the base option (3 year lookback life annuity) and lookback life annuity option with times to maturity of respectively 2, 4 and 5 year.

Figure 4: **Variation in Initial Rate**



This figure displays the effect of a change in initial conversion rate. Option prices are expressed in a percentage of wealth at the option issuance date, for both men and women. Prices are provided for the base option (3 year lookback life annuity with initial rate of 4.42%) and for a lookback life annuity option with initial rates of 4%, 5% and 6% respectively.

Figure 5: **Variation in Asset allocation**



This figure displays the effect of a change in asset allocation. Option prices are expressed in a percentage of wealth at the option issuance date, for both men and women. Prices are provided for the base option (3 year lookback life annuity with equal allocation in the three investment categories (stocks, two year bond and 10 year bond)) and for a lookback life annuity option with investments restricted to one of the three categories.

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Table 1: **Base Option Prices**

This table presents prices for the lookback life annuity option with a time to maturity of three years. Prices for both men and women are expressed in percentages of the wealth at the issuance date of the option (W_{t_a}). In parentheses we display the standard deviation across all scenarios of the discounted option prices by σ_m and σ_w. We present option prices based on 90,000, 100,000 and 110,00 scenarios. The last row reports the average across all scenarios of the deflated cumulative stock returns.

	90,000	**100,000**	**110,000**
Men	8.02	7.98	7.97
(σ_m)	(55.06)	(53.71)	(55.23)
Women	9.22	9.17	9.17
(σ_w)	(63.94)	(62.33)	(64.11)
$E[MR_{s,T}]$	0.997	0.999	0.995

Table 2: **Return Summary Statistics**

This table presents summary statistics of the annualized stock, two and ten year bond returns. For each return series, different rows display the mean, standard deviation, skewness and kurtosis. The statistics are computed on a data period ranging from January 1952 until January 2004.

	r_s	r_{2b}	r_{10b}
Mean	0.108	0.059	0.065
Standard Deviation	0.149	0.100	0.093
Skewness	-0.708	0.840	0.713
Kurtosis	2.79	0.762	0.188

Table 3: **VAR Estimation Results**

This table presents parameter estimates and the corresponding T-statistics of the VAR(1)-model for stock and bond returns and for δ_0 and δ_1 as specified in section 3. Monthly stock, two year and ten year bond returns are respectively denoted by $\boldsymbol{r_{s,t}}$, $\boldsymbol{r_{2b,t}}$ and $\boldsymbol{r_{10b,t}}$ and **c** represents a constant. The one month log bond yield is represented by $\boldsymbol{y^{(1)}_{1m_t}}$. We approximate the one month maturity bond yield by the monthly yield on a three month maturity bond, for data reasons. T-statistics are reported in parentheses. The estimations are performed on a data period ranging from January 1952 until January 2004.

	c	$\boldsymbol{r_{s,t-1}}$	$\boldsymbol{r_{2b,t-1}}$	$\boldsymbol{r_{10b,t-1}}$
$\boldsymbol{r_{s,t}}$	0.0055	0.0490	0.5806	0.0705
	(2.5604)	(1.2202)	(1.6162)	(0.4917)
$\boldsymbol{r_{2b,t}}$	0.0040	-0.0330	0.2101	0.0116
	(10.7840)	(-4.7172)	(3.3589)	(0.4628)
$\boldsymbol{r_{10b,t}}$	0.0037	-0.0653	0.4208	-0.0485
	(3.8897)	(-3.6423)	(2.6217)	(-0.7573)
	c	$\boldsymbol{r_{s,t-1}}$	$\boldsymbol{r_{2b,t-1}}$	$\boldsymbol{r_{10b,t-1}}$
$\boldsymbol{y^{(1)}_{1m_t}}$	0.0036	-0.0044	0.1912	-0.0455
	(35.0430)	(-2.2660)	(11.0330)	(-6.5832)