SWAP DERIVATIVES AND BOUNDS FOR THE HEDGE ACCOUNTING EFFECTIVENESS TEST

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ABSTRACT

The aim of this paper is to determine and analyze the bounds for the hedge ratio in the prospective (*ex-ante*) effectiveness test as specified in IAS 39 with regards to swap derivatives, and to investigate the role of the first and second order interest rate risk measures in the hedge accounting framework. The cash flow hedge and the "theoretical swap" method are studied and lower and upper bounds for the prospective hedge ratio are derived under the parallel shift assumption for the market yield curve. These bounds are functions of the first and second order risk measures and empirical evidence is provided to show that they do not allow to control the retrospective (*ex-post*) test effectiveness.

KEYWORDS

IAS39, Hedge accounting, Hedge effectiveness test, Interest rate risk

1. DERIVATIVES AND HEDGE ACCOUNTING

Economically speaking, an operation in derivatives for hedging purposes intends to neutralize possible losses recognized in a hedged item caused by a specified risky event, with gains deriving from a hedging instrument contract in case such an event should occur.

The rules for hedge accounting as set out in IAS 39 allow accounting representation to hedge operations (ABI (2004) and IASB (2004)). In particular, the problem of how to deal with economic hedged items arises from IAS 39 rules on the measurement of fair value in derivative contracts and the recognition in profit and loss of fair value variations of the derivative, even when it is purchased to hedge assets or liabilities subject to different accounting treatment (for example, measured at cost or at fair value with gain or loss recognized directly in equity). Following the hedge accounting rules it is possible to derogate the standard evaluation criteria provided for the hedging instrument, thus eliminating possible evaluation asymmetries between the hedged item and the hedging derivative.

2. THE HEDGE EFFECTIVENESS VALUATION

The most important requirement of hedge accounting is concerned with hedge effectiveness evaluation. The hedge effectiveness measure is the hedge ratio, generically defined by IAS 39 as the ratio between the fair value changes of the hedging instrument and the hedged item (or vice versa). Given the time of valuation, the hedge ratio has to be calculated periodically by a prospective (*ex-ante*) and a retrospective (*ex-post*) test. The prospective test deals with hedge effectiveness in future time, so it is a model dependent test as it depends on the adopted market model. The retrospective test instead deals with the observed hedge effectiveness in a specific period of observation.

In this paper the problem of measuring a cash flow hedge with the so called "theoretical swap" method is analyzed by considering a cash flow hedge rather common in corporate financial management and financial intermediation: the hedging of the variability in expected future cash flows attributable to interest rate risk of a variable interest rate instrument (hedged item) through a pay fixed/receive variable interest rate swap contract (hedging item) [1]. In this case the theoretical swap will be characterized by a variable leg equal to the cash flow stream of the hedged item and by a fixed leg whose structure depends on the strategy set by the risk management [2].

The hedge ratio over a given time interval is defined as:

$$E_{t,s} = {}^{R}\Delta V_{t,s} \big/ {}^{T}\Delta V_{t,s}$$

where

t is the time of valuation,

[1] A cash flow hedge is "a hedge of the exposure to variability in cash flows that (i) is attributable to a particular risk associated with a recognized asset or liability (such as all or some future interest payments on variable rate debt) or a highly probable forecast transaction and (ii) could affect profit or loss.". One of the more widely diffused methods for the effectiveness test is the "theoretical swap" method (change in fair value of cash flows): once established the risk management strategy and given the hedged item, the theoretical swap is defined as the derivative contract which guarantees a perfect hedge of the hedged item.

[2] In particular, if the risk management objective is to build a fixed rate debt equivalent to the variable rate debt as if it was fixed at the inception date of the debt, the leg of the theoretical swap to be paid by the firm will have a fixed rate equal to the corresponding market swap rate at the inception date. If, instead, the objective is to have fixed rate payments equal to the fixed rate stream of the hedging instrument, the fixed leg of the theoretical swap will be equal to the fixed leg of the realized swap. See Gheno and Mottura (2007) regarding interest rate risk management strategies and hedge accounting. For interest rate swaps and hedge accounting see Kawaller (2007).

s is a different date (future or past),

${}^{R}\Delta V_{t,s}$ is the fair value change of the realized swap,

${}^{T}\Delta V_{t,s}$ is the fair value change of the theoretical swap.

According to IAS 39, if $0,8 \le E_{t,s} \le 1,25$ the hedge is declared "highly effective"; in particular, if $E_{t,s} = 1$, the hedge is declared a "fully effective" hedge.

In the retrospective test $s < t$ and therefore this test does not involve evaluation problems. In this case, in fact, the hedge ratio $E_{t,s}$ is observable since the fair value change of the realized and theoretical swaps are equal to the difference between their respective values (clean prices) at time *t* and *s*.

In the prospective test $s > t$. In this case the ratio $E_{t,s}$ is a random variable dependent on the future values, in *s*, of the realized and theoretical swaps; these values depend on the yield curve observed on the market at time *s*. Since in order to value the hedge ratio in *t* a term structure model is needed, $E_{t,s}$ is an interest rate model dependent measure.

Formally, if it is assumed that at the time of valuation *t* a yield curve shift occurs in an instant *s* after *t* and $\Delta V_{t,s} = V_N(s) - V_N(t)$ is the difference between the values at time *s* and *t* of a generic interest rate swap, then the hedge ratio for the prospective test is given by the expression:

$$E_{t,s} = \frac{{}^{R}V_N(s) - {}^{R}V_N(t)}{{}^{T}V_N(s) - {}^{T}V_N(t)}. \tag{1}$$

3. BOUNDS FOR THE PROSPECTIVE HEDGE RATIO

If with $v(t,t_k) = \left[1 + i(t,t_k)\right]^{-(t_k - t)}$ and $\delta(t,t_k) = -\frac{\partial}{\partial t_k}\log v(t,t_k)$, $t \le t_k$ $k = 1,...,m$, are denoted respectively the discount factor and the force of interest for the generic time interval (t,t_k), being $i(t,t_k)$ the spot interest rate, the value in *t* of a generic cash flow stream $\mathbf{x} = \{x_1, x_2, ..., x_m;\ t_1, t_2, ..., t_m\}$, where x_k is the sum payable at time t_k $(k = 1, 2, ..., m$ and $t_1 < t_2 < ... < t_m)$ known at time *t*, can be expressed as:

$$V(t) = \sum_{k=1}^{m} x_k v(t,t_k) = \sum_{k=1}^{m} x_k \, \mathrm{e}^{-\int_t^{t_k} \delta(t,u)du}. \tag{2}$$

Let

$$\tilde{D}(t) = \sum_{k=1}^{m} (t_k - t)\, x_k v(t,t_k) \tag{3}$$

and

$$\tilde{D}^{(2)}(t) = \sum_{k=1}^{m} (t_k - t)^2\, x_k v(t,t_k) \tag{4}$$

be respectively the first and the second order interest rate risk measures of **x** at time *t* (*i.e.* the numerator of the Macaulay duration and of the second order duration of **x** at time *t*).
For valuation purposes, each swap contract is seen as an asset-liability portfolio comprising an asset leg and a liability leg. In presence of a stochastic cash flow stream, a corresponding deterministic replicating portfolio is considered. For instance for a variable rate bond a zero coupon bond with same face value and maturity equal to the next coupon payment date is considered as its replicating portfolio at issue. Doing so allows defining the following deterministic cash flow streams of the theoretical swap:

$$ {}_c^T\mathbf{x} = \left\{ {}_c^T x_1, {}_c^T x_2, \dots, {}_c^T x_m;\ t_1, t_2, \dots, t_m \right\}, \quad {}_d^T\mathbf{x} = \left\{ {}_d^T x_1, {}_d^T x_2, \dots, {}_d^T x_m;\ t_1, t_2, \dots, t_m \right\} $$

and of the realized swap:

$$ {}_c^R\mathbf{x} = \left\{ {}_c^R x_1, {}_c^R x_2, \dots, {}_c^R x_m;\ t_1, t_2, \dots, t_m \right\}, \quad {}_d^R\mathbf{x} = \left\{ {}_d^R x_1, {}_d^R x_2, \dots, {}_d^R x_m;\ t_1, t_2, \dots, t_m \right\} $$

where ${}_c^T x_k$, ${}_d^T x_k$, ${}_c^R x_k$ and ${}_d^R x_k$ are respectively the amounts to be received and to be paid at time t_k from the theoretical and the realized swap $\left(k = 1, 2, \dots, m \text{ and } t_1 < t_2 < \dots < t_m\right)$.
The value, the first and second order risk measures functions of the theoretical swap at time *t* can be expressed as:

$$ {}^T V_N(t) = {}_c^T V(t) - {}_d^T V(t), \quad {}^T\tilde{D}_N(t) = {}_c^T\tilde{D}(t) - {}_d^T\tilde{D}(t), \quad {}^T\tilde{D}_N^2(t) = {}_c^T\tilde{D}^2(t) - {}_d^T\tilde{D}^2(t); $$

analogously, for the realized swap, is:

$$ {}^R V_N(t) = {}_c^R V(t) - {}_d^R V(t), \quad {}^R\tilde{D}_N(t) = {}_c^R\tilde{D}(t) - {}_d^R\tilde{D}(t), \quad {}^R\tilde{D}_N^2(t) = {}_c^R\tilde{D}^2(t) - {}_d^R\tilde{D}^2(t). $$

PROPOSITION
Under the assumption $\delta(t^+, t_k) = \delta(t, t_k) + Y$, being $t^+ = t + dt$ and Y a random variable representing an additive infinitesimal shift, if the following conditions hold:

$$ {}^T\tilde{D}_N(t) \neq 0 $$

$$ \frac{1}{{}^T\tilde{D}_N(t)} \left[\frac{{}^R\tilde{D}_N(t)}{{}^T\tilde{D}_N(t)} {}^T\tilde{D}_N^{(2)}(t) - {}^R\tilde{D}_N^{(2)}(t) \right] \neq 0 $$

then

$$ E_{t,t^+} = \frac{{}^R\tilde{D}_N(t)}{{}^T\tilde{D}_N(t)} + G_t Y \tag{5} $$

where

$$G_t = \frac{1}{2\,{}^T\tilde{D}_N(t)}\left[\frac{{}^R\tilde{D}_N(t)}{{}^T\tilde{D}_N(t)}\,{}^T\tilde{D}_N^{(2)}(t) - {}^R\tilde{D}_N^{(2)}(t)\right] \tag{6}$$

■

PROOF

Since equations (2), (3) and (4) are linear in x_k, the value, the first and the second order interest rate risk measures of the theoretical and realized swap can be expressed in terms of net pay-offs. If with ${}^{T/R}_{N}x_k$ is denoted the net pay-off at time t_k of the theoretical or of the realized swap, the corresponding post shift values can be written as:

$$
\begin{aligned}
{}^{T/R}V_N\left(t^+\right) &= \sum_{k=1}^{m} {}^{T/R}_{N}x_k v\left(t^+, t_k\right) = \sum_{k=1}^{m} {}^{T/R}_{N}x_k e^{-\int_{t^+}^{t_k}\delta\left(t^+,u\right)du} = \\
&= \sum_{k=1}^{m} {}^{T/R}_{N}x_k e^{-\int_{t}^{t_k}\delta(t,u)du} e^{-Y\left(t_k-t\right)} = \sum_{k=1}^{m} {}^{T/R}_{N}x_k v\left(t,t_k\right)e^{-Y\left(t_k-t\right)}\,.
\end{aligned}
$$

Expanding the above equation up to the second order in Y yields:

$$
\begin{aligned}
{}^{T/R}V_N(t^+) &= \sum_{k=1}^{m} {}^{T/R}_{N}x_k v(t,t_k)\left[1 - Y(t_k - t) + \frac{1}{2}Y^2(t_k-t)^2 + o(Y^2)\right] = \\
&= {}^{T/R}V_N(t) - {}^{T/R}\tilde{D}_N(t)Y + \frac{1}{2}\,{}^{T/R}\tilde{D}_N^{(2)}(t)Y^2 + o\left(Y^2\right)
\end{aligned}
$$

which inserted into (1) gives:

$$E_{t,t^+} = \frac{-{}^R\tilde{D}_N(t) + \frac{1}{2}\,{}^R\tilde{D}_N^{(2)}(t)Y + o(Y)}{-{}^T\tilde{D}_N(t) + \frac{1}{2}\,{}^T\tilde{D}_N^{(2)}(t)Y + o(Y)} =$$

$$= \frac{{}^R\tilde{D}_N(t) - \frac{1}{2}\,{}^R\tilde{D}_N^{(2)}(t)Y + o(Y)}{{}^T\tilde{D}_N(t)}\;\frac{1}{1 - \frac{1}{2}\frac{{}^T\tilde{D}_N^{(2)}(t)}{{}^T\tilde{D}_N(t)}Y + o(Y)}\,.$$

Expanding the second factor of the previous relation up to the first order in Y yields:

$$E_{t,t^+} = \frac{\left[{}^R\tilde{D}_N(t) - \frac{1}{2}\,{}^R\tilde{D}_N^{(2)}(t)Y + o(Y)\right]\left[1 + \frac{1}{2}\frac{{}^T\tilde{D}_N^{(2)}(t)}{{}^T\tilde{D}_N(t)}Y + o(Y)\right]}{{}^T\tilde{D}_N(t)}$$

and ignoring the influence of the higher order terms gives:

$$E_{t,t^+} = \frac{{}^{R}\tilde{D}_N(t)}{{}^{T}\tilde{D}_N(t)} + \frac{1}{2\,{}^{T}\tilde{D}_N(t)}\left[\frac{{}^{R}\tilde{D}_N(t)}{{}^{T}\tilde{D}_N(t)}{}^{T}\tilde{D}_N{}^{(2)}(t) - {}^{R}\tilde{D}_N{}^{(2)}(t)\right]Y\ .$$

From equation (6), it follows that

$$E_{t,t^+} = \frac{{}^{R}\tilde{D}_N(t)}{{}^{T}\tilde{D}_N(t)} + G_t Y\ .$$

■

COROLLARY

If

$${}^{R}\tilde{D}_N(t) = {}^{T}\tilde{D}_N(t) \neq 0$$

then

$$E_{t,t^+} = 1 + G_t Y$$

where

$$G_t = \frac{1}{2}\frac{{}^{T}\tilde{D}_N^{(2)}(t) - {}^{R}\tilde{D}_N^{(2)}(t)}{{}^{T}\tilde{D}_N(t)}\ .$$

■

REMARK

If ${}^{R}\tilde{D}_N(t) = {}^{T}\tilde{D}_N(t) \neq 0$ and ${}^{R}\tilde{D}_N^2(t) = {}^{T}\tilde{D}_N^2(t)$ then, ignoring the influence of the higher order terms, $E_{t,t^+} = 1$.

■

REMARK

In the Proposition it is not assumed that ${}^{R}V_N(t) = {}^{T}V_N(t) = 0$, hence equation (5) holds both for par and non-par swaps.

■

REMARK

From a practitioner's point of view the Proposition can be interpreted in the sense of the traditional "what-if" analysis. By doing so from equations (5) and (6) the following hedge ratio upper and lower bounds can be defined:

$$\begin{cases} \frac{{}^{R}\tilde{D}_N(t)}{{}^{T}\tilde{D}_N(t)} + G_t \underline{Y} \leq E_{t,t^+} \leq \frac{{}^{R}\tilde{D}_N(t)}{{}^{T}\tilde{D}_N(t)} + G_t \overline{Y} & \text{if } G_t > 0 \\ \frac{{}^{R}\tilde{D}_N(t)}{{}^{T}\tilde{D}_N(t)} + G_t \overline{Y} \leq E_{t,t^+} \leq \frac{{}^{R}\tilde{D}_N(t)}{{}^{T}\tilde{D}_N(t)} + G_t \underline{Y} & \text{if } G_t < 0 \end{cases} \quad (7)$$

where $\overline{Y}$ and $\underline{Y}$ respectively are "small" positive and negative additive shifts exogenously fixed.

If $\underline{Y} = -\overline{Y}$ and ${}^{R}\tilde{D}_N(t) = {}^{T}\tilde{D}_N(t) \neq 0$, from equation (7) it follows that in order to have $0,8 \leq E_{t,t^+} \leq 1,25$ (*i.e.* a "highly effective" prospective hedge):

$$\left| \frac{{}^{T}\tilde{D}_N^{(2)}(t) - {}^{R}\tilde{D}_N^{(2)}(t)}{{}^{T}\tilde{D}_N(t)} \right| \leq \frac{0,4}{\overline{Y}}.$$

■

REMARK

The parallel shift assumption for the yield curve is characteristic of interest rate risk management strategies based on the results of the immunization theory [3].

■

EXAMPLE

The issue of a 40 years bullet bond and a corresponding long term amortizing swap pay fixed/receive fixed are considered [4]. The asset leg of the swap is equal to the bond cash flow stream (face value 100 euros, coupon rate 4% paid annually). The liability leg has constant payments of 5.05 euros, paid annually for 40 years. To perform the analysis of the prospective hedge effectiveness at issue, it is assumed that this swap contract represents the theoretical swap (*i.e.* the risk management objective of the issuer). To compare at the inception time different hedging situations in terms of the G_t function and of lower and upper bounds (equations (6) and (7)), five different swaps have been considered, all having the same maturity (40 years) and asset leg (equal to the bond stream) but different liability legs. If a 4% flat interest rate term structure is assumed, it results that all these different hedge situations have the same value but different first and second order risk measures.

In Table 1, the different liability leg structures and the corresponding values for their G_t functions are reported as well as lower and upper bounds considering $\pm 0,01\%$ positive and negative additive shifts of the term structure.

Table 1

Hedge	*Swap Liability leg*	G_t	*Hedge ratio Lower bound*	*Hedge ratio Upper bound*
A	€ 10.31 every 2 years	-0,98	90,01%	90,03%
B	€ 21.45 every 4 years	-2,87	69,65%	69,70%
C	€ 27.37 every 5 years	-3,78	59,27%	59,35%
D	€ 46.55 every 8 years	-6,31	27,39%	27,52%
E	€ 60.66 every 10 years	-7,82	5,51%	5,66%

From Table 1 it results that Hedge A is the only "highly effective" prospective hedge.

■

[3] Details on immunization theory development can be found in De Felice (2000).

[4] This kind of long term swaps is very popular, for example, in Italian regional markets where long term bonds are issued by municipalities (*e.g.* Buoni Ordinari Comunali, BOC).

4. EMPIRICAL EVIDENCE

An interest rate risk hedge rather common in corporate financial management is considered: a firm, in $t_0 = 01/01/99$, issues a variable rate bond with face value of 100 euros and maturity $t_m = 01/01/02$; the bond pays a coupon every semester, indexed at the six-month Euribor rate. To fix the debt cost, the firm enters into a pay fixed/receive variable interest rate swap with a notional amount equal to the face value of the bond issue with the same maturity as that of the bond. Since a cash flow hedge is considered, the hedge effectiveness test according to IAS 39 rules can be set up with the "theoretical swap" method. Two alternative hedge situations for the debt cost have been considered. In the standard case (Hedge 1), the hedge instrument is a plain vanilla interest rate swap in which the firm pays semi-annually the market swap rate with maturity 3 years, quoted 3,31% on annual basis at inception. In the alternative case (Hedge 2) the swap has a variable leg equal to the coupon payments of the hedged item and a fixed leg defined by the cash flow stream $\{4{,}853\,,105{,}076\,;\ 01/07/99\,,01/01/02\}$.

In Figure 1, the interest rate term structure (euros) in $t_0 = 01/01/99$, obtained by bootstrapping from the swap rates quoted on the market for each maturity from 1 to 10 years, is represented [5].

Figure 1

Given the market situation, the first and the second order risk measures of the hedged item and of the swaps have been calculated in $t = t_0$ by equations (3) and (4). The results are shown in Table 2 [6].

[5] For maturity less than one year, the curve was completed with the corresponding Euribor rates.
[6] In order to make the comparison between hedged and hedging items more clear, the fixed and the variable leg of the realized swap are respectively considered as a fixed rate bond and a variable rate bond with face value equal to the notional of the swap. This, of course, does not affect the result of the analysis.

Table 2

Stream	$\tilde{D}(t)$	$\tilde{D}^{(2)}(t)$
Variable rate bond	50	25
<u>Realized swap - Hedge 1</u>	<u>-240</u>	<u>-837</u>
fixed leg (-)	290	862
variable leg (+)	50	25
<u>Realized swap - Hedge 2</u>	<u>-240</u>	<u>-845</u>
fixed leg (-)	290	870
variable leg (+)	50	25

It results that, in $t = t_0$, the realized swaps in Hedge 1 and in Hedge 2 have the same net first order risk measure and different second order risk measures.

Description of the theoretical swap. If the risk management objective is to build a fixed rate debt equivalent to the variable rate debt as if it was fixed at the inception date of the variable rate debt, it follows that the theoretical swap will have a fixed leg equal to the fixed leg of the realized swap in Hedge 1 and a variable leg equal to the variable rate bond stream. The theoretical swap, given the management objective, is therefore contractually equal to the realized swap described in Hedge 1.

Bounds for the prospective effectiveness test. The values of the G_t function and lower and upper bounds are calculated by equations (6) and (7), assuming a shift of the term structure equal to ± 0.01%. The results are reported in Table 3.

Table 3

Stream	G_t	*Hedge ratio Lower bound*	*Hedge ratio Upper bound*
Hedge 1	0,00000	100,00000%	100,00000%
Hedge 2	-0,01650	99,99984%	100,00017%

From Table 3 it results that Hedge 1 is a “fully effective” hedge and Hedge 2 is an “highly effective” hedge in terms of prospective test.

Prospective and retrospective effectiveness tests. In order to analyze how lower and upper bounds, as previously derived, work with respect to the prospective and the retrospective effectiveness test, the two hedge situations at inception have been considered and prospective and retrospective effectiveness tests with the theoretical swap method have been performed daily ($\Delta t = 1 \text{ day}$), during the period January 1999-December 2001. It has been assumed, in other terms, a “frozen” hedge portfolio in the observation period.
The term structure of interest rates has been derived for each day of the observation period by bootstrapping from the daily swap rates quoted on the market for each maturity from 1 to 10 years and completed with the Euribor rates for maturity less than one year.
The development of the prospective effectiveness test has been carried out always assuming for the term structure evolution an additive shift equal to $+0,01\%$. The course of daily

prospective and retrospective tests results in each day of the first five semesters of the life of the hedge is shown in Figures 2 and 3, respectively referred to Hedge 1 and Hedge 2.

Figure 2

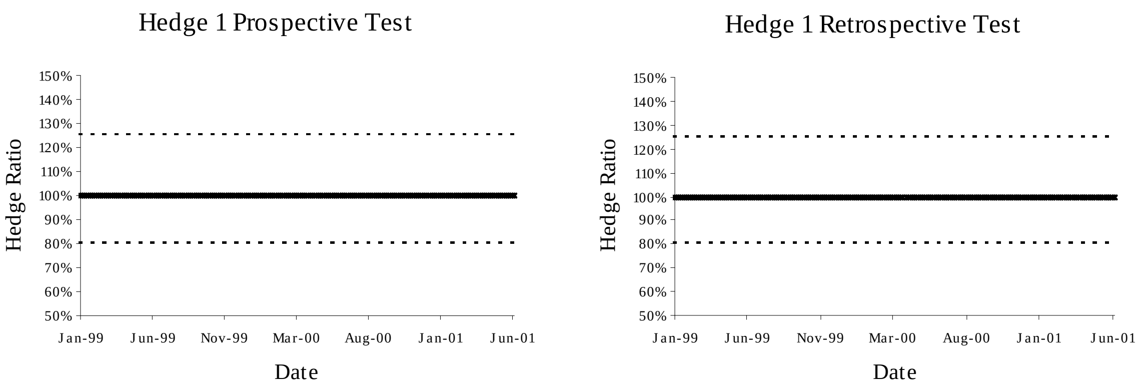
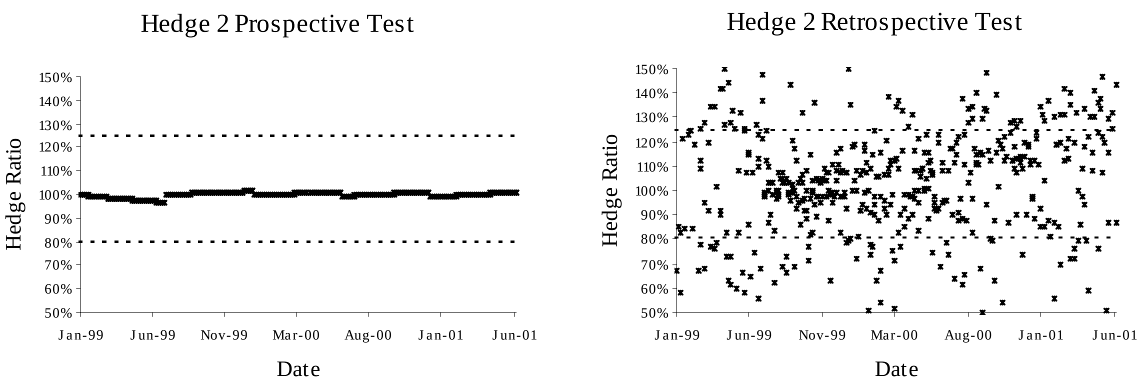


Figure 3



It results that, as expected, Hedge 1 determines "fully effective" prospective and retrospective hedges in each day of the observation period: $E_{t,t^+} = E_{t,t-\Delta t} = 1, \forall t$.

In Hedge 2, the prospective test is "highly effective" in each of the 643 days of the observation period ($0,8 \le E_{t,t^+} \le 1,25 , \forall t$) but, given the non additive observed term structure evolution, the retrospective test is "not effective" 316 times (about once every two days on average). In this case and in the observation period, it results that lower and upper bounds derived at the inception date under the additive shift market assumption are hence relevant for the prospective test but do not allow to control the retrospective hedge ratio.

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