Risk-adjusted Performance Indicators in Life Insurance*

Authors:

Rosa Cocozza, Università di Napoli "Federico II", Dipartimento di Economia Aziendale, Napoli, Italy, rosa.cocozza@unina.it, phone. +39(0)81675083, fax +39(0)81675058. ***Corresponding author.***

Emilia Di Lorenzo, Università di Napoli "Federico II", Dipartimento di Matematica e Statistica, Napoli, Italy, diloremi@unina.it, phone. +39(0)81675102, fax +39(0)81675058.

Albina Orlando, Consiglio Nazionale delle Ricerche, Istituto per le Applicazioni del Calcolo "Mauro Picone", via P. Castellino 80131 Napoli, Italy, a.orlando@na.iac.cnr.it, phone +39(0)816132395, fax +39(0)816132597. ***Presenting author.***

Marilena Sibillo, Università di Salerno, Dipartimento di Scienze Economiche e Statistiche, via Ponte Don Melillo, campus universitario, 84084 Fisciano (SA), Italy, msibillo@unisa.it, phone +39(0)89962001, fax+39(0)89962049..

Abstract:

The paper approaches the potential of risk-adjusted performance indicators in life insurance, with special reference to a structured policy. The final issue is the computation of risk adjusted indicators as a tool to evaluate the portfolio given a policy structure. The computation of such indicator could be suitable for the appraisal of both portfolio optimization and potential profits of the structured policy. The selection tool is put into an asset and liability management decision making context, where the relationship between expected surplus and capital at risk are compared.
The analysis is applied to a structured temporary annuity and is treated by means of Monte Carlo simulations.

Keywords: Life insurance, Risk-adjusted performance.
JEL Classification: G22, G28.

[*] Although the paper is the result of a common work, sections 1 and 2 are by R. Cocozza and the remaining ones are by E. Di Lorenzo, A. Orlando and M. Sibillo.

1. Introduction

One of the most recent innovation in insurance's management process has been the introduction of Value-at-Risk methodologies (Cocozza et al. 2006). Typically, VaR is defined as "the predicted worst-case loss at a specific confidence level over a certain period of time". This definition is based on the identification of the probability density function for the one-period profit or loss of a portfolio and on the capability of summarizing the distribution with a single statistic, often reported as the maximum loss that can occur within a given confidence interval. VaR methodologies serve many applications. One of the most interesting is the opportunity to compare a profit or margin measure with the capital absorbed by a unit or a portfolio, so as to evaluate the profitability/risk combination and build a risk-adjusted performance measure. The easiest way to assess the risk adjusted performance is to relate the profit measure to the capital at risk, that is to say to relate the profit margin to the value at risk. Alternatively, profit can be compared with return required by shareholders and defined by the product of the capital at risk by the rate set by top management for capital at risk. The difference between profits and required earnings can be regarded as a measure of economic added value. The first approach can be directly applied to an insurance portfolio thanks to the possibility to measure the Value-at-risk of the mathematical provision and to consider it as a measure of the capital at risk, since it includes both the financial and the demographic risk factor. Therefore, this is the approach followed in the paper, whose issue is a first step towards the computation and the application of such risk adjusted indicators as tools to evaluate the portfolio performance given a policy structure.

To this aim the asset and liability risk structure is recalled in the second section. Then the model is applied to a structured policy (GEILA) whose features are given in the third section, while the final one refers to the computational application.

2. The reference scheme

Life insurance business is characterised by a complex system of risks that can be split into two main type of drivers: actuarial and financial. Both the aspects can be regarded at the same time as risk drivers and value drivers, since they can give rise to a loss – or a profit – if the ex ante expected values prove to be higher – or lower – than the ex post actual realizations. If we regard the value of the reserve as the net present value of the residual debt towards the policyholders evaluated at current interest rates and, eventually, at current mortality rates, the VaR of the mathematical provision can be regarded as a first proxy of the capital absorbed by the portfolio under analysis. This guess can be explained by the consideration that the mathematical provision, and more precisely its value, is directly influenced by the demographic and financial risk factors and therefore can be properly regard as a capital at a risk on a specific portfolio of policies, also because its value is the value of the residual debt towards policy holders and therefore is the amount that has to be repaid.

As far as the VaR application is concerned, some specifications are needed. To begin with, worst cases coincide with an increases in the value of the liability, because these rises are the counterparty of expenses or, better, additional costs which may result in either a profit shrinkage or a proper loss. Therefore, the classical portfolio return distribution can be redesigned as a liability cost distribution, where critical values lie in the right-hand tail.

Moreover, for a liability the cost distribution has to be centered on the expected cost. In the case of the mathematical provision the expected cost can be easily linked to the expected value of the reserve at the end of the risk horizon which is the value of the reserve without any modification of relevant risk factors (Cocozza et al., 2004c; 2005), secluding the time passage. In this perspective the expected reserve is the predicted value of the reserve at end of the risk horizon calculated using the informative set available at the beginning of the risk horizon.

Finally, there is the risk horizon. In a risk management perspective the risk horizon could either coincide with the duration of the policy portfolio or with any other intermediate period that cam be regarded as significant for performance appraisal. Therefore, the VaR of the mathematical provision can be defined as the predicted worst case additional cost at a specific confidence level over a period of time consistent with the risk analysis.

As far as the risk factors are concerned, a full VaR estimation should consider both financial and actuarial risk factors. Setting apart the actuarial risk components, a first order approximation of the performance variation due to a change in the evaluation interest rate can be obtained through the cash flow mapping of the reserve. The mathematical provision can be assimilated to a portfolio of zero coupon bonds – whose nominal value is the certain-equivalent of the conditional payment – with maturity equal the each single critical (remaining) instant and with portfolio contribution equal to ratio of the individual present value of each cash flow to the total present value.

Once the VaR of the reserve over a certain risk horizon is estimated, the ratio of the portfolio expected consistent profit and the VaR of the reserve gives rise to a risk-adjusted performance indicator that can be used as a measure of overall profitability of the portfolio under observation.

In evaluating the problems linked to the definition of risk-adjusted performance measures, it is necessary to bear in mind that they can be applied to different objects and used for different purposes. In the case of insurance portfolios, a company may wish to know the return on capital at risk obtained by issuing a certain policy or by serving a certain market segment. In this case the risk-adjusted performance measure can be useful in orienting top management decisions and thus it should be as precise and accurate as possible. At the same time, this kind of measures can also be calculated with reference to a specific business unit. In this case it can have a double function. On the one hand, it can be used to support top management decision and, on the other hand, it can be used to “influence” the business unit manager decisions. Therefore, this kind of measures could be useful to decide which business areas are preferable on the base of the contribution they provide to the economic result. In this perspective, for example, the observation of a “low” risk-adjusted performance indicator can give rise either to a reduction (or even the exclusion) of this portfolio through a reinsurance agreement or an improvement of the policy design by re-structuring the contract itself.

3. The contract

An equity indexed life annuity (EILA) is an annuity due in case of life and corresponding the interest linked to a stock or an equity index. The contract is considered here in the case of the existence of a guarantee floor for the interest earned at the payment time (GEILA). The product payoff, structured as a call option, is hence a combination of two elements: the base component, a fixed-rate annuity paying real income protecting the investor against inflation risk and providing a stable income, and the variable component, linked to an equity index.

Term by term the company calculates the payment amount reflecting the index changes and pays a benefit not lower than that one assigning the minimum interest rate guaranteed in the contract. The product desirability is due to the interest rate risk covering offered to the owner, this last being a policyholder or a pensioner as well. The agreement is so that he thinks improbable losing money rather catching the safety aspect of the position; these elements make the GEILA a good vehicle for retirement planning. In the framework pertaining the pension annuities, the GEILA secures to the beneficiary a guaranteed periodic income not blocking the possibility of better earnings: the contract assumes the shape of a safety tool aiming welfare purposes.

In what follows we consider the contract beginning time coinciding with the accumulation phase ending time, meaning the end of the period during which the annuity is being funded. This time is fixed at the beneficiary's retirement age x birthday and from this term on the accumulated amount is turned into the GEILA. At this time the operations of valuation and reserving are made.

It is opportune to notice that the guaranteed annuity option embedded in the contract places it within the wide variety of life insurance products characterized by a complex structure. That is, the payoff will be non-linear. Of course, the success of this product will depend on the shrewd construction of the guarantee rate of interest that replicates the average consumption pattern of retired individuals.

The instability of financial market and the presence of accidental and systematic errors in mortality forecasting leads to a careful valuation and reserving, considering that both have to follow the guidelines indicated by IASB prospect.

Obviously the issue of the guaranteed life annuity implies the pensioner's existence in life at the time of valuation and the knowledge of the total amount F_0 at disposal for the GEILA planning.

3.1 Notation and cash flow structure in the pure financial case

Posing in T the contract maturity, with T=1,2,…,w-x and w the ultimate lifetime, we consider the fund F_0 invested in the reference asset at time 0, this time coinciding with the annuity beginning and contextual with the time of valuation. The payments to the survivals are made at the beginning (the end) of each year till T years and guarantee at least the annual interest rate g.

Indicating by F_t the value of the reference portfolio at time t, t=1,2,…,T, the interest rate maturing on it in the time interval t-1,t is:

$$i_t = \frac{F_t - F_{t-1}}{F_{t-1}}$$

In a purely financial perspective, that is neglecting the uncertainty in the insured's or pensioner's survival at time t, the cash at time t can be described as follows:

$$B_t = \left[R_{g,t} + \max\left(0, R_t - R_{g,t}\right)\right] \tag{1}$$

with:

$$R_t = \frac{F_0}{\ddot{a}_{x:\overline{T|}\,i_t}}$$

and the payment due in t calculated at the guarantee rate g (constant with t):

$$R_{g,t} = \frac{F_0}{\ddot{a}_{x:\overline{T|}\,g}}$$

The cash in t can be alternatively expressed as:

$$B_t^{'} = \left[R_t + \max\left(0, R_{g,t} - R_t\right)\right] \tag{2}$$

The second term on the right hand side of formulas (1) and (2) represent respectively a European call and a European put payoff, both issued on R_t with strike $R_{g,t}$. In particular, at the time of valuation we can recognize T call (put) options, the first a plain vanilla one with maturity 1 and the other T-1 path dependent ones with maturity 2,3,…,T and duration one.

As usually observed, if the European calls (puts) existing in the contract are traded at each term and if zero coupon bonds with maturity t (t=1,2,…,T) are traded as well, the financial risk embedded in is completely hedged. For example, in the first approach involving calls (formula 1) the strategy consists in buying $R_{g,t}$ unit face zero coupon bonds respectively with maturity t=1,2,…,T and T European call options on R_t with strike $R_{g,t}$.

Still neglecting the demographic risk, if $c_0(t, R_{g,t})$ is the market price at time 0 of a European call plain vanilla on R_1 if t=1 and of a European call path dependent on R_t with maturity t, t=2,3,…,T, we can write the market value p_0 of the contract:

$$p_0 = \sum_{t=1}^{T} R_{g,t} v_0(t) + \sum_{t=1}^{T} c_0\left(t, R_{g,t}\right) \tag{3}$$

3.2 Inserting the demographic risk

The cash flow develops in a more complex form inserting the contractor's survival uncertainty at the payment terms.

The scheme consists in T call options, each one expiring at the time of death if this happens before maturity. In this sense the calls are like barrier options in relation to the demographic factor, specifically knock-out barrier options. In this hypotheses formula 3 becomes:

$$p_0 = \sum_{t=1}^{T} 1_{\{K_x > t\}} \left[R_{g,t} v_0(t) + c_0\left(t, R_{g,t}\right)\right] \tag{4}$$

where the indicator function $1_{\{K_x > t\}}$ takes value 1 if the curtate future lifetime k_x of the pensioner aged x at issue is greater than t, 0 otherwise.

Taking into account the risk arising from the uncertainty in survival, this implies premium amounts lower than those calculated in the case of pure financial risk scheme; but it is necessary to notice that the incompleteness of the insurance market makes impossible acting hedging strategies replicating the claim contingencies as in the pure financial risk case.

The demographic risk component has the accidental nature of the deviations of the number of deaths from the expected values (mortality risk) and the systematic nature due to the betterment in the evolution in time of the industrialized country mortality behaviour (longevity risk). Pooling strategies for the mortality risk can avoid its influence and, if the pooling aim is reached with a sufficiently large homogeneous portfolio, the mortality risk can be considered completely hedged and, as a consequence, the insurer or the Institution becomes risk neutral with respect to mortality accidental risk (cf. **[1]**).

Formula (4) can be rewritten as follows:

$$p_0 = \sum_{t=1}^{T} {}_t p_x \left[R_{g,t} v_0(t) + c_0\left(t, R_{g,t}\right)\right] \tag{5}$$

with ${}_t p_x$ as usual the probability that x lives after t years.

The mathematical provision at time t calculated in 0 are given by the following formula:

$$L_t^{(0)} = \sum_{j=1}^{T-t} {}_t p_x \;\; {}_j p_{x+t} \left[R_{g,t+j} \; v_0(t, t+j) + c_t\left(t+j, R_{g,t+j}\right)\right] \tag{6}$$

in which $v_0(t, t+j)$ is the market price expressed in 0 of one unit face zero coupon bond issued in t with maturity after j years.

In the asset-liability perspective, indicating with:

$N^{(k)}$ the number of survivors at time k belonging to the cohort of $N^{(0)}$ pensioners aged x at the benefit flow beginning (time 0) i(k-1,k) the rate of interest for the k[th] year maturing on the invested fund

B_k the call payoff at time t as in formula (1)

the portfolio fund at time t is expressed as:

$$F_t = F_0 \prod_{k=1}^{t} \left[1 + i(k-1,k)\right] - \left[\sum_{k=1}^{t-1} N^{(k)} B_k \prod_{h=k}^{t-1} \left[1 + i(h,h+1)\right] + N^{(t)} B_t\right] \tag{7}$$

with t=2,3,…, and with t=1

$$F_1 = F_0 \left[1 + i(0,1)\right] - N^{(1)} B_1 \tag{8}$$

The class of contracts we are considering are long term ones. In these cases definitely crucial is the hedging problem of the systematic risk known as longevity risk, influencing all the contracts in the same direction and hence being not a pooling risk

Projected model for the survival functions are the most used tools for fronting this question and a wide literature on this subject points out the extremely deep interest of academic and practitioner world.

4. Quantile based risk measures: The Conditional VaR

The *Conditional Var* or *Expected Tail Loss* (ETL) is a "Var-like" risk measure in the sense that it reflects the quantiles of the loss distribution. It is a risk measure retaining the benefits of VaR in terms of probabilistic contents while avoiding its limits. In particular, it satisfies the desirable properties of coherence better than VaR, having in particular the attraction of being subadditive. Moreover it has the advantage over Var to present a convex risk surface. This is important in problems concerning optimization, in the sense that it ensures that the risk minimum is a unique global one (Dowd and Blake, 2004).

Referring to the GEILA scheme described in the previous section, our aim is the computation of the ETL for its proper use in a risk-adjusted assessment perspective.

Indicating by $L_t^{(0)}$ the stochastic mathematical provision of the GEILA at time *t* as in formula (6), let us consider the time interval [t,t+h] and the financial positions at its extremes, say $l^{(0)}(t)$ and $L_{t+h}^{(0)}$ respectively; the potential periodic loss is defined as :

$$PPL = l^{(0)}(t) - L_{t+h}^{0} \tag{9}$$

At the confidence level α, the Value at Risk $VaR(\alpha)$ is given by:

$$P\{PPL > VaR(\alpha)\} = 1 - \alpha \tag{10}$$

The ETL is the expected value of the worst 1-α losses, that is the average of losses exceeding VaR:

$$\text{ETL} = \text{E}[\text{PPL}|\text{PPL}>\text{VaR}] \tag{11}$$

5. Numerical results

5.1 Financial and mortality scenario

The example of application we propose is referred to a GEILA contract with duration T=15 issued on a life aged 40. The time of valuation is t=5. We pose the guaranteed fixed rate g=0.02 and F_0 =1000.

Our goal is the computation of the ETL at time t at a given confidence level of the stochastic mathematical provision given by formula (6). To this aim we introduce the stochastic background for the interest rate and reference portfolio distribution.

We consider a term structure of interest rates based on the Cox-Ingersoll-Ross stochastic differential equation:

$$dr_t = -\alpha(r_t - \mu)dt + \sigma\sqrt{r_t}\,dW_t \tag{12}$$

with α and σ positive constants, μ the long term mean and W_t a Wiener process.

In line with much of the work on path dependent options, we assume that the underlying asset price follows a geometric Brownian motion with constant volatility:

$$dF_t = \delta F_t dt + \sigma F_t dW_t \tag{13}$$

with δ and σ the long term mean and the volatility parameters respectively and W_t a Wiener process.

The solution is:

$$F_t = F_0 e^{(\delta - \sigma^2/2)t + \sigma dW_t} \tag{14}$$

5.2 The computational procedure

The computation of the Expected Tail Loss requires the knowledge of the of the potential periodic loss distribution function described in the previous section. To this aim we use a Monte Carlo simulation procedure.

In order to perform the simulation procedure it is necessary to consider the discrete time equation for the chosen SDE describing the evolution in time of the interest rates and the reference portfolio. On the basis of the first order Euler scheme the discrete version of equation (11),considering the time interval [0,T], is:

$$r_{t_k} = r_{t_{k-1}} + \alpha(\mu - r_{t_k})\Delta + \sigma\sqrt{r_{t_{k-1}}\Delta} \cdot \varepsilon_k \quad k=1,2,..,n(T\text{-}1) \tag{15}$$

where Δ=T/*n* is the sampling interval, ε_k the increment ΔW_k of the Wiener process between $t_{k+1} = (k+1)\Delta$ and $t_k = k\Delta$. The increments ΔW_k are $N(0,\Delta)$ distributed random variables.

In a risk neutral context, the discounting term:

$$\exp[-\int_t^T r(k)dk)] \tag{16}$$

is approximated by the following expression

$$\exp(-\sum_{k=1}^{n-1} r_{t_k} \cdot \Delta) \tag{17}$$

Expression (17) allows us to derive the forward prices $v^{(0)}(t,t+j)_{j=1(T-t)}$ of equation (6).

The simulation of the (T-t) embedded path dependent call option payoffs:

$$\max\left(0, R_{g,t+j} - R_{t+j}\right)_{j=1,..,T-t}$$

requires the discrete version of equation (14):

$$F_{t_k} = F_{t_{k-1}} * e^{(\delta - \sigma^2/2)\Delta + \sigma\sqrt{\Delta}\varepsilon_k}$$

In order to obtain an accurate approximation, Δ have to be sufficiently small and we choose a monthly sapling interval.

By means of formulas (6) and (9), we proceed simulating N values of the potential periodic loss PPL at time *t*. The simulated *{PPL(j)}, j=1,2,…,N* can be treated as a sample from a normal distribution as N increases. In our example we consider N=10000.

At this point we can estimate the ETL slicing the tail of the loss distribution into *n* slices. Each slice has the same probability mass. Estimating the Value at Risk associated with each slice, we can take the average of the VaRs, obtaining the ETL. We should use a value of *n* large enough to give accurate results.

For the survival probabilities we use the mortality Italian data for the period 1947-1999 to evaluate the projection of the mortality factor in a Lee Carter context. We assume for the CIR process α= 0.0452, σ= 0.0053, estimated on the 3-month T-Bill January 1996 – January 2006. We pose $\delta = 0.03$, $\theta = 0.020$ for the evolution in time of the reference portfolio.

In the following table 1 the characteristic values of the simulated distribution of the reserve and of the potential periodic loss over one year, obtained by means of the proposed simulation, are reported.

Table1.: ***GEILA*** **Reserve and Loss distribution** results obtained for *N*=10000 simulation paths.

	Mean	**St. Dev.**	**Skewness**	**Kurtosis**
$L_t^{(0)}$	2310.352	0.635	0.0004	3.04
PPL(t)	26.17347	0.405	0.0001	3.03

Table 2. gives the ETL values at a 95% confidence level.

Table2: ***95% ETL as a function of the Number of Tail Slices.*** N=10000 simulation paths

n	**5**	**10**	**100**	**1000**	**5000**
ETL	26.9422	26.9898	27.0598	27.0688	27.0708

References

Bacinello A.R., 2006. *Fair valuation of life insurance contracts with embedded options* Assembléè générale 200s6 Association Suisse des Actuaries, Lausanne September 2006.

Brigo D., Mercurio F., 2004. *Interest rate models- Theory and Practice.* Springer Second Edition.

Cleur E.M. & Manfredi P. 'One dimensional SDE models, low order numerical methods and simulation based estimation: a comparison of alternative estimators' *Computational Economics Vol. 13 N. 2 April 1999, pp.177-197(21) Kluver Academic Publisher.*

Cocozza, R., 2000. *La gestione del rischio di tasso di interesse nelle imprese di assicurazione sulla vita*. CEDAM, Padova.

Cocozza, R., Di Lorenzo, E., 2003. *Risk profiles of life insurance business: a combined approach*. Proceedings of the 6th Spanish-Italian Meeting on Financial Mathematics, Dipartimento di Matematica Applicata Bruno de Finetti, Trieste, pp.157-170.

Cocozza, R., Di Lorenzo, E., 2006. *Solvency of life insurance companies: methodological issues*, in Journal of Actuarial Practice.

Cocozza R., Di Lorenzo E., Orlando A., Sibillo M. (2006a), *A liability adequacy test for mathematical provision*. Atti del convegno Metodi Matematici e Statistici per le Assicurazioni e la Finanza MAF2006, Salerno, October 11-13, 2006. http://193.205.179.11/maf2006/index.html

Cocozza R., Di Lorenzo E., Sibillo M., Orlando A. (2006b), *The VaR of the mathematical provision: critical issues*. Proceedings of The 28th International Congress of Actuaries, 28 May – 2 June 2006, Paris. http://papers.ica2006.com/2002.html .

Cocozza, R., Di Lorenzo, E., Sibillo, M., 2004a. *Risk Profiles of Life Insurance Business: quantitative analysis in a managerial perspective*. Atti del convegno Metodi Matematici e Statistici per l'Analisi dei Dati Assicurativi e Finanziari - MAF2004 -, CUSL, Salerno, pp. 81-86. http://www.labeconomia.unisa.it/matstatsalerno

Cocozza R., Di Lorenzo, E., Sibillo, M., 2004b. *Methodological problems in solvency assessment of an insurance company*, Investment Management and Financial Innovations, Issue 2, pp. 95-102, 2004.

Cocozza R., Di Lorenzo, E., Sibillo, M., 2004c. *Life insurance risk indicators: a balance sheet approach*, proceedings of the VIII International Congress on Insurance: Mathematics & Economics, Rome, I, http://www.imerome2004.com

Cocozza R., Di Lorenzo, E., Sibillo, M., 2005. *The current value of the mathematical provision: a financial risk prospect*, technical report.

Coppola, M., Di Lorenzo, E., Sibillo, M., 2000. *Risk Sources in a Life Annuity Portfolio: Decomposition and Measurement Tools*. Journal of Actuarial Practice, 8, pp. 43-61.

Coppola, M., Di Lorenzo, E., Sibillo, M., 2002. *Further Remarks on Risk Sources Measuring in the case of a Life Annuity Portfolio.* Journal of Actuarial Practice, 10, pp. 229-242.

Cox, J.C., Ingersoll, J.E., Ross, S.A., 1985. *A theory of the term structure of the interest rate*. Econometrica 53, pp. 385-407.

Di Lorenzo, E., Sibillo, M., 2002. *The Longevity Risk: Measurement and Application Perspectives*. Proceedings di 2nd Conference in Actuarial Science and Finance on Samos, Karlovassi (Grecia), http://www.stat.ucl.ac.be/Samos2002/proceedSibillo.pdf

Dowd K., Blake D., 2006. After VaR: The theory, estimation, and insurance applications of quantile based risk measures . *The journal of risk and insurance*, 2006, Vol. 73, No.2, 193-229.

Frees, E. W., 1990. *Stochastic life contingencies with solvency considerations*. Transactions of the Society of Actuaries, XLII, pp. 91-129.

Melsa, J.L., Sage, A.P., 1973. *An Introduction to Probability and Stochastic Processes*. Prentice-Hall, New Jersey.

Parker, G., 1997. *Stochastic analysis of the interaction between investment and insurance risks*. North American Actuarial Journal 1 (2), pp. 55-84.

Rubinstein Y.R., 1981, *Simulation and Monte Carlo method,* Wiley& Sons, Chichester.

Talay D. & Zheng Z, *2003, Quantile of the Euler scheme for diffusion processes and financial application,* Mathematical Finance*, Vol.13 pp.187-199.*