16TH INTERNATIONAL AFIR COLLOQUIUM
JUNE 12-15, 2007

STOCKHOLM, SWEDEN

# MODELING AND MANAGEMENT OF NONLINEAR DEPENDENCIES– COPULAS IN DYNAMIC FINANCIAL ANALYSIS

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Abstract: The aim of this paper is to study the influence of nonlinear dependencies on a non-life insurer's risk and return profile. To achieve this, we integrate several copula models in a dynamic financial analysis (DFA) framework and conduct numerical tests within a simulation study. We also test several management strategies in response to adverse outcomes generated by nonlinear dependencies. We find that nonlinear dependencies have a crucial influence on the insurer's risk profile that can hardly be affected by the analyzed management strategies. Depending on the copula concept employed, we find large differences in risk assessment for the ruin probability and for the expected policyholder deficit. This has important implications for regulators and rating agencies that use these risk measures as a foundation for capital standards and ratings.

Keywords: Enterprise Risk Management, Asset Liability Management, Dynamic Risk Modelling, Dynamic Financial Analysis, Solvency Analysis, Risk-Based Capital, Financial Performance Measurement, Simulation, Copulas

## 1. INTRODUCTION

Dynamic financial analysis (DFA) is a financial modeling approach that projects financial results under a variety of possible scenarios, showing how outcomes might be affected by changing internal and external conditions (see Casualty Actuarial Society, 2006). DFA has become an important tool for decision making and an essential part of enterprise risk management (ERM), particularly within the field of non-life insurance.

DFA results and the quality of decisions derived from them are dependent on an appropriate modeling of the stochastic behavior of assets and liabilities. In this context, the correct mapping of nonlinear dependencies is of central concern. Although many DFA models still use linear correlation, the literature suggests that linear correlation is not appropriate in modeling dependence structures between heavy-tailed and skewed risks, which are frequent in the insurance context (see, e.g., Embrechts/McNeil/Straumann, 2002). These risks are especially relevant in case of extreme events, e.g., the September 11, 2001, terrorist attacks that resulted in insurance companies experiencing large losses both from their underwriting business and the related capital markets plunge (see, e.g., Achleitner/Biebel/Wichels, 2002).

In this paper we evaluate the influence of such extreme events on a non-life insurer's risk and return profile. We integrate nonlinear dependencies in a DFA framework using the copulas concept and evaluate their effects on the insurer's risk and return distribution within a simulation study. As one cannot generally say which copula describes reality best, we compare different forms of copulas and evaluate the possible impact in a stress-testing sense.

In our simulation study, we find that nonlinear dependencies have a strong influence on the insurer's default risk and performance. We also find different impacts of nonlinear dependencies on ruin probability and expected policyholder deficit, a result that is of special relevance for policyholders and regulators. For example, for some kinds of nonlinear dependencies, the expected policyholder deficit cannot be reduced by increasing equity capital. It thus seems that these tail dependencies are relevant not only for low-capitalized companies but also for well-capitalized companies. Furthermore, we test several management strategies implemented in response to adverse outcomes generated by nonlinear dependencies. Our simulation results show that simple risk-reduction strategies are of little use. For example, a reinsurance strategy can delimit the high ruin probability generated by nonlinear dependencies, but not necessarily the expected policyholder deficit.

Our paper builds upon two branches of literature—DFA and the copulas concept. In the late 1990s, the Casualty Actuarial Society introduced simulation models for property-casualty insurers, calling them "DFA" (see Casualty Actuarial Society, 2006). Since then, several surveys and applications of DFA have been published in academic journals. Lowe/Stanard (1997), as well as Kaufmann/Gadmer/Klett (2001), provide an introduction to DFA by presenting a model framework and an application of their model. Lowe/Stanard (1997) develop a DFA model for the underwriting, investment, and capital management process of a property-catastrophe reinsurer and Kaufmann/Gadmer/Klett (2001) provide an up-and-running model for a non-life insurance company. Blum et al. (2001), D'Arcy/Gorvett (2004), and Eling/Parnitzke/Schmeiser (2006) use DFA to examine specific decision-making situations. Blum et al. (2001) investigate the impact of foreign exchange risks on reinsurance decisions within a DFA framework and D'Arcy/Gorvett (2004) apply DFA to search for an optimal growth rate in the property-casualty insurance business. Eling/Parnitzke/Schmeiser (2006) investigate the influence of management strategies on an insurer's risk and return position within a DFA framework.

The copulas concept and the problem of mapping nonlinear dependencies in an insurance context was first introduced by Wang (1998), who discussed models and algorithms for the aggregation of correlated risk portfolios. Frees/Valdez (1998) also provide an introduction to the use of copulas in risk measurement by describing the basic properties of copulas, their relationships to measures of dependence, and several families of copulas. Klugman/Parsa (1999) and Da Costa Dias (2004) both develop appropriate models to analyze finance and insurance data by fitting copulas to empirical data. Blum/Dias/Embrechts (2002) discuss the use of copulas to handle the measurement of dependence in alternative risk transfer products. Embrechts/McNeil/Straumann (2002) present properties, pitfalls, and simulation algorithms for correlation and dependence in risk management and analyze the effect of dependence structures on the value at risk. Pfeifer/Nešlehová (2004) propose approaches for modeling and generating dependent risk processes in the framework of collective risk theory.

We contribute to this literature by integrating copulas in an extended version of the DFA model presented by Eling/Parnitzke/Schmeiser (2006) and by evaluating their influence on the insurer's risk and return position. Our results indicate that it is crucial to consider the copulas concept in order to improve DFA and decision making in enterprise risk management. Our findings are important for regulators and rating agencies because we find large differences in risk assessment for the expected policyholder deficit and for the ruin probability. As these measures are the basis of many capital standards and ratings, it is important to integrate nonlinear dependencies in the regulatory framework and in rating assessment, e.g., in stress testing and scenario analysis.

The rest of the paper is organized as follows. In Section 2, we present a DFA framework containing the essential elements of a non-life insurance company. In Section 3, we describe the copulas concept, different types of copulas, and how these might be integrated within the DFA framework. In Section 4, we define financial ratios, reflecting both risk and return in a DFA context. A DFA simulation study to examine the effects of the copulas on risk and return is presented in Section 5. In Section 6, we measure the influence of management strategies implemented as a response to adverse outcomes generated by the copulas. Section 7 concludes.

## 2. Model Framework

We build on the DFA framework used by Eling/Parnitzke/Schmeiser (2006) to investigate the influence of management strategies on an insurer's risk and return position. In this model, a management rule changes the portion of risky investments and the market share in the underwriting business depending on the insurer's financial situation. We extend this framework with a modified underwriting cycle following an autoregressive process of order two and a modified claims process consisting of noncatastrophe and catastrophe losses.

Let $EC_t$ be the equity capital of the insurance company at the end of time period $t$ ($t \in 1,...,T$) and $E_t$ the company's earnings in $t$. Then, development of the equity capital over time can be written as:

$$EC_t = EC_{t-1} + E_t. \tag{1}$$

The financial statement earnings $E_t$ in period $t$ consist of the investment result $I_t$ and the underwriting result $U_t$ In case of positive earnings, taxes are paid. We denote $tr$ as the tax rate and obtain the company's earnings in $t$ as:

$$E_t = I_t + U_t - \max(tr \cdot (I_t + U_t), 0). \quad (2)$$

The assets can be divided in high-risk investments, such as stocks or high-yield bonds, and low-risk investments, such as government bonds or money market instruments. We denote $\alpha_{t-1}$ as the portion of high-risk investment in time period $t$ and $r_{1t}$ ($r_{2t}$) as the return of the high-risk (low-risk) investment in $t$. The return of the company's investment portfolio in $t$ ($r_{pt}$) can be calculated as:

$$r_{pt} = \alpha_{t-1} \cdot r_{1t} + \left(1 - \alpha_{t-1}\right) \cdot r_{2t}. \quad (3)$$

By multiplying the portfolio return by the funds available for investments, we calculate the company's investment results. The funds available for investments are the equity capital and the premium income $P_{t-1}$, less the upfront expenses $Ex_{t-1}^P$:

$$I_t = r_{pt} \cdot (EC_{t-1} + P_{t-1} - Ex_{t-1}^P). \quad (4)$$

To model the underwriting business, we denote $\beta_{t-1}$ as the company's portion of the relevant market in $t$. The whole underwriting market accessible to the insurer (given by $MV$) is obtained with $\beta = 1$. The premium rate level achievable in the market has been observed to exhibit a cyclical pattern. Following Cummins/Outreville (1987), we model the underwriting cycle using an autoregressive process of order two:

$$\Pi_t = \varphi_0 + \varphi_1 \Pi_{t-1} + \varphi_2 \Pi_{t-2} + \varepsilon_t. \quad (5)$$

The current rate level $\Pi_t$ depends on the premium levels of the two previous periods and a random error term $\varepsilon_t$ following a white noise process. Depending on the parameterization, the process produces cycle lengths that can be calibrated according to observed data. The premium income $P_t$ in period $t$ thus also depends on the premium rate level $\Pi_t$.

Based on an experiment, Wakker/Thaler/Tversky (1997) showed that a rise in default risk leads to a rapid decline in the achievable premium level. The premium income should thus not only be connected to the underwriting cycle but also to a consumer response function. The consumer response function (denoted by the parameter $cr$) represents the link between the premiums written and the company's safety level. We determine the safety level by considering the equity capital at the end of the previous period. The premium income in our model is given as:

$$P_{t-1} = cr_{t-1}^{EC_{t-1}} \cdot \Pi_{t-1} \cdot \beta_{t-1} \cdot MV. \quad (6)$$

Two types of expenses are integrated in the model: upfront costs ($Ex_{t-1}^P$) and claim settlement costs ($Ex_t^C$).The upfront expenses depend linearly on the level of written market volume (modeled with the factor $\gamma$), and nonlinearly on the change in written market volume (modeled with the factor $\eta$, e.g., because of increased advertising and promotion efforts). The upfront costs $Ex_{t-1}^{P}$ are thus calculated as:

$$Ex_{t-1}^{P} = \gamma \cdot \beta_{t-1} \cdot MV + \eta \cdot ((\beta_{t-1} - \beta_{t-2}) \cdot MV)^2 . \tag{7}$$

Claim settlement costs are calculated as a portion $\delta$ of the claims (denoted by $C$) incurred ($Ex_t^C = \delta C_t$). The claims consist of noncatastrophe losses and catastrophe losses ($C=C_{ncat}+C_{cat}$). The underwriting result is thus given by:

$$U_t = P_{t-1} - C_t - Ex_{t-1}^{P} - Ex_t^{C} . \tag{8}$$

A summary, in table form, of the model parameters used in this paper can be found in Appendix I.

## 3. Integration of Copulas in DFA

A crucial part of DFA is the modeling of dependencies between risk categories, i.e., between different asset classes (high-risk vs. low-risk investments), different kinds of liabilities (noncatastrophe losses vs. catastrophe losses), and between assets and liabilities. The dependencies between these risk categories can be integrated in DFA by generating correlated random numbers.

As the literature suggests that linear correlation is not appropriate in modeling dependence structures between heavy-tailed and skewed risks (see, e.g., Embrechts/McNeil/Straumann, 2002), we use copulas to model nonlinear dependencies. Copulas provide a means of separating the description of a dependence structure from the marginal distributions.

To investigate the effects of different copulas, we model correlations between high-risk investments, low-risk investments, noncatastrophe losses, and catastrophe losses using the Gauss copula, the *t* copula, and three nonexchangeable Archimedean copulas (Gumbel copula, Clayton copula, Frank copula). The Gauss and the *t* copulas have been studied extensively in risk management literature and are widespread in practice (see, e.g., McNeil/Frey/Embrechts, 2005). The Gauss copula is contained in the multivariate normal distribution and does not exhibit tail dependence:

$$C_P^{Gauss}(\mathbf{u}) = \boldsymbol{\Phi}_P(\Phi^{-1}(u_1), \Phi^{-1}(u_2), \Phi^{-1}(u_3), \Phi^{-1}(u_4)) . \tag{9}$$

$\Phi$ denotes the standard univariate normal density function and $\boldsymbol{\Phi}_P$ is the joint density function of a four-dimensional Gaussian vector **u** with correlation matrix $P$. Another copula that arises from the multivariate Student *t* distribution is the *t* copula. In contrast to the Gauss copula, the *t* copula exhibits upper and lower tail dependence:

$$C_{\nu,P}^{t}(\mathbf{u}) = \mathbf{t}_{\nu,P}(t_\nu^{-1}(u_1), t_\nu^{-1}(u_2), t_\nu^{-1}(u_3), t_\nu^{-1}(u_4)) , \tag{10}$$

where $t_\nu$ is the density function of a standard univariate *t* distribution and $\mathbf{t}_\nu$ is the joint density function of a four-dimensional vector with correlation matrix $P$.

McNeil/Frey/Embrechts (2005) propose a method to calibrate elliptical copulas such as the Gauss and the *t* copula using the relationship between Kendall's rank correlation $\rho_\tau$ and the off-diagonal elements $\rho_{ij}$ of the correlation matrix $P$, where $\rho_{ij}$ stands for the correlation between the two random variables $X_i$ and $X_j$. We follow their approach and calibrate the Gauss and the *t* copula parameters according to:

$$\rho_\tau(X_i, X_j) = (2/\pi)\arcsin\rho_{ij}\,. \tag{11}$$

In addition to the Gauss and *t* copulas, we implement three Archimedean copulas. The key characteristic of Archimedean copulas is that they can be easily constructed by the use of generator functions $\phi(t)$ (see Nelsen, 1999). We use three different copulas so as to account for different types of tail dependence. The Gumbel copula shows upper tail dependence, the Clayton copula has dependence in the lower tail, and the Frank copula exhibits no tail dependence. The generator functions $\phi(t)$ for the three Archimedean copulas are given in Table 1. $\theta$ denotes the respective copula parameter for each of the copulas.

Table 1: Generator functions for Archimedean copulas

| Copula | Tail Dependence | Generator $\phi(t)$ | Kendall's tau $\rho_\tau$ |
|---|---|---|---|
| $C_\theta^{Gumbel}$ | upper | $(-\ln t)^\theta$ | $1-1/\theta$ |
| $C_\theta^{Clayton}$ | lower | $\frac{1}{\theta}(t^{-\theta}-1)$ | $\theta/(\theta+2)$ |
| $C_\theta^{Frank}$ | none | $-\ln(\frac{e^{-\theta t}-1}{e^{-\theta}-1})$ | $1-4\theta^{-1}(1-\theta^{-1}\int_0^\theta t/(\exp(t)-1)dt)$ |

Archimedean copulas can be calibrated to data based on the functional relationship between Kendall's rank correlation $\rho_\tau$ and the copula parameter $\theta$. These relationships are summarized in the last column of Table 1 for the copulas used in our model. For example, Kendall's tau $\rho_\tau$ equals $1-1/\theta$ for the Gumbel copula. By inverting this relationship, the parameter value $\theta$ can be obtained for any given value of $\rho_\tau$.

We will use the generator function $\phi_1$ and its corresponding parameter $\theta_1$ to model the correlation between high-risk and low-risk investments, $\phi_2$ with parameter $\theta_2$ for the correlation between noncatastrophe losses and catastrophe losses, and $\phi_3$ with $\theta_3$ to correlate assets and liabilities. The copula parameter values $\theta_1$, $\theta_2$, and $\theta_3$ are calibrated based on the correlations $\rho_{\tau 1}$ (high-risk investments and low-risk investments), $\rho_{\tau 2}$ (noncatastrophe losses and catastrophe losses), and $\rho_{\tau 3}$ (assets and liabilities).

The family of Archimedean copulas contains both exchangeable and nonexchangeable copulas. As exchangeable copulas impose restrictive conditions on the dependence structure, especially in a multivariate context (e.g., exchangeable copulas result in the same correlation within liabilities as between assets and liabilities), we will use nonexchangeable copulas in order to avoid these unfavorable features. We choose a four-dimensional nonexchangeable construction, described in McNeil/Frey/Embrechts (2005), consisting of three strict Archimedean generators with completely monotonic inverses and composite functions $\phi_3 \circ \phi_1^{-1}$ and $\phi_3 \circ \phi_2^{-1}$:

$$C(u_1, u_2, u_3, u_4) = \phi_3^{-1}(\phi_3 \circ \underbrace{\phi_1^{-1}(\phi_1(u_1) + \phi_1(u_2))}_{\text{high-risk and low-risk investments}} ) + \phi_3 \circ \underbrace{\phi_2^{-1}(\phi_2(u_3) + \phi_2(u_4))}_{\text{non-catastrophe and catastrophe losses}} ). \qquad (12)$$

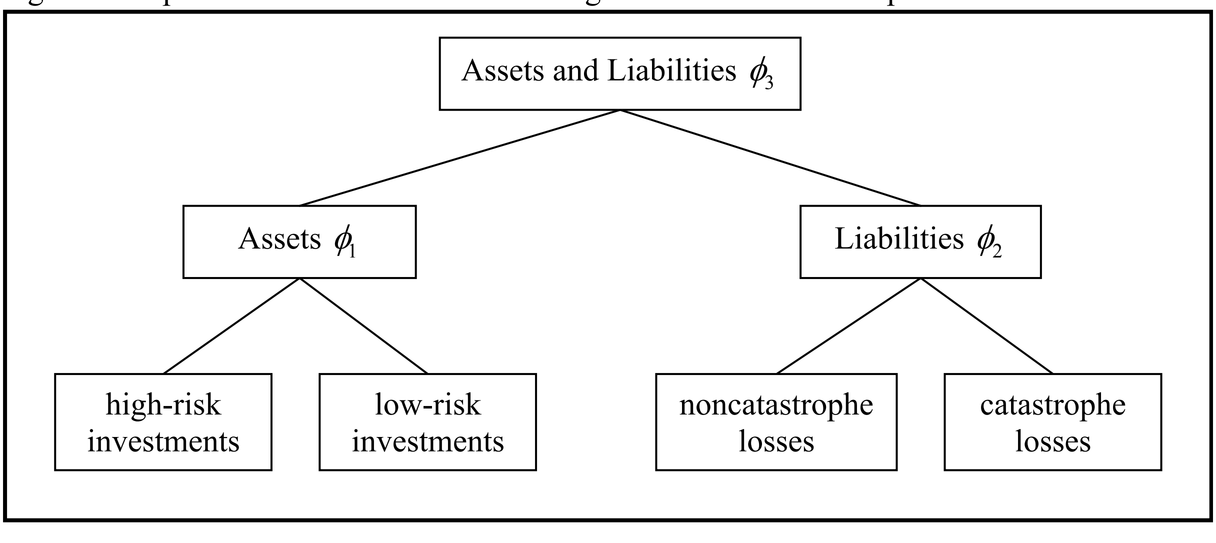
There are other possible four-dimensional nonexchangeable constructions, but this one proves helpful since it results in two exchangeable groups. The first group consists of the high-risk and the low-risk investments and the second group consists of noncatastrophe losses and catastrophe losses. Thus, we are able to calibrate the copulas according to different correlations for assets and liabilities.

Although it would also be possible to combine the copulas shown in Table 1 in the four-dimensional construction, we will concentrate on the same copula for all three generating functions in the construction scheme (Equation (12)) in order to analyze the pure effects of different types of tail dependence. Thus, the generators $\phi_1$, $\phi_2$, and $\phi_3$ differ only in their respective parameter values, which are calibrated using Kendall's rank correlation $\rho_\tau$.

To generate random deviates from the Archimedean copulas, we apply the inverse transform method to the conditional distributions using numerical rootfinding techniques, following the algorithm described in Embrechts/Lindskog/McNeil (2001). An application of this algorithm to a financial market context can be found in Savu/Trede (2006). Nonexchangeable Archimedean copulas are computationally demanding and usually result in clumsy expressions. Therefore, we restrict ourselves to the basic description in Table 1 and refer the reader to Appendix II for the full mathematical expressions.

Nonexchangeable Archimedean copulas following the construction scheme of Equation (12) result in a hierarchical dependence structure that can be represented by a tree diagram, as shown by Figure 1.

Figure 1: Dependence structure of nonexchangeable Archimedean copulas



One of the technical requirements in the construction of nonexchangeable Archimedean copulas results in higher correlations for copulas on a lower level in the hierarchical structure. This technical condition limits the level of correlation at higher hierarchical levels (see Joe, 1997, pp. 89–91). For example, the correlation between assets and liabilities must be smaller than the minimum of correlations of different asset classes and the correlations of different liability classes.

## 4. Measurement of Risk, Return, and Performance in DFA

In the DFA simulation study, we measure risk, return, and performance considering seven financial ratios used in Eling/Parnitzke/Schmeiser (2006). As a measure of return, we consider the expected gain per annum. We denote the expected gain from time 0 to time $T$ as $E\left(EC_T\right)-EC_0$. The expected gain *E(G)* per annum can be written as:

$$E(G)=\frac{E\left(EC_T\right)-EC_0}{T}. \tag{13}$$

We analyze three risk measures: standard deviation, ruin probability, and expected policyholder deficit. The standard deviation of the gain per annum $\sigma(G)$ takes into account both positive and negative deviations from the expected value and thus is a measure of total risk:

$$\sigma(G)=\frac{\sigma(EC_T)}{T}. \tag{14}$$

However, in the insurance context, risk is often measured using downside risk measures such as the ruin probability (*RP*) or the expected policyholder deficit (*EPD*). Downside risk measures differ from total risk measures in that only negative deviations from a certain threshold are taken into account. In this context, the ruin probability can be defined as:

$$RP=Pr\left(\hat{\tau}\leq T\right), \tag{15}$$

where $\hat{\tau}=\inf\left\{t>0;EC_t<0\right\}$ with $t=1,\ 2,\ \ldots,\ T$ describes the first occurrence of ruin (i.e., a negative equity capital).

The ruin probability does not provide any information regarding the severity of insolvency (see Butsic, 1994; Barth, 2000) or the time value of money (see Powers, 1995; Gerber/Shiu, 1998). To take these into account, the expected policyholder deficit (*EPD*) can be considered:

$$EPD=\sum_{t=1}^{T}E\left[max\left(\text{-}EC_t,0\right)\right]\cdot\left(1+r_f\right)^{-t}, \tag{16}$$

where $r_f$ stands for the risk-free rate of return.

Following Eling/Parnitzke/Schmeiser (2006), we consider three performance measures. The Sharpe ratio measures the relationship between the risk premium (mean excess return above the risk-free interest rate) and the standard deviation of returns (see Sharpe, 1966):

$$SR_{\sigma} = \frac{E(EC_T) - EC_0 \cdot (1+r_f)^T}{\sigma(EC_T)}. \tag{17}$$

In the numerator, the risk-free return is subtracted from the expected value of the equity capital in $T$. Using the standard deviation as a measure of risk, the Sharpe ratio also measures positive deviations of the returns in relation to the expected value. Since risk is often calculated by downside measures, either the ruin probability or the *EPD* can be used in the denominator of the Sharpe ratio:

$$SR_{RP} = \frac{E(EC_T) - EC_0 \cdot (1+r_f)^T}{RP}, \tag{18}$$

$$SR_{EPD} = \frac{E(EC_T) - EC_0 \cdot (1+r_f)^T}{EPD}. \tag{19}$$

$SR_{RP}$ denotes the Sharpe ratio based on ruin probability. $SR_{EPD}$ is the Sharpe ratio based on expected policyholder deficit.

## 5. Measuring the Influence of Copulas in DFA

### 5.1. Model Specifications

In the simulation study, we consider a typical German non-life insurance company, using corresponding data and German solvency rules. We consider a time period of $T$ = 5 years. The market volume *MV* (i.e., $\beta = 1$) of the underwriting market accessible to the insurance company is €1,000 million. In $t$ = 0, the insurer has a share of $\beta_0$ = 0.2 in the insurance market, so that the premium income for the insurer at a premium rate level equal to 1 in the underwriting cycle is €200 million. Applying the empirical findings of Cummins/Outreville (1987), we parameterize the underwriting cycle (see Equation (5)) according to the German all-lines underwriting profit ratios given by $\varphi_0 = 1.191$, $\varphi_1 = 0.879$, and $\varphi_2 = -0.406$. This leads to an underwriting cycle length of $2\pi / \arccos(\varphi_1 / 2\sqrt{-\varphi_2}) = 7.76$ years (see Cummins/Outreville, 1987).

The expenses for the premiums written are $Ex_{t-1}^P = 0.05 \cdot \beta_{t-1} \cdot MV + 0.001 \cdot ((\beta_{t-1} - \beta_{t-2}) \cdot MV)^2$. The tax rate is $tr = 0.25$. The consumer response parameter *cr* is 1 (0.95) if the equity capital at the end of the last period is above (below) the company's safety level. The company's safety level is determined by the minimum capital required (*MCR*), which is based on the Solvency I rules currently used in Germany. The Solvency I minimum capital is calculated as:

$$MCR_t = \max\begin{pmatrix} 0.18 \cdot (\min(P_{t-1}; €50 \text{ million}) + 0.16 \cdot (\max(P_{t-1} - €50 \text{ million}; 0)); \\ 0.26 \cdot (\min(C_t; €35 \text{ million}) + 0.23 \cdot (\max(C_t - €35 \text{ million}; 0) \end{pmatrix} \tag{20}$$

(see § 53c of the German Insurance Supervision Act (VAG)). Using these rules, we calculate a minimum capital requirement of €40.15 million in $t$ = 1 (max(0.18·€50 million + 0.16·€150 million; 0.26·€35 million + 0.23·€135 million)). To comply with these rules, the insurance company is capitalized with €75 million in $t$ = 0. This corresponds to an equity to premium ratio of 37.5%, which is typical for German non-life insurance companies (see BaFin, 2005, Table 520).

Asset returns are normally distributed. The continuous rate of return of the high- (low-) risk investment has a mean of 10% (5%) and a standard deviation of 20% (5%). To calculate the asset allocation, we use data from the German regulatory authority (BaFin). German non-life insurance companies typically invest approximately 40% of their wealth in high-risk investments; the remaining 60% is invested in low-risk investments (see BaFin, 2005, Table 510). We thus fix $\alpha_0 = 0.40$. The risk-free return $r_f$ is 3%.

For the underwriting business, noncatastrophe losses are log-normally distributed, with a mean of $0.85 \cdot \beta_{t-1} \cdot MV$ and a standard deviation of $0.085 \cdot \beta_{t-1} \cdot MV$ (see BaFin, 2005, Table 541). The catastrophe claims are modeled using a Pareto distribution for the overall claim amount with a mean parameter of 0.5 and a dispersion parameter of 4.5. The claim settlement expenses are 5% of the claims ($Ex_t^C = 0.05 \cdot C_t$; see BaFin, 2005, Table 541).

We use random numbers with the following correlation structure. Kendall's rank correlation between high-risk and low-risk investments is 0.2. The correlation between catastrophe losses and noncatastrophe losses is 0.2, and between assets and liabilities Kendall's rank correlation is –0.1. There is no clear empirical evidence concerning these correlation values (see Lambert/Hofflander, 1966, Haugen, 1971, Kahane/Nye, 1975, and Li/Huang, 1996); however, in the robustness test we will present results for alternative parameter settings.

All model parameters, their meanings, and their initial values are summarized in Appendix I.

## 5.2. Simulation Results

Table 2 sets out the simulation results for six different dependence structures. All results have been calculated on the basis of a Monte Carlo simulation with 500,000 iterations (for details on Monte Carlo simulation, see, e.g., Glassermann, 2004).

Table 2: Results for the input parameters given in Table A1

| Dependence structure | No corr. | Gauss | *t* | Gumbel | Clayton | Frank |
|---|---|---|---|---|---|---|
| Tail dependence | none | none | upper and lower | upper | lower | none |
| E(G) in million € | 30.81 | 30.39 | 30.39 | 30.47 | 30.10 | 30.45 |
| σ(G) in million € | 14.33 | 16.69 | 16.77 | 17.56 | 19.02 | 16.45 |
| RP | 0.07% | 0.36% | 0.58% | 0.24% | 0.87% | 0.25% |
| EPD in million € | 0.01 | 0.04 | 0.08 | 0.85 | 1.78 | 0.03 |
| $SR_\sigma$ | 1.98 | 1.68 | 1.67 | 1.60 | 1.46 | 1.71 |
| $SR_{RP}$ | 210.87 | 39.04 | 24.18 | 57.59 | 15.95 | 56.57 |
| $SR_{EPD}$ | 22.53 | 3.67 | 1.81 | 0.17 | 0.08 | 4.12 |

E(G): expected gain per annum, σ(G): standard deviation of the gain per annum, RP: ruin probability, EPD: expected policyholder deficit, $SR_\sigma$: Sharpe ratio based on standard deviation, $SR_{RP}$: Sharpe ratio based on ruin probability, $SR_{EPD}$: Sharpe ratio based on expected policyholder deficit.

In the case without correlations, we find an expected gain of €30.81 million per annum with a standard deviation of €14.33 million. The ruin probability amounts to 0.07%, which is far below the requirements of many regulatory authorities, e.g., the Solvency II framework planned in the European Union requires a ruin probability below 0.50% (see European Commission, 2005).

Comparing the case without correlations and the Gauss copula, we observe minor effects on the mean returns. *E(G)* is reduced about 1.37%, from 30.81 to 30.39. However, we find much larger changes in risk. *σ(G)* rises from 14.33 to 16.69 (+16.46%) and *RP* from 0.07% to 0.36% (+432.05%). Obviously, the extreme changes in risk are especially due to the lower partial moments, as the increase in ruin probability (the measure for downside risk) is 26 times higher than the increase in standard deviation (the measure for total risk). Therefore, the performance is much lower than in the case without correlations: $SR_{\sigma}$ is reduced about 15.43% and $SR_{RP}$ by about 81.49%.

We find large differences when comparing the copulas. Looking at the results of the *t*, the Gumbel, the Clayton, and the Frank copulas, we again observe only minor effects on the mean returns and extreme effects on the risk. However, the change in risk depends on the form of nonlinear dependence. Looking at copulas with upper or no tail dependence (the Gumbel and Frank copulas), the ruin probability is lower than with the Gauss copula, whereas these values are much higher with lower tail dependent copulas (the *t* and Clayton copulas). It is also noteworthy that in this example the ruin probability for the *t* (0.58%) and the Clayton copulas (0.87%) is above most regulatory requirements (e.g., 0.50% in Solvency II).

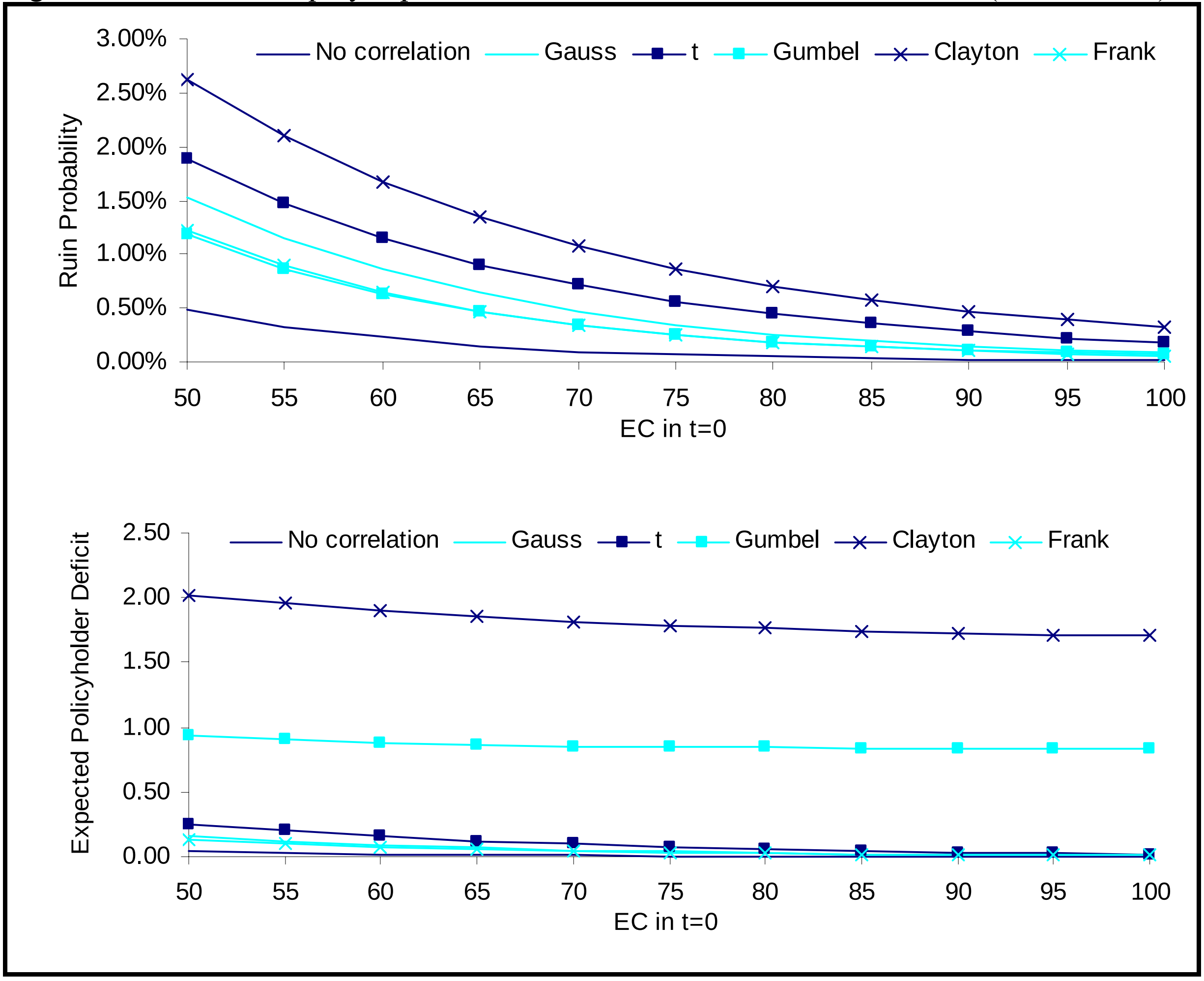
The large impact of nonlinear dependencies is illustrated by the results for ruin probability and expected policyholder deficit. The Gumbel copula has a smaller *RP* compared to the Gauss and the *t* copulas, but the *EPD* is higher; comparing the performance measures based on downside risk, we find that the Gumbel copula has a higher $SR_{RP}$ than the Gauss and the *t* copulas, but a lower $SR_{EPD}$. These differences can be explained by the extreme levels of the higher moments of the return distribution (skewness, kurtosis) with the Gumbel copula, as the *EPD* is more sensitive to higher moments than is the ruin probability. The results illustrate the importance of modeling nonlinear dependencies in DFA, as the integration of these features has extreme effects on the risk and performance of the insurance company. The results found with the *RP* and the *EPD* might be of special relevance for regulators and rating agencies, because, depending on the copula concept employed, we find large differences in risk assessment for different risk measures. It thus appears important to integrate nonlinear dependencies in the rating framework, e.g., in stress testing and scenario analysis.

### 5.3. ROBUSTNESS OF FINDINGS

We check the robustness of our findings by varying the level of equity capital and the correlation settings. The results can be considered robust if the results presented in the last section and the basic relations between the analyzed copulas are independent of the given input parameters setting.

In the first step, we vary the level of equity capital in $t = 0$, which determines the company's safety level, leaving everything else constant. In Section 5.2, the level of equity capital was set at €75 million. To test the implications of different levels of equity capital, we vary the equity capital in $t = 0$ from €50 to €100 million in €5-million intervals. The results are shown in Figure 2, where the ruin probability and the expected policyholder deficit are displayed as a function of the equity capital.

Figure 2: Variation of equity capital in $t = 0$ between €50 mill. and €100 mill. (*RP* and *EPD*)

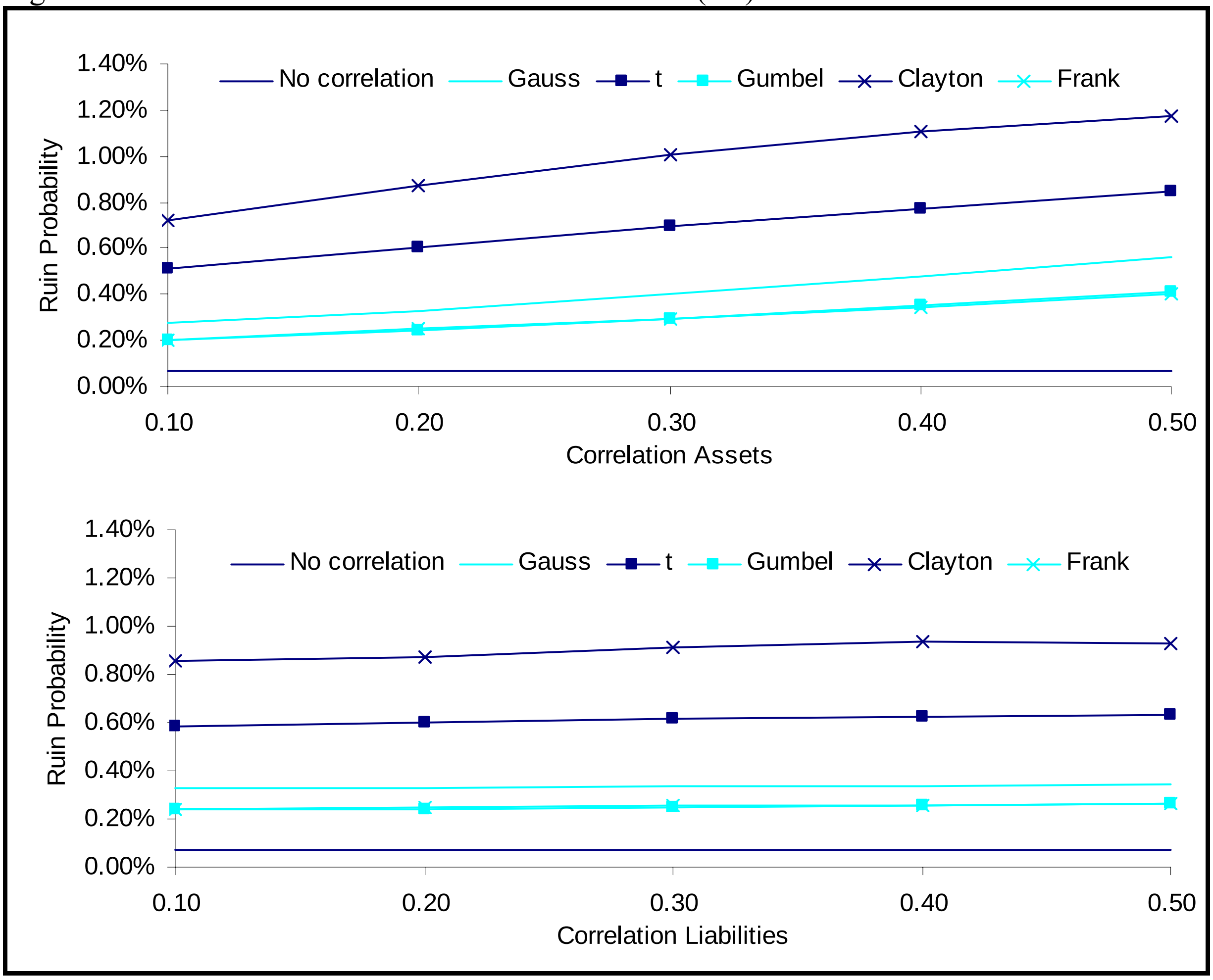



As the level of equity capital increases, the ruin probability and the expected policyholder deficit decrease because the company's safety level is improved. The ruin probability is greatly reduced with all copulas. However, the relative difference between the copulas increases with an increasing level of equity capital. For example, with $EC_0 = 50$, the ruin probability of the Clayton copula (*RP* = 2.62%) is five times higher than the ruin probability without correlation (*RP* = 0.49%), but with $EC_0 = 100$, the ruin probability of the Clayton copula (*RP* = 0.316%) is 26 times higher than in the case without correlation (*RP* = 0.012%). The fact that the influence of nonlinear dependencies increases with an increasing level of equity capital is an important result because it indicates that copulas are relevant not only for low-capitalized companies but also for well-capitalized companies.

Looking at the expected policyholder deficit, we find a relatively small risk reduction with the Gumbel and the Clayton copulas, where the *EPD* does not decrease as fast as with the other copulas. This is because the Gumbel and the Clayton copulas are not symmetric. The results of the simulation indicate that the risks generated by tail dependencies are not much reduced by an increasing level of equity capital. Therefore, it again seems that copulas are important for well-capitalized companies. This result is also relevant for policyholders and regulators because the expected policyholder deficit is more important for the policyholders than for the equityholders (see Bingham, 2000), given that policyholders have to bear the amount of loss, while the shareholders have a limited downside risk.

The basic correlation setting in our simulation study is relatively low. Kendall's rank correlation between high-risk and low-risk investments is 0.2, between catastrophe losses and noncatastrophe losses 0.2, and between assets and liabilities –0.1. To test the implications of different correlation assumptions, we vary the correlation between the high-risk and low-risk investments from 0.1 to 0.5 in 0.1 intervals (upper part of Figure 3) and between catastrophe losses and noncatastrophe losses also from 0.1 to 0.5 in 0.1 intervals (lower part of Figure 3). The resulting ruin probability is shown in Figure 3.

Figure 3: Variation of correlation between 0.1 and 0.5 (*RP*)




We find that ruin probability increases with an increasing correlation between the assets. This occurs because the higher the correlation, the higher the likelihood that negative outcomes are generated for both types of assets (i.e., low returns with the high- and the low-risk investments). The increase in ruin probability is much smaller when the correlation between the liabilities is varied. The reason for this is that with the given parametrization, the underwriting business is more profitable than the insurer's investments on the capital market. Comparing the copulas, we find that the basic relations remain unchanged and thus the main results of the last section appear to be very robust.

The expected policyholder deficit yields similar results. Figure A1 in Appendix III shows the results for the expected policyholder deficit.

## 6. Measuring the Influence of Management Strategies
### 6.1. Model Specifications

In this section, we investigate whether management can effectively influence the risk introduced by tail dependencies. To this end, in our model, decisions concerning the portion of risky investments ($\alpha$) and the market share in the underwriting business ($\beta$) can be made at the beginning of each year. We first consider two heuristic strategies introduced by Eling/Parnitzke/Schmeiser (2006): the solvency strategy and the growth strategy.

The solvency strategy is aimed at risk reduction. For each point in time ($t$ = 1,…, $T$-1), we decrease the portion of risky investments $\alpha$ by 0.05 and the insurance market share $\beta$ by 0.02 as soon as the equity capital falls below the critical value defined by the minimum capital required (*MCR*) plus a safety loading of 50%. The growth strategy combines the solvency strategy with a growth target for the underwriting business. Should the equity capital drop below the minimum capital required (*MCR*), including a safety loading of 50%, the same rules apply as in the solvency strategy. If the equity capital is above the trigger, there is a growth of 0.02 in $\beta$.

In addition to the strategies used in Eling/Parnitzke/Schmeiser (2006), we consider a reinsurance strategy, which might be of special benefit in the context of extreme events. Under this strategy, the insurer signs a stop-loss reinsurance contract on its whole book of business with an attachment point of €200 million, a limit of €40 million, and a premium of €2 million at the beginning of each year. The payment from the reinsurer at the end of each year can thus be calculated by min (max ($C_t$-200,0);40).

### 6.2. Simulation Results

The simulation results for the solvency, growth, and reinsurance strategies are presented in Table 3.

When comparing the results of Table 3 with those of Table 2, we see that downside risk is much reduced under the solvency strategy. Although the return remains almost unchanged, we find much lower values for the downside risk measures. Thus, the solvency strategy reduces the ruin probability without having much effect on the return. However, risk is not as much reduced when nonlinear dependencies are taken into account. The solvency strategy is thus not effective in reducing downside risk in the case of nonlinear dependencies.

Under the growth strategy, we obtain a completely different risk and return profile—a higher return is accompanied by higher risk. Again, the level of return is not affected by the integration of nonlinear dependencies, but large differences are found for downside risk measures. Risk is much increased with all copula models. Therefore, the performance numbers for the growth strategy are mostly lower than those in the situation where no management strategy is applied.

Table 3: Results for the solvency, growth, and reinsurance strategies

| Dependence structure | No corr. | Gauss | t | Gumbel | Clayton | Frank |
|---|---|---|---|---|---|---|
| Tail dependence | no | no | upper and lower | upper | lower | no |
| Solvency strategy | | | | | | |
| E(G) in million € | 30.63 | 30.12 | 30.14 | 30.22 | 29.83 | 30.19 |
| σ(G) in million € | 14.44 | 16.81 | 16.90 | 17.67 | 19.21 | 16.58 |
| RP | 0.06% | 0.31% | 0.52% | 0.20% | 0.78% | 0.20% |
| EPD in million € | 0.00 | 0.03 | 0.07 | 0.84 | 1.77 | 0.03 |
| $SR_{\sigma}$ | 1.96 | 1.65 | 1.64 | 1.58 | 1.43 | 1.68 |
| $SR_{RP}$ | 255.82 | 45.46 | 26.87 | 68.82 | 17.53 | 68.14 |
| $SR_{EPD}$ | 30.66 | 4.70 | 2.08 | 0.16 | 0.08 | 5.02 |
| Growth strategy | | | | | | |
| E(G) in million € | 35.05 | 34.38 | 34.42 | 34.52 | 34.04 | 34.47 |
| σ(G) in million € | 16.64 | 19.37 | 19.47 | 20.12 | 21.63 | 19.10 |
| RP | 0.09% | 0.45% | 0.71% | 0.31% | 1.04% | 0.30% |
| EPD in million € | 0.01 | 0.05 | 0.10 | 0.86 | 1.84 | 0.04 |
| $SR_{\sigma}$ | 1.96 | 1.65 | 1.64 | 1.60 | 1.46 | 1.68 |
| $SR_{RP}$ | 183.50 | 35.76 | 22.70 | 52.46 | 15.20 | 53.01 |
| $SR_{EPD}$ | 19.31 | 3.25 | 1.56 | 0.19 | 0.09 | 3.96 |
| Reinsurance strategy | | | | | | |
| E(G) in million € | 29.51 | 29.12 | 29.14 | 29.18 | 28.89 | 29.16 |
| σ(G) in million € | 14.02 | 16.28 | 16.34 | 17.21 | 18.50 | 16.09 |
| RP | 0.03% | 0.22% | 0.32% | 0.16% | 0.53% | 0.16% |
| EPD in million € | 0.00 | 0.02 | 0.03 | 0.83 | 1.70 | 0.02 |
| $SR_{\sigma}$ | 1.93 | 1.64 | 1.64 | 1.56 | 1.43 | 1.66 |
| $SR_{RP}$ | 431.83 | 60.20 | 42.19 | 84.57 | 25.14 | 82.83 |
| $SR_{EPD}$ | 55.15 | 6.71 | 4.36 | 0.16 | 0.08 | 7.91 |

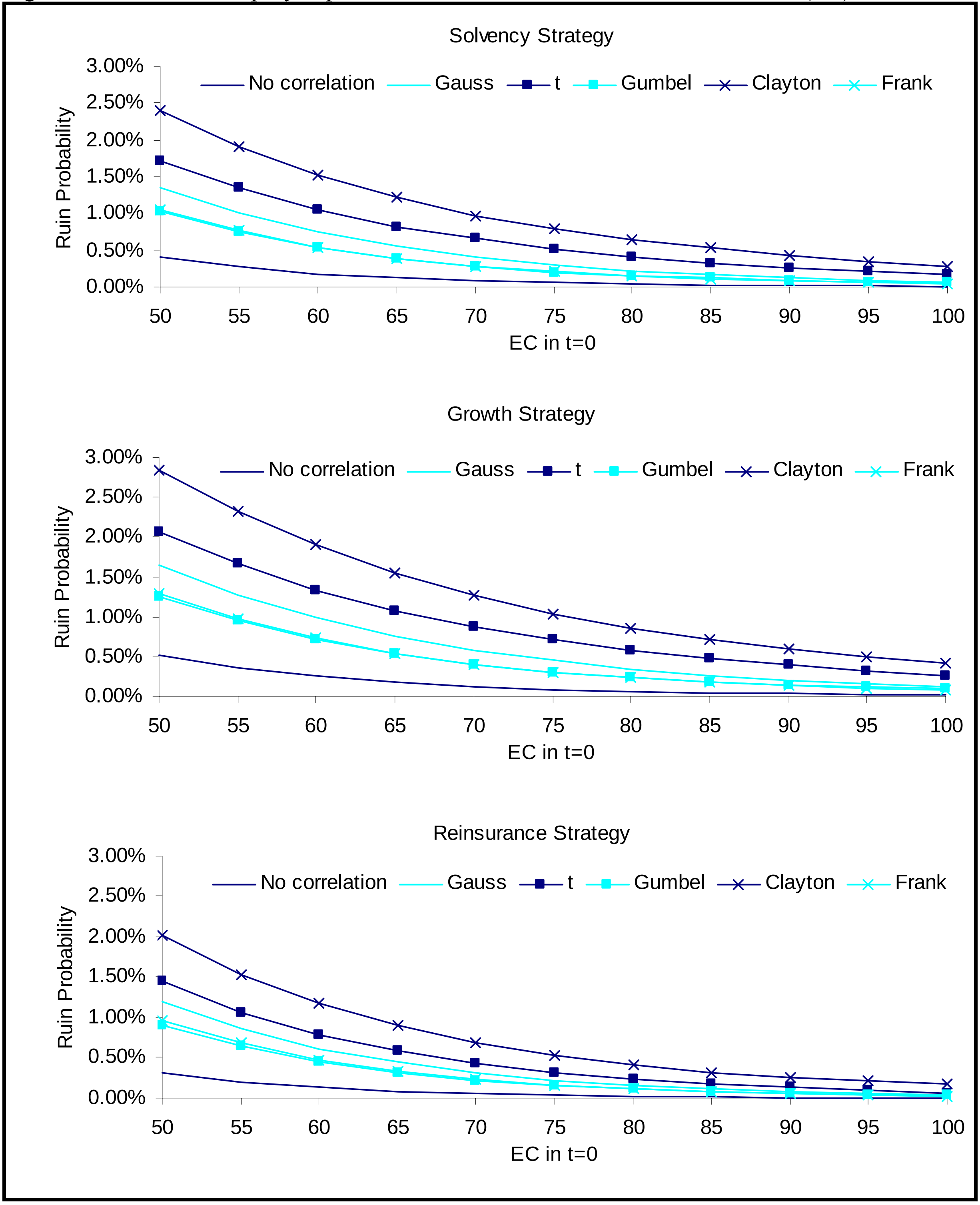
E(G): expected gain per annum, σ(G): standard deviation of the gain per annum, RP: ruin probability, EPD: expected policyholder deficit, $SR_{\sigma}$: Sharpe ratio based on standard deviation, $SR_{RP}$: Sharpe ratio based on ruin probability, $SR_{EPD}$: Sharpe ratio based on expected policyholder deficit.

In contrast to the other two strategies, the reinsurance strategy leads to a lower return, but we again find large differences for the downside risk measures. By buying reinsurance, the ruin probability is in almost all cases kept within the regulatory limits suggested by the Solvency II framework (the maximum ruin probability is 0.53% for the Clayton copula). It thus seems that reinsurance is an efficient method to limit the risks generated by nonlinear dependencies. However, this is again only true from the equityholders' perspective because we find that the *EPD* is very little reduced by purchasing reinsurance.

### 6.3. Robustness of Findings

We checked the robustness of our findings using the tests described in Section 5.3. Figure 4 shows the ruin probability for different levels of equity capital under the solvency strategy (upper part of the figure), the growth strategy (middle part of the figure), and the reinsurance strategy (lower part of the figure).

Figure 4: Variation of equity capital in $t = 0$ between €50 mill. and €100 mill. (*RP*)



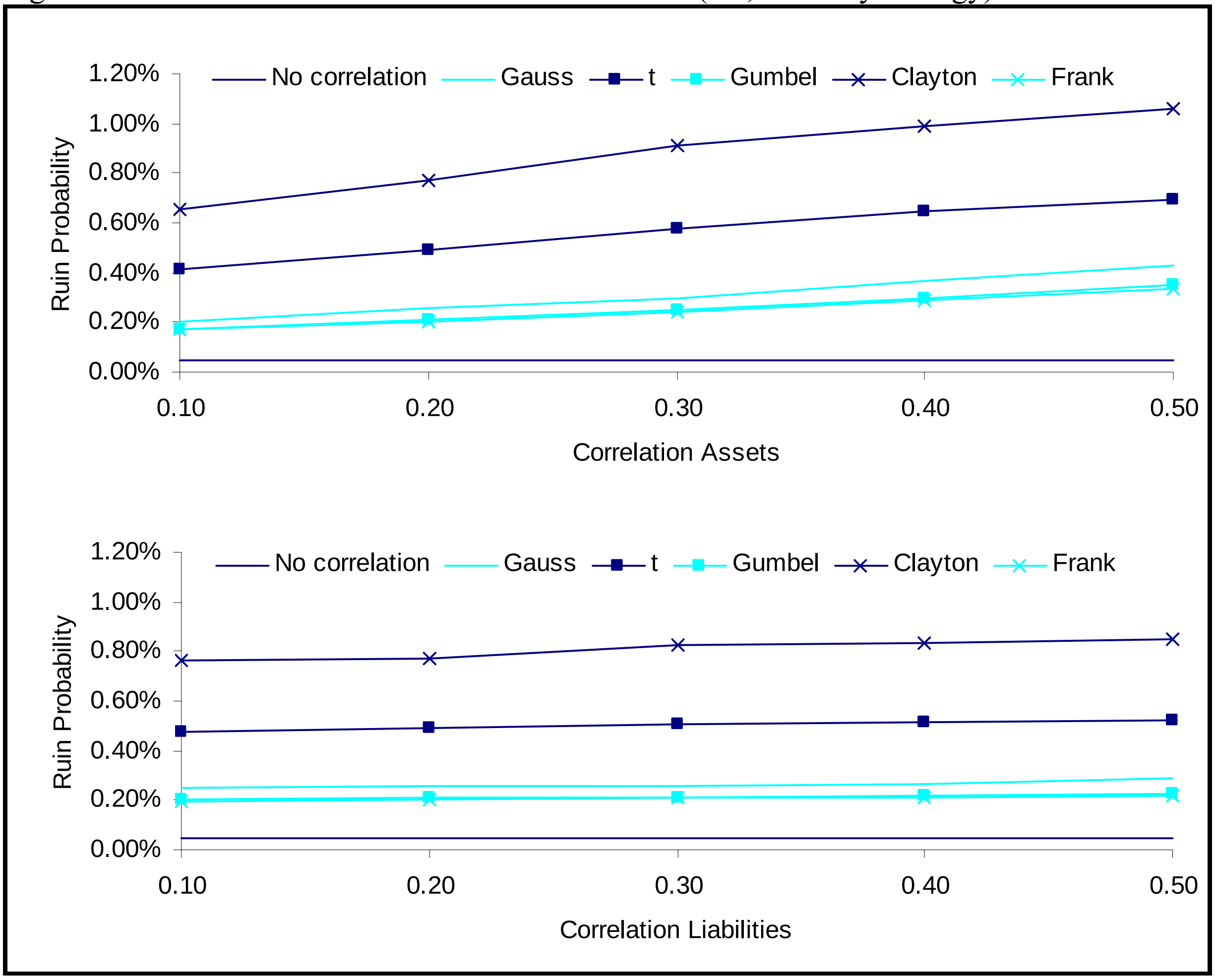

The results displayed in Figure 4 are comparable to the results presented in Figure 2. As equity capital increases, ruin probability decreases, for both the solvency strategy and the growth strategy. The only difference is the base level of the ruin probability. Both strategies cannot influence the relative difference between the copulas described in Section 5.2, i.e., the fact that the relative difference between the copulas increases with the level of equity capital.

However, overall these results indicate that the main conclusions presented in Section 6.2 are very robust.

The expected policyholder deficit yields the same conclusions as in Section 5.3; the results are displayed in Figure A2 in Appendix III.

We also investigated the implications of different correlation assumptions and varied the correlation between the assets and between the liabilities from 0.1 to 0.5 in 0.1 intervals (all else being constant). Again, the results remain robust. The ruin probability for the solvency strategy is shown in Figure 5; in the upper part of the figure the correlation between the assets is varied, in the lower part of the figure the correlation between the liabilities.

Figure 5: Variation of correlation between 0.1 and 0.5 (*RP*, solvency strategy)




The ruin probability of the growth and reinsurance strategies and the expected policyholder deficit for all three management strategies are displayed in Appendix III (see Figures A3–A7). As before, all results are very robust.

## 7. Conclusion

We study the influence of nonlinear dependencies on a non-life insurer's risk and return profile by integrating several copula models in a DFA framework. Nonlinear dependencies are especially relevant in the case of extreme events that might induce tail dependencies between different asset classes, different kinds of liabilities, or between assets and liabilities. One example of such an extreme event are the terrorist attacks of September 11, 2001, which resulted in insurers experiencing large losses from the underwriting business and on the capital markets.

We find that such events are especially relevant for policyholders and regulators because nonlinear dependencies do not affect the return level but, instead, the ruin probability and the expected policyholder deficit. Depending on the copula, the ruin probability increases up to a factor of 10 in our simulation study compared to a situation without dependencies. We observe the highest levels of risk in the case of lower tail dependent copulas such as the *t* and the Clayton copulas. Another key result is that while in general the ruin probability decreases when equity capital increases, there are nonlinear dependencies where the expected policyholder deficit cannot be reduced by increasing equity capital. Moreover, the fact that the influence of nonlinear dependencies increases with an increasing level of equity capital indicates that copulas are relevant not only for low-capitalized companies but also for well-capitalized companies.

We also check the effectiveness of certain management strategies used in response to adverse outcomes generated by nonlinear dependencies, but we find that the risk profile cannot be affected by simple risk reduction strategies. A reinsurance strategy can delimit the ruin probability, but not the expected policyholder deficit. In our simulation study, the reinsurance strategy thus proves to be an useful instrument for securing the position of equityholders, but not necessarily for policyholders.

All these results highlight the importance of considering nonlinear dependencies, especially for regulators and rating agencies. Depending on the copula concept employed, we find large differences in the risk assessment of policyholder deficit and ruin probability. As these measures are the foundation of capital standards and ratings, it is important to consider nonlinear dependencies in the regulatory framework and in rating assessment, e.g., in stress testing and scenario analysis.

However, we cannot say which one of the copulas describes reality best. Due to the rare occurrence of extreme events, there is a lack of data, which makes most kinds of empirical studies impossible. For that reason, we evaluated different forms of copulas in a stress-testing sense. Nevertheless, it would be important to have more empirical studies on the correlations between different asset classes, different liabilities, and between assets and liabilities, not only to calibrate the DFA model but also to provide further empirical insights on the nature of the extreme events.

## APPENDIX I: BASE PARAMETER CONFIGURATION

Table A1: Base parameter configuration

| Parameter | Symbol | Initial value at $t$ = 0 |
|---|---|---|
| Time period in years | $T$ | 5 |
| Equity capital at the end of period $t$ | $EC_t$ | €75 million |
| Tax rate | $tr$ | 0.25 |
| Portion invested in high-risk investments in period $t$ | $\alpha_{t-1}$ | 0.40 |
| Normal high-risk investment return in period $t$ | $r_{1t}$ | |
| Mean return | $E(r_{1t})$ | 0.10 |
| Standard deviation of return | $\sigma(r_{1t})$ | 0.20 |
| Normal low-risk investment return in period $t$ | $r_{2t}$ | |
| Mean return | $E(r_{2t})$ | 0.05 |
| Standard deviation of return | $\sigma(r_{2t})$ | 0.05 |
| Risk-free return | $r_f$ | 0.03 |
| Underwriting market volume | $MV$ | €1,000 million |
| Company's underwriting market share in period $t$ | $\beta_{t-1}$ | 0.20 |
| Premium rate level in period $t$ | $\Pi_t$ | 1 |
| Autoregressive process parameter for lag 0 | $\varphi_0$ | 1.191 |
| Autoregressive process parameter for lag 1 | $\varphi_1$ | 0.879 |
| Autoregressive process parameter for lag 2 | $\varphi_2$ | -0.406 |
| Consumer response function | $cr_{t-1}^{EC_{t-1}}$ | 1 |
| Upfront expenses linearly depending on the written market volume | $\gamma$ | 0.05 |
| Upfront exp. nonlinearly depending on the change in written market vol. | $\eta$ | 0.001 |
| Log-normal noncatastrophe claims as portion underwriting market share | $C_{ncat}$ | |
| Mean claims | $E(C_{ncat})$ | €170 million |
| Standard deviation of claims | $\sigma(C_{ncat})$ | €17 million |
| Claim settlement costs as portion of claims | $\delta$ | 0.05 |
| Pareto catastrophe claims | $C_{cat}$ | |
| Mean claims | $E(C_{cat})$ | €0.5 million |
| Dispersion parameter | $D(C_{cat})$ | 4.5 |
| Kendall's rank correlation between high-risk and low-risk investments | $\rho_{\tau 1}$ | 0.2 |
| Kendall's rank correlation between noncatastrophe losses and cat. losses | $\rho_{\tau 2}$ | 0.2 |
| Kendall's rank correlation between assets and liabilities | $\rho_{\tau 3}$ | -0.1 |

## Appendix II: Full Mathematical Expressions for the Nonexchangeable Archimedean Copulas

Gumbel copula:

$$C^{Gumbel}(u_1,u_2,u_3,u_4) = \exp(-[(-\ln(\exp(-[(-\ln u_1)^{\theta_1} + (-\ln u_2)^{\theta_1}]^{\frac{1}{\theta_1}}))^{\theta_3} + (-\ln(\exp(-[(-\ln u_3)^{\theta_2} + (-\ln u_4)^{\theta_2}]^{\frac{1}{\theta_2}}))^{\theta_3}]^{\frac{1}{\theta_3}}) \tag{A1}$$

Clayton copula:

$$C^{Clayton}(u_1,u_2,u_3,u_4) = ([(u_1^{-\theta_1} + u_2^{-\theta_1} - 1)^{-\frac{1}{\theta_1}}]^{-\theta_3} + [(u_3^{-\theta_2} + u_4^{-\theta_2} - 1)^{-\frac{1}{\theta_2}}]^{-\theta_3} - 1)^{-\frac{1}{\theta_3}} \tag{A2}$$

Frank copula:

$$C^{Frank}(u_1,u_2,u_3,u_4) = -\frac{\ln[\frac{([\frac{(\exp(-\theta_1 u_1)-1)(\exp(-\theta_1 u_2)-1)}{\exp(-\theta_1)-1}+1]^{\frac{\theta_3}{\theta_1}}-1)([\frac{(\exp(-\theta_2 u_3)-1)(\exp(-\theta_2 u_4)-1)}{\exp(-\theta_2)-1}+1]^{\frac{\theta_3}{\theta_2}}-1)}{\exp(-\theta_3)-1}+1]}{\theta_3} \tag{A3}$$

## Appendix III: Robustness of Findings

Figure A1: Variation of correlation between 0.1 and 0.5 (*EPD*)

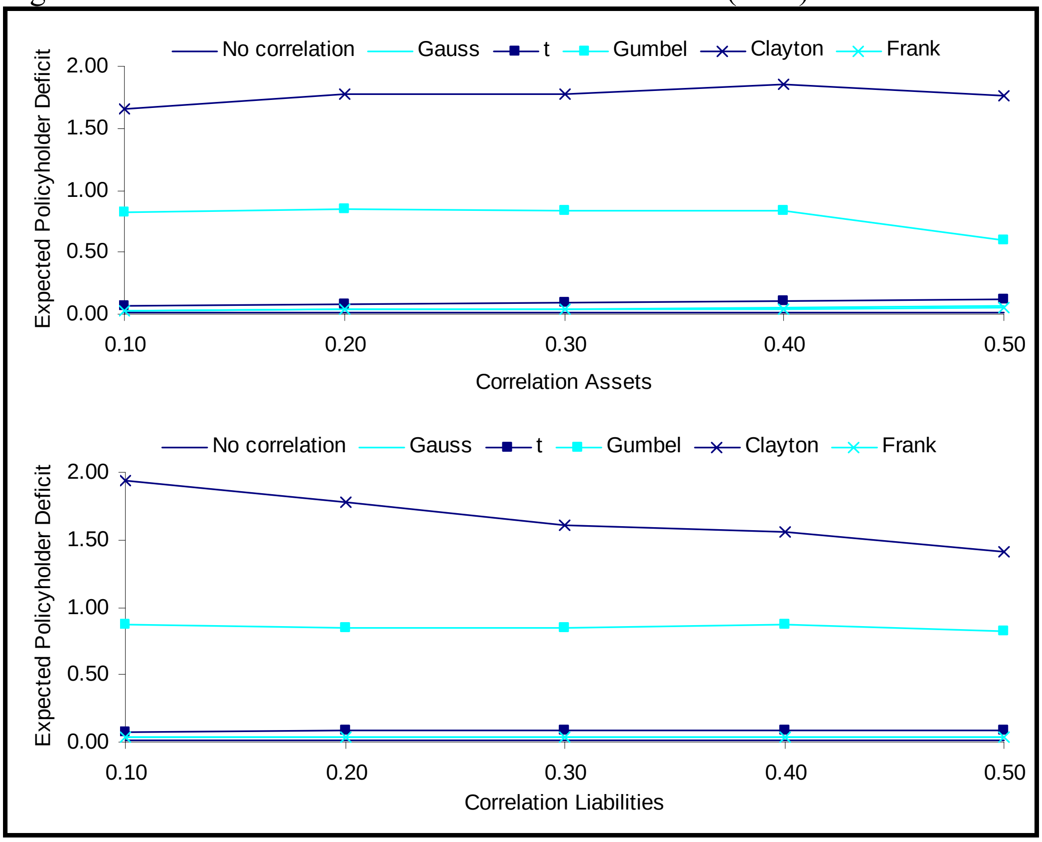


Figure A2: Variation of equity capital in $t$= 0 between €50 mill. and €100 mill. (*EPD*)

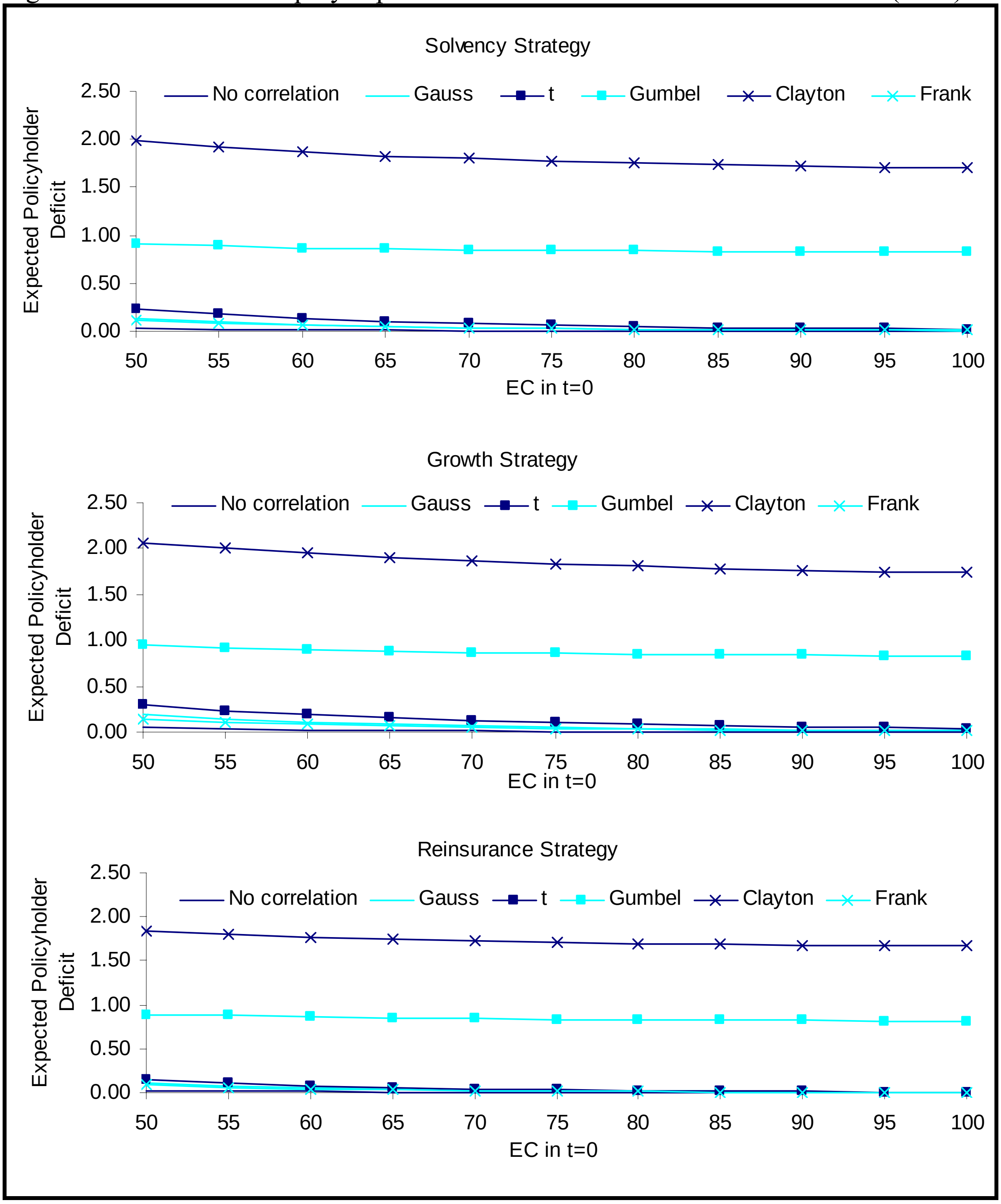


Figure A3: Variation of correlation between 0.1 and 0.5 (*RP*, growth strategy)

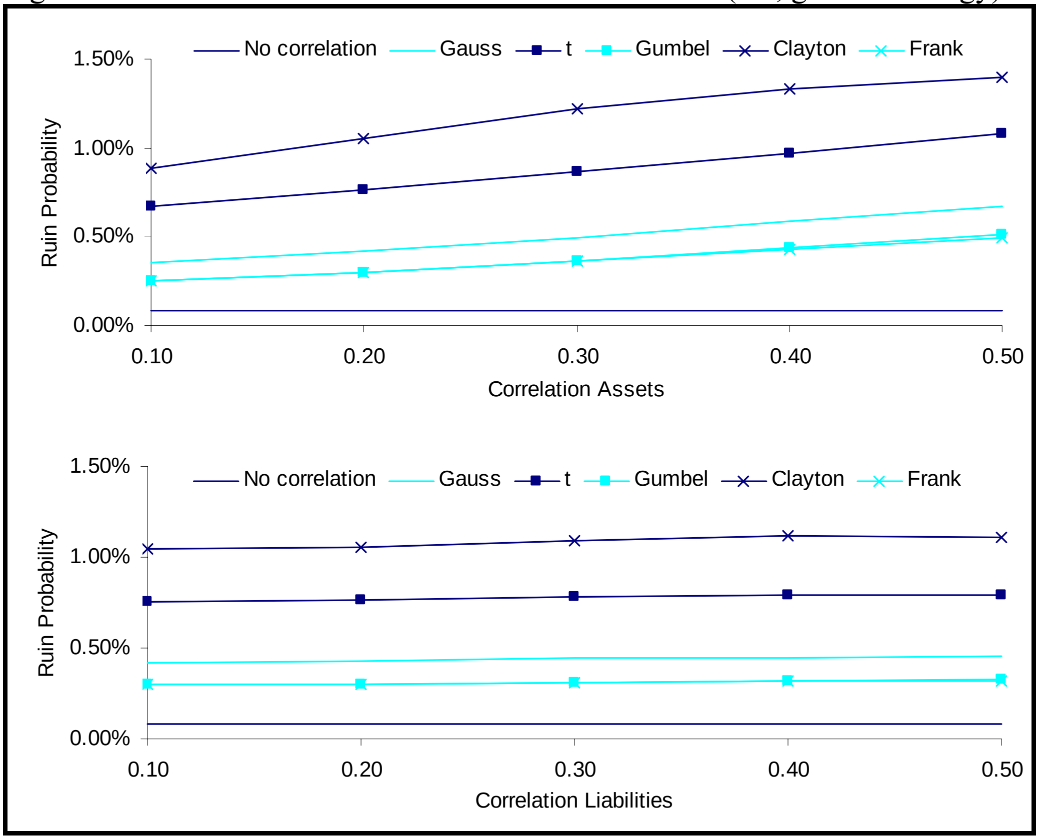



Figure A4: Variation of correlation between 0.1 and 0.5 (*RP*, reinsur. strategy)

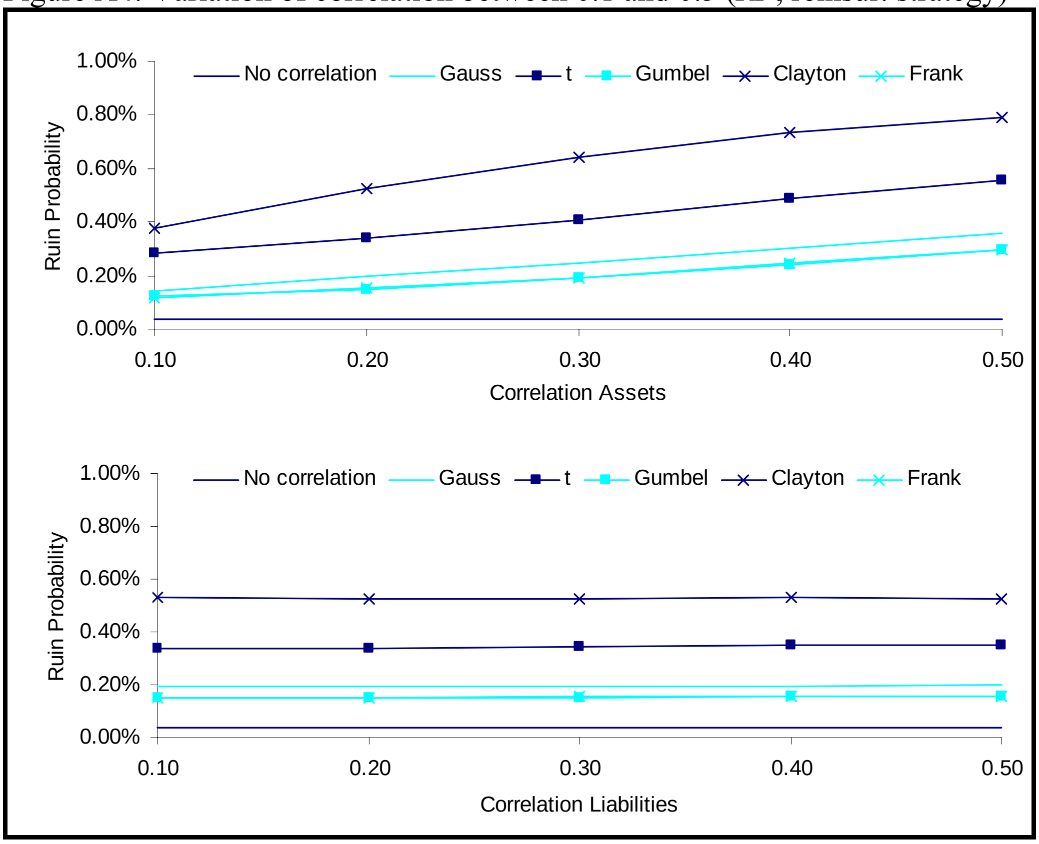


Figure A5: Variation of correlation between 0.1 and 0.5 (*EPD*, solvency strategy)

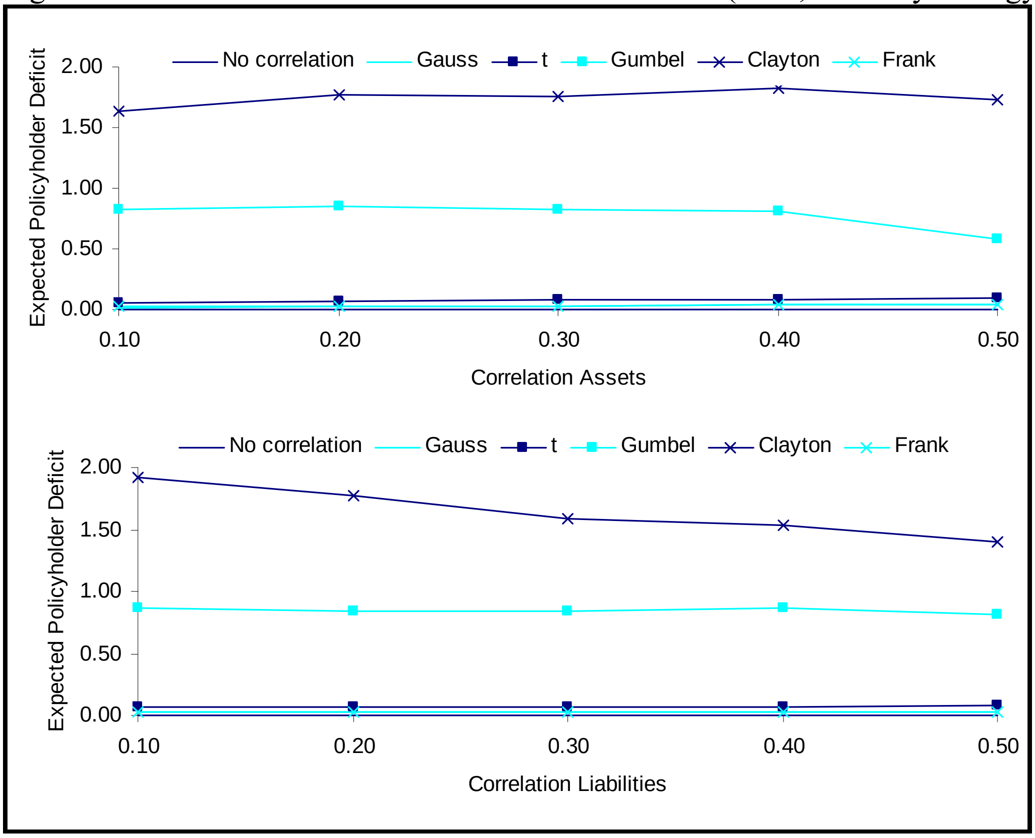



Figure A6: Variation of correlation between 0.1 and 0.5 (*EPD*, growth strategy)

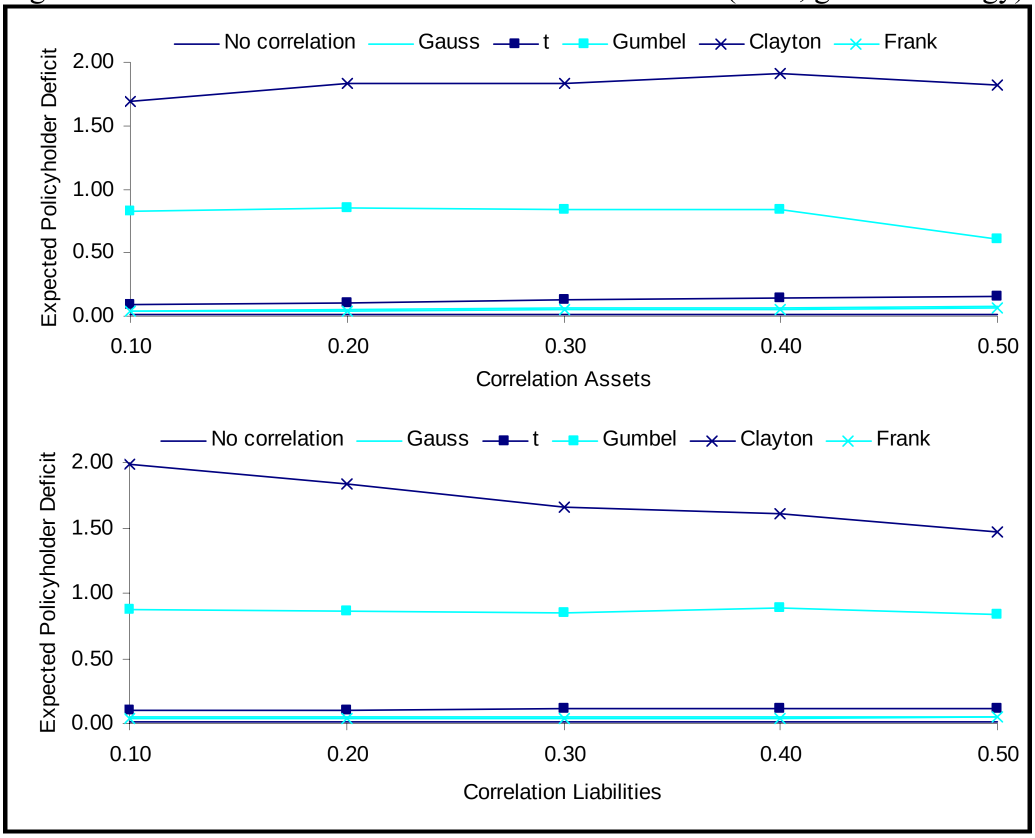
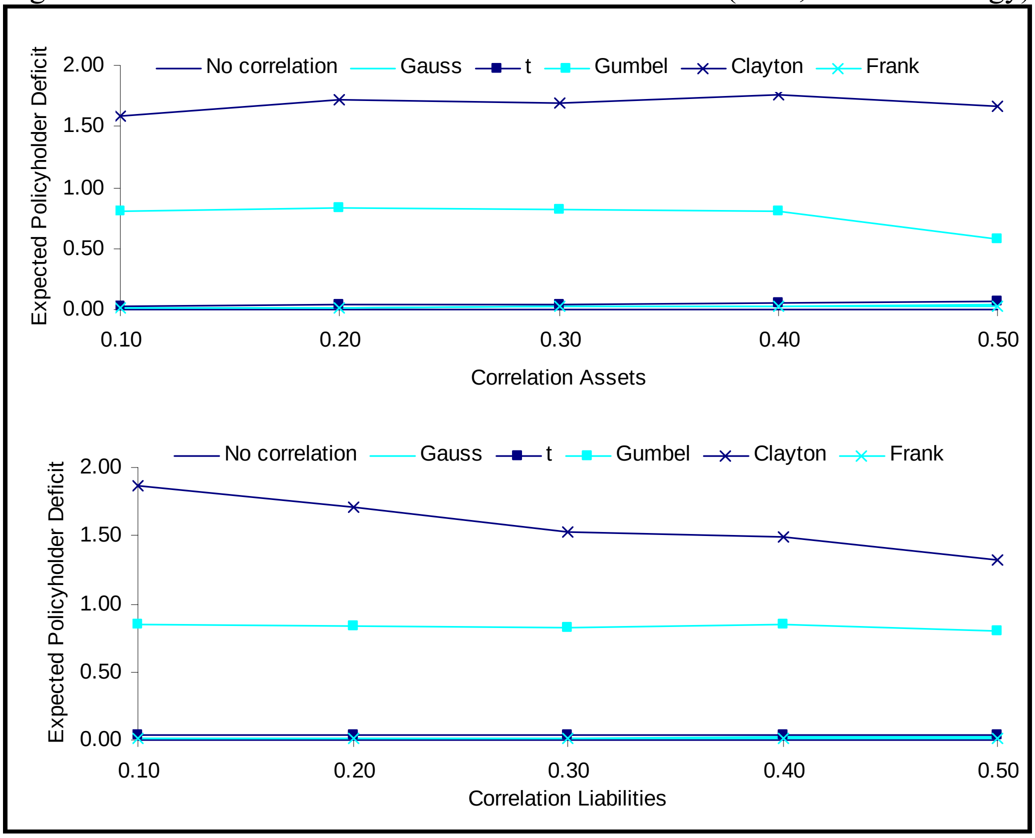


Figure A7: Variation of correlation between 0.1 and 0.5 (*EPD*, reinsur. strategy)



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