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REINSURANCE, ACTUARIAL CONCEPTS AND FINANCIAL VALUES

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RESUME

Cet article montre comment des concepts actuariels - tels que la probabilité de ruine, l'approximation de puissance normale et la variation structurelle - peuvent être utilisés pour évaluer la valeur financière d'une compagnie d'assurance primaire pour ses propriétaires. Nous examinons en outre comment la réassurance peut effectuer la valeur de compagnies d'assurances dans les cas de deux statuts : société par actions et mutuelle.

REINSURANCE, ACTUARIAL CONCEPTS AND FINANCIAL VALUES

J. H. VON EIJE*

SUMMARY

This paper shows how actuarial concepts (like the probability of ruin, the normal power approximation and structure variation) can be used in assessing the financial value of a primary insurance company to its owners. In addition we will discuss how reinsurance can affect the value of both stock insurers and mutual insurance companies.

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In this paper we will show that actuarial concepts can be used in assessing the value of a primary insurance company. This is done by discussing how reinsurance can influence the value of a primary insurance company to its owners. We start with an analysis of the impacts of limited liability and of the probability of ruin on the value of the stakes of the shareholders of a primary insurance company, We then discuss the discount rate used in valuing the primary insurance company. It is assumed that the shareholders require a compensation for systematic risk and that the price of systematic risk is given. Therefore the rate used to discount future cash flows in the primary insurer increases with systematic risk.

Because the claims of individual insurers are in general positively skewed, we incorporate skewness explicitly into our analysis. This, however, also demands an explicit analysis of the possibility that the market rate of return is skewed and that such market skewness may be priced. This implies that the so-called co-skewness also becomes relevant in discounting future values.

In section 1 we present the value of a primary insurance company to its shareholders. The impact of reinsurance on the market value of the shares is discussed in section 2. Inasmuch as the articles of association of a mutual allow limitations in personal liability of the members, the analysis may also be used for the owners of a mutual. Mutual members are, however, not only the owners, but also the clients. We therefore develop in section 3 an evaluation model for the stakes of the clients of an insurance company. By adding the value of the stakes of the clients and of the owners, we find in section 4 the value of a mutual insurance company to its members. The impact of reinsurance on that value can then be analysed. The conclusions are, finally, presented in section 5.

1. THE MARKET VALUE OF A STOCK INSURANCE COMPANY

We may discern four economic processes in a primary insurance company. In the insurance process, a fixed amount of premiums P is received, while claims S and commission for insurance agents B_p are paid (stochastic variables are indicated with italics). The amount of the premiums is given and will not change, irrespective of the solvency characteristics of the insurance company and irrespective of the use the shareholders make of their rights of limited liability. The total claim amount S is stochastic. In the reinsurance process, premiums J are paid to the reinsurer, while reinsurance claims *K* and reinsurance commission B are received We will frequently speak of "the reinsurer", though we do not exclude situations where more reinsurers are involved in reinsurance arrangements. The providers of the factors of production (in the following sometimes denoted as "employees") receive a fixed amount of income C, which is a cash outflow to the primary insurer. Investment income is finally generated in the investment process by investing the assets risk free ($I=r_f.Q$). The assets Q equal the sum of equity, of ordinary debt and of premium and loss reserves. In this paper it is assumed that the clients do not receive an explicit remuneration on the provision of debt in the form of reserves. Implicitly, the premiums received from the clients are assumed to be relatively small because of the interest income from these reserves. Also the premiums paid to the reinsurer are assumed to be relatively small because the primary insurer provides the reinsurer with technical reserves. The providers of *ordinary* debt will however be entitled to an explicit remuneration. That remuneration will be indicated by $r_o.D_o$; i.e. the ordinary debtholders accept a given rate of return of r_o because the amount of ordinary debt is D_o. During normal operations the cash flows Y to the shareholders *equal* the *sum* of the results of the four economic processes:

$$(1)\quad Y = P - S - B_p - J + K + B - C + I - r_o.D_o$$

where:
P = insurance premiums
S = stochastic gross claim amount
B_p = commission paid
J = reinsurance premiums
K = reinsurance claims received
B = reinsurance commission received
C = operation costs
I = investment income
r_o = the interest rate on ordinary debt
D_o = the amount of ordinary debt

The density function of the cash flows is indicated by $f(Y)$. We calculate the expected cash flow to the shareholders by integrating between $-\infty$ and $+\infty$ though we recognise that the maximal amount of cash flows generated in the insurance company equals $P + I - C - B_p - r_o.D_o$, which is the case with zero claims and no reinsurance cover. We assume that the insurance process is profitable, i.e. that the expected cash flow to the shareholders is positive. The expected *cash* flow to the shareholders in one year in case of complete liability is:

$$(2)\quad E(Y) = \int_{-\infty}^{\infty} Y\, f(Y)\, dY$$

By introducing limited liability we find that the obtained insurance cover may still be risky to the clients of primary insurance companies [Schlesinger and Von der Schulenburg, 19871. The insurer will not in all circumstances fulfil the promises to the policyholders. As long as the losses to the shareholders are smaller than a threshold value $-A^*$, the company may remain solvent and the shareholders will then not use limited liability but they will pay the claims in full. The *end* of period value of the firm to the shareholders V^f_y is then:

$$(3)\quad \begin{aligned} V^f_y &= Y + V_y \qquad &\text{if } Y \geq -A^* \\ V^f_y &= 0 \qquad &\text{if } Y < -A^* \end{aligned}$$

where:
V^f_y = the end of period value of the firm to the shareholders
V_y = the value of the firm to the shareholders at the beginning of the period
A^* = the threshold amount indicating the amount of losses above which the insurer is ruined.

This gives:

$$(4)\quad V_y = \int_{-A*}^{\infty} Y\, f(Y)\, dY + \int_{-A*}^{\infty} V_y\, f(Y)\, dY$$

$$- \int_{-\infty}^{+\infty} Y\, f(Y)\, dY - \int_{-\infty}^{-A*} Y\, f(Y)\, dY + \int_{-A*}^{\infty} V_y\, f(Y)\, dY$$

$$- E(Y) + \Pi.E(UC) + (1 - \Pi).V_y$$

$$- E(Y^*) + (1 - \Pi).V_y$$

where: Π = the probability of ruin
$E(UC)$ = the expected amount of unindemnified claims with ruin
$E(Y^*)$ = the expected cash flow to the shareholders with limited liability

The present value of the firm can now be found by discounting the end of period value by the relevant rate of return $E(r)$.

$$(5)\quad V_y = \{E(Y^*) + (1 - \Pi).V_y\} \;/\; \{1 + E(r)\}$$

$$= E(Y^*) \;/\; \{\Pi + E(r)\}$$

$$= E(Y^*) \;/\; E(r_a)$$

where: $E(r)$ = the rate of return used for discounting cash flows of the company
r_a = the actual rate of return $(Y^*)/V_y$ for which
$E(r_a)$ = $\Pi + E(r)$

Equation 5 indicates the stock market value to depend on expected cash flows of shareholders who use their limited liability rights $E(Y^*)$, on the probability of being ruined Π and on the relevant rate $E(r)$. Related expressions are found amongst others by Scott [1976, p. 38] and von Eije [1989, p. 100]. A linear market relation is here assumed to exist for the rate $E(r)$:

$$(6)\quad E(r) = r_f + p_\beta.\beta + p_\xi.\xi$$

Not only the risk free rate r_f but also the systematic risk β and the co-skewness ξ of the rate of return of the primary insurance company influence the expected rate of retum of the insurer. β is measured by the covariance of the rate of return of the insurance company with the rate of return of the market portfolio and normalised by the variance of the market portfolio (see equation 8). The co-skewness gauges the skewness the shares of the primary insurance company tend to bring to the rate of return on its shareholders' portfolio (see equation 9). The price of systematic risk p_β is generally positive. The sign of the co-skewness price depends on the sign of market skewness. As positively skewed market rates of return are favourable for investors, the co-skewness price will have a sign opposite to the third central moment of the market rate of return [Kraus and Litzenberger, p. 10881 and thus to market skewness. This means that, assuming positively skewed market rates of return, the expected rate of return of a security which shows positive co-skewness will be smaller than the expected rate of return of a security with negative co-skewness. Market skewness is in general positive [Kraus and **Litzenberger,** p. **1097;** Francis and Archer, 1979, p. 365]. **Therefore** p_ξ will be negative. Investors will then be **satisfied** with a smaller expected rate of return of the insurance **company** if the skewness of the rate of return is positively related **to** that of the **positively** skewed market portfolio. For the **market** portfolio both β_m and ξ_m equal 1 and $(E(r_m) - r_f = p_\beta + p_\xi)$ [Kraus and **Litzenberger,** p. **1089].**

The prices for systematic risk and for **co-skewness** presented in equation 6 are not **necessarily** overall market prices. They **should** be considered as prices that are paid for the systematic risk and the co-skewness in the rate of return of the primary insurer considered. We assume that the price for systematic risk p_β and the price for co-skewness pe are given for the relevant interest groups of the primary insurance **company** in question.

Inserting equation 6 in equation 5 gives:

$$(7)\quad V_y = E(Y^*)/(\Pi + r_f + p_\beta.\beta + p_\xi.\xi)$$

$$= \{E(Y^*) - V_y.(p_\beta.\beta + p_\xi.\xi)\}/\{\Pi + r_f\}$$

For the actual rate of return in a year $(r_a = Y^*/V_y)$ we **find** by **definition** for a given stock market value V_y equations 8 and 9:

$$(8)\quad \beta = E[\{r_a - E(r_a)\}.\{r_m - E(r_m)\}]/\mu_{m2}$$

$$= \{1/(V_y.\mu_{m2})\}.E[\{Y^* - E(Y^*)\}.\{r_m - E(r_m)\}]$$

$$= \{1/(V_y.\mu_{m2})\}.Cov(Y^*, r_m)$$

$$(9)\quad \xi = \{1/(V_y.\mu_{m3})\}.E[\{Y^* - E(Y^*)\}\{r_m - E(r_m)\}^2]$$

$$= \{1/(V_y.\mu_{m3})\}.Cov(Y^*, r_m^2)$$

where: μ_{m2} = the second central moment of the market rate of return
μ_{m3} = the third central **moment** of the market rate of return

Substituting equations 8 and 9 in 7 yields:

$$(10)\quad V_y = [E(Y^*) - (p_\beta/\mu_{m2}).Cov(Y^*, r_m) - (p_\xi/\mu_{m3}).Cov(Y^*, r_m^2)]/\{\Pi + r_f\}$$

where: V_y = the stock market value of the primary **insurance company,**
Y^* = the stochastic cash flow **to** the **shareholders who use** limited liability
p_β = the price of systematic risk
p_ξ = **the price of co-skewness**
μ_{m2} = the second central moment of the market rate of return

μ_{m3} = the third central moment of the market rate of return
r_m = the stochastic rate of return on the market portfolio
Π = the probability of ruin
r_f = the risk free rate of return

Equation 10 can be useful in analysing the impact of reinsurance cover. The equation shows some desirable characteristics in comparison with other evaluating measures. The first improvement is that primary insurance company managers do not have to make explicit statements on utility. Another positive aspect of the stock market value criterion is that it is related to the overall perfamance of the primary insurer. Traditional reinsurance theory analyses reinsurance cover per line of business. In order to evaluate the impact of reinsurance, it is useful to know what the combined effect is of reinsurance cover obtained in all non-life lines. In addition to the insurance process and the reinsurance process, the production and investment processes should also be taken into account [Farny, 1984; Daykin e.a., 1987]. This was done by defining the stochastic cash flows according to equation 1. The third improvement is that the equation not only incorporates systematic risk and co-skewness but also the probability of ruin

2. REINSURANCE AND THE VALUE OF THE COMPANY TO THE SHAREHOLDERS

Given equation 10, reinsurance may affect the expected cash flow to the shareholders, both covariance terms and the probability of ruin. The impact of reinsurance on these variables will be discussed in the following subsections.

1. Reinsurance and expected cash flows

By taking the expectations of equation 1 we find:

$$\text{(11)} \quad E(Y) = P - E(S) - B_p - J + E(K) + B - C + I - r_o.D_o$$

The difference between the claims of the insurance process and those of the reinsurance process are the net claims Z ($Z = S - K$). $E(Y^*)$ of equation 4 can thus be rewritten as:

$$\text{(12)} \quad E(Y^*) = E(Y) + \Pi.E(UC)$$
$$= P - E(S) - B_p - J + E(K) + B - C + I - r_o.D_o + \Pi.E(UC)$$
$$= P - B_p - J + B - C + I - r_o.D_o - E(Z) + \Pi.E(UC)$$

It is common practice that reinsurance companies use a mark up on expected reinsurance claims. The absolute value of reinsurance premiums J minus reinsurance commission B is then higher than expected reinsurance claims $E(K)$. The direct impact of reinsurance during namal operations is thus a reduction of the expected cash flows which equals $E(K) + B - J$.

In the preceding sections we used the assumption that shareholders might use their rights of limited **liability.** The question of how **reinsurance** affects the expected cash flows to shareholders using these rights is thus important As we assume that the reinsurer will pay all claims which can be recovered under the reinsurance agreement, the mark up of the reinsurer $J - B - E(K)$ will not be affected by limited liability. The direct impact of reinsurance on expected claims of a company does not differ between shareholders who use and shareholders who do not use limited liability. The negative impact is thus still equal to $E(K) + B - J$ as found under equation 12. Reinsurance may however also influence the second term after the first "=" sign, i.e. $\Pi.E(UC)$. In general these expected unindemnified claims will be reduced by reinsurance: firstly because reinsurance cover is very likely to cause a reduction in the probability of ruin Π and, secondly, because it is plausible that the cover will also reduce the expected unindemnified amount of claims with bankruptcy $E(UC)$. Reinsurance cover thus protects the interests of the policyholders, while it reduces at the same time the possibility for the shareholders of using limited liability. The conclusion is that the effect of reinsurance on expected cash flows is negative for the shareholders.

2. Reinsurance and covariance terms

Equation 10 showed that the covariance of the cash flows with the market portfolio is relevant in calculating the stock market value of a company. Here a relationship is assumed between the rate of return on the market portfolio and the expected number of yearly claims. The actual number of claims is Poisson distributed. Because of interactions with the business cycle (which cause a correlation with the *market* rate of return), the parameter, which characterises the Poisson distribution, variates. The central density parameter n – which indicates the expected number of claims of the Poisson distribution – is thus changing over the years. This is called structure variation [Beard, Pentikäinen and Pesonen, 1984, p. 32)]. Such structure variation can be presented as follows:

$$(13) \quad n = n_0 . q$$

where:

where:	n	=	the stochastic central density parameter for the number of claims of the Poisson distribution in one year (expected number of claims in one year)
	n_0	=	the average number of claims over a long period of time
	q	=	the (stochastic) structure variation variable.

Equation 13 indicates that the central density parameter for the number of claims underlying the Poisson distribution is nu necessarily the same every year. On average during a long period of time there are a number of n_0 claims. We assume the structure variation variable q to be normally distributed $N(1, \sigma_q)$. The expected number of claims in one year used as the central density measure n of the Poisson distribution is thus a random, normally distributed, variable $N(n_0, n_0.\sigma_q)$.

The two covariance terms of equation 10 yield f a given P, B_p, J, B, C, I, $r_0.D_0$ and A^*:

$$(14) \quad \mathrm{Cov}(Y^*, r_m) = E[\{Y^* - E(Y^*)\}.\{r_m - E(r_m)\}]$$

$$= - \mathrm{E}[\{Z^* - \mathrm{E}(Z^*)\}.\{r_m - \mathrm{E}(r_m)\}]$$
$$= - \mathrm{Cov}(Z^*, r_m^2)$$

$$(15) \quad \mathrm{Cov}(Y^*, r_m^2) = \mathrm{E}[\{Y^* - \mathrm{E}(Y^*)\}.\{r_m - \mathrm{E}(r_m)\}^2]$$
$$= - \mathrm{E}[\{Z^* - \mathrm{E}(Z^*)\}.\{r_m - \mathrm{E}(r_m)\}^2]$$
$$= - \mathrm{Cov}(Z^*, r_m^2)$$

where: Z^* = the net claim amount of a reinsured insurer whose shareholders use their limited liability rights.

Through the years, a relation may exist between the market rate of return and the aggregate amount of net claims Z^*. We assume that this is not a relation between the claim size and the market rate of return, but between the central density parameter n –indicating the expected number of claims per year –and the business cycle (which may be correlated with the market rate of return). In fact we assume:

$$(16) \quad \mathrm{Cov}(Z^*, r_m) = \alpha_z.\mathrm{Cov}(n, r_m)$$

and

$$(17) \quad \mathrm{Cov}(Z^*, r_m^2) = \alpha_z.\mathrm{Cov}(n, r_m^2)$$

where: α_Z indicates the average retained (net) claim size.

The covariance of the net claim amount Z^* with the market rate of return is thus connected with the covariance of the expected number of claims in one year and the market rate of return. It will be higher if the average retained claim size increases. We now assume that the reinsurance process does not affect the number of claims, but only the moments of the net claim size distribution. As reinsurance reduces the average retained claim size α_Z, the absolute value of the covariance of the claim amount with the market rate of return will diminish. By substituting equation 16 in 14 and 17 in 15 we find:

$$(18) \quad \mathrm{Cov}(Y^*, r_m) = - \alpha_z.\mathrm{Cov}(n, r_m)$$
$$(19) \quad \mathrm{Cov}(Y^*, r_m^2) = - \alpha_z.\mathrm{Cov}(n, r_m^2)$$

As reinsurance reduces a_r, an increase in reinsurance cover will thus affect both covariance terms of shareholders' cash flows positively if the covariance terms of the expected number of claims in one year with the (squared) market rate of return are positive. Given a positive price of systematic risk and assuming for the moment the price of co-skewness to be zero implies that reinsurance increases the stock market value if there is a negative correlation between the expected number of claims **n** in one year and the rate of **return** *on* the market portfolio r_m. **This** can be **concluded** from equations 18 and 10. If the central density parameter for the number of claims is positively **correlated** with the market rate of return, reinsurance **should** not be sought. Of *course* such notions have to be reevaluated if **co-skewness** is also taken into account.

3. Reinsurance and the probability of ruin

If the *cash* flows to the **shareholders** presented in equation 1 are **distributed normally,** the distribution function can be **characterised** by the **first** two moments. We then **analyse** the average results μ_Y and the standard **deviation** of these results σ_Y. The distance between the average results and a complete loss of the threshold amount A* can then be expressed in terms of the number of standard deviations. We find a solvency measure Φ for the *insurance* **company:**

$$(20)\quad \Phi = (A^* + \mu_Y)/\sigma_Y$$

where: Φ = a solvency measure based *on* the **first** two moments (my and sy) of the *cash* flows and
A* = the threshold amount used by the shareholders

It may be noticed that the probability of ruin is calculated for a company that is operating normally, indicating that **ruin** can only be caused by the **random occurrences** during the time the primary insurer is not ruined. Therefore calculations of the occurrence of ruin should be based on the characteristics of the **company** during normal **operations.** In **particular,** expected income should not be revised for profits expected to originate out of limited liability.

If Φ is 1, the probability of ruin will be **0.1587.** Because we consider annual results, one sixth of the insurers will then be **ruined** each **year.** In practice insurers are more **solid,** implying Φ is > 1. **This** is a convenient observation if the **first** three moments of the **distribution** of results are relevant. In such a case the normal power approximation [Beard, **Pentikäinen** and **Pesonen,** 1984, p. **108** ff] can be **used.** When Φ is greater than 1 we can calculate a solvency measure Ω as:

$$(21)\quad \Omega = -(3/\gamma_Y).[\sqrt{\{\gamma_Y^2/9 - (2/3).\gamma_Y.\Phi + 1\}} - 1]$$

$$= -(3/\gamma_Y).(\sqrt{\Psi} - 1)$$

where: $\gamma_Y = \mu_{Y3}/\sigma_Y^3$,
$\Psi = \gamma_Y^2/9 - (2/3).\gamma_Y.\Phi + 1$ and
Φ = defined in equation 20.

In general, the claim distribution of primary insurance companies will be positively skewed, **i.e.** $\gamma_Z = \mu_{Z3}/\sigma_Z^3$ is positive. The cash flows to **shareholders** will then be negatively skewed and γ_Y is negative ($\gamma_Y = -\gamma_Z$). As Φ is in **general** positive, the value of ψ is positive **and** greater than **1, implying** that Ω will also be positive.

Equation 21 indicates that the more negative the third central moment of Y, the smaller the solvency measure Ω. The more positively skewed the claim distributions are, the more the solvency of the primary insurer reduces [see also Beard, Pentikäinen and Pesonen, 1984, Figure 3.11.2, p. 118]. Equations 20 and 21 also show that Ω depends on $\mu_Y, \sigma_Y, \gamma_Y$ and A*. The value of A* is given. The impact of reinsurance on μ_Y, σ_Y and γ_Y can be calculated from the impact of reinsurance on the first three moments of Z. These moments can be derived by using the formulae of Beard, Pentikäinen and Pesonen [1984, p. 54] for the aggregate (after reinsurance) claim distribution in case of a normally distributed structure variation variable.

$$(22) \quad \mu_Z = n_0.\alpha_z$$

$$(23) \quad \mu_{Z2} = n_0.\alpha_{z2} + (n_0.\alpha_z.\sigma_q)^2$$

$$(24) \quad \mu_{Z3} = n_0.\alpha_{z3} + 3n_0^2.\alpha_z.\alpha_{z2}.\sigma_q^2$$

where: μ_Z, μ_{Z2} and μ_{Z3} are the mean and the second and third central moments of the distribution of the retained claim amounts respectively and α_Z, α_{Z2} and α_{Z3} are the mean and the *second* and third raw moments of the net claim size distribution.

Equations 22 - 24 thus represent the first three characteristics of the distribution of the' net claim amounts. These characteristics depend on the average number of claims of the Poisson distribution over a long period n_0, on the variance σ_q^2 of the structure variation variable and on the first three moments of the net claim size distribution As already indicated, the expected number of claims in one year is assumed to be unaffected by reinsurance cover. Reinsurance influences - depending on the form and conditions - α_Z, α_{Z2} and α_{Z3}. Equations 22, 23 and 24 show that reinsurance thus also influences μ_Z, μ_{Z2} and μ_{Z3}. We can rewrite equations 22, 23 and 24 and we find the following equations:

$$(25) \quad \mu_Y = E(Y) = P - B_P - J + B - C + I - r_o.D_o - E(Z)$$
$$= P - B_P - J + B - C + I - r_o.D_o - \mu_Z$$
$$= P - B_P - J + B - C + I - r_o.D_o - n_0.\alpha_z$$

$$(26) \quad \mu_{Y2} = \mu_{Z2} = n_0.\alpha_{z2} + (n_0.\alpha_z.\sigma_q)^2$$

$$(27) \quad \mu_{Y3} = -\mu_{Z3} = -(n_0.\alpha_{z3} + 3n_0^2.\alpha_z.\alpha_{z2}.\sigma_q^2)$$

We can now distinguish the values for the first three months of the cash flow distribution of equation 25 - 27 in two situations. One is the situation of an insurer whose shareholders use limited liability and the other the situation when the shareholders do not use that facility. The characteristics of the claims are of course changed by ruin; however, ruin originates from the original characteristics of the claims. Therefore, a correct calculation of the moments necessary for calculating the probability of ruin must be based on the value of the latter. This implies that we can use equations 25 - 27 directly in calculating the impact of reinsurance on the probability of ruin.

As reinsurance premiums J exceed expected reinsurance claims $E(K)$, μ_Y will decrease. This will according to equations 20 and 21 diminish Ω and solvency. In general, this negative impact will be surpassed by the reduction of the absolute values of σ_Y and γ_Y through reinsurance cover. If this is effectuated reinsurance improves solvency, which results in a protracted period of receiving dividends implying - ceteris paribus - an increase in the stock market value of the insurance company to the shareholders.

3. REINSURANCE AND THE VALUE OF THE COMPANY TO THE OTHER STAKEHOLDERS

Until now we have studied the value of reinsurance to the shareholders. Other stakeholder groups may also be affected by reinsurance. These are the groups of the clients, of the providers of production factors, of the reinsurers, of the insurance agents and of the ordinary debtholders. It is still assumed that future cash flow distributions do not change. In addition, all the stakeholders have the same beliefs on future cash flows. In section 3.1 we go into the value of the primary insurance company to all the stakeholders taken together. In Section 3.2 we discuss the value which the reinsurer receives from the company. In section 3.3 the value of the interests of the employees, of the insurance agents and of the ordinary debtholders is derived. The value of the primary insurance company to the clients is finally analysed in section 3.4.

1. The value of *the* interests of *all the* stakeholders together

We will now discuss the value accruing to all the stakeholder groups. The following relations are found during normal operations.

1) the shareholders receive Y
2) the clients receive S - P
3) the providers of production factors receive C
4) the reinsurers receive J - K - B
5) the insurance agents receive B_p
6) the ordinary debtholders receive $r_o.D_o$

All the stakeholders together thus receive during normal operations : $Y + S - P + C + J - K - B + B_p + r_o.D_o$. Because Y is according to equation 1 defined as : $P - S - B_p - J + K + B - C + I - r_o.D_o$, the stakeholders together in fact receive : I. This is the investment income gathered from the risk free investments of assets Q. During normal operations the investment income will flow to the primary insurance company.

If the primary insurer is ruined, the stakeholders will distribute the assets amongst each other. This can be the total value of the company, but also less. The total value will be transferred, if there are no ruining costs. The stakeholders will receive less if bankruptcy costs, for example fees to the trustee, exist. The amount of the direct bankruptcy costs may be relatively small [Warner, 1977], though the indirect costs caused by difficulties of operating the company during the receivership phase may be high [Baxter, 1967]. Considering the insurance company at hand, the bankruptcy costs will at least be zero and at most be equal to the sum of investment income I and the company value at the beginning of the period V. At the end of the period the company value to all the stakeholders together V^f equals:

$$(28)\quad \begin{aligned} V^f &= I + V && \text{if } Y \geq -A^* \\ V^f &= \lambda_I . I + \lambda_V . V && \text{if } Y < -A^* \end{aligned}$$

where:	V	=	the value of the company at the beginning of the period
	V^f	=	the value of the company at the end of the period
	I	=	the investment income
	λ_I	=	the remaining fraction of investment income after bankruptcy
		=	the remaining fraction of firm value after bankruptcy
	Y	=	the stochastic cash flow to the shareholders
	A^*	=	the ruining threshold used by the owners

The present value of the company to all the stakeholders can be found by discounting the expected end of period value. Because all the stakeholders together receive investment income, which originates from investments in risk free assets, the relevant discount rate for the stakeholders together is the risk free rate. The discounted expected value of the company at the beginning of the period for a given ruining threshold A* then yields:

$$(29)\quad V = \{\int_{-A^*}^{\infty} (I + V)\ f(Y)\ dY\}/(1 + r_f) + \{\int_{-\infty}^{-A^*} (\lambda_I . I + \lambda_V . V)\ f(Y)\ dY\}/(1 + r_f)$$

or

$$(30)\quad V = \{(I + V).(1 - \Pi) + \Pi.(\lambda_I . I + \lambda_V . V)\}/\{1 + r_f)\}$$

which gives:

$$(31)\quad V = \{I.(1 - \Pi + \Pi.\lambda_I)\}/\{r_f + \Pi.(1 - \lambda_V)\}$$

If $\lambda_I = \lambda_V = 1$, then $V = I/r_f$: in the event of bankruptcy the stakeholders lose no value and the value of the firm equals the discounted present value of investment income. In such a case we can think of the transfer of the company to new owners, without any loss to the old stakeholder groups together: no fees have to be paid to the official receiver, to lawyers and accountants and no administration and court costs must be paid The value of the company to the ex- stakeholders is then equal to the assets Q (i.e. V = Q). If, however, the value of the firm is reduced by bankruptcy costs, the value of the firm to the stakeholder groups diminishes. Equation 31 indicates that the value of the firm to all the stakeholders taken together reduces if the probability of ruin increases and if the parameters λ_I and A„ diminish In particular a high probability of ruin and a small value of λ_V will quickly reduce the value of the company.

If the relevant stakeholders will be able to retain exactly the value of the primary insurance company at the beginning of the period, a high probability of ruin will not harm the combined stakeholders tremendously. This is the case where $\lambda_I = 0$ and A„ = 1. In such a situation, only investment income is at risk with ruin. If the bankruptcy costs exactly equal the value of the firm at the beginning of the period ($\lambda_I = 1$ and $\lambda_V = 0$) then the value of V equals the investment income I discounted by $r_f+\Pi$. A high probability of ruin will then hurt the stakeholders taken together more. If the bankruptcy costs are higher ($\lambda_I = 0$ and A„ = 0), then there is no value left to all the stakeholders after ruin. The bankruptcy costs then equal V + I and the value of the firm at the beginning of the period becomes $I.(1-\Pi)/(r_f+\Pi)$. With bankruptcy costs, the value of the firm to all the stakeholders diminishes if the probability of ruin increases. Then also the other stakeholders may be interested in a small probability of ruin.

2. The value of the interests of the reinsurer

The value of reinsurance operations to the reinsurer can be calculated in a similar way. It is assumed that bankruptcy will be caused by a huge amount of claims. Assuming that the reinsurer is involved in these claims, the reinsurer will then face a loss, which - by assumption - cannot be reduced by a limitation of the liability of a perfectly solvent reinsurer. After bankruptcy, the reinsurer will still be able to reap the reinsurance premiums, because these can be subtracted from the higher amount of the sum of reinsurance claims and reinsurance commission. The reinsurer will therefore always face a net cash flow equal to J - K - B, which in the ruination period will be negative. With ruin, the future cash flows - originating from reinsuring this particular primary insurer - will cease to exist for the reinsurer. If the shareholders use their rights of limited liability, the reinsurer will lose his stake in the company. Moreover, in case of ruin the reinsurer will not be entitled to any of the possible remaining company value (which is $\lambda_I.I + \lambda_V.V$), because the loss of reinsurance value is only a loss based on future profits and not on debt provided by the reinsurer in the past The end of period value of the reinsurer V^f_h is:

$$
(32) \quad \begin{array}{ll} V^f_h = J - K - B + V_h & \text{for } Y \geq -A^* \\ V^f_h = J - K - B & \text{for } Y < -A^* \end{array}
$$

where:

V^f_h = the end of the period value of the cash flows to the reinsurer
V_h = the present value of the cash flows to the reinsurer
J = the reinsurance premiums received by the reinsurer
K = the reinsurance claims paid by the reinsurer
B = the commission paid by the reinsurer

We now find by calculating the present value based on the expected rate of return to the reinsurer $E(r_h)$:

(33) $$V_h = \{\int_{-\infty}^{\infty} (J - K - B)\, f(Y)\, dY\}/\{1 + E(r_h)\}$$

$$+ \{\int_{-A^*}^{\infty} V_h\, f(Y)\, dY\}/\{1 + E(r_h)\}$$

or

(34) $$V_h = \{E(J - K - B) + V_h.(1 - \Pi)\}/\{1 + E(r_h)\}$$

$$= E(J - K - B)/\{\Pi + E(r_h)\}$$

$$= E(J - K - B)/E(r_{ah})$$

where: r_{ah} is the actual rate of return on the reinsurer's present value defined as $(J - K - B)/V_h$, for which $E(r_{ah}) = \Pi + E(r_h)$

The expected rate of return of the reinsurer used for discounting $E(r_h)$ is not the risk free rate, because the reinsurer may accept systematic risk and co-skewness from the primary insurance company (see the discussion in subsection 2.2). This implies that a market relation also exists for the reinsurer. This market relation is by assumption:

(35) $$E(r_h) = r_f + p_\beta.\beta_h + p_\xi.\xi_h$$

where:

$\beta_h = Cov(r_{ah}, r_m)/\mu_{m2} = -Cov(K, r_m)/(V_h.\mu_{m2}) = -\alpha_k.Cov(n, r_m)/(V_h.\mu_{m2})$
$\xi_h = Cov(r_{ah}, r_m^2)/\mu_{m3} = -Cov(K, r_m^2)/(V_h.\mu_{m3}) = -\alpha_k.Cov(n, r_m^2)/(V_h.\mu_{m3})$
α_k = the average amount paid out by the reinsurer per claim insured by the primary insurance company

After substitution of the relations above in equation 34 we find:

(36) $$V_h = \{J - E(K) - B + (p_\beta/\mu_{m2}).\alpha_k.Cov(n, r_m) + (p_\xi/\mu_{m3}).\alpha_k.Cov(n, r_m^2)\}/(\Pi + r_f)$$

According to equation 36 the value of the interest of the reinsurer in the primary insurance company depends on the mark up amount levied by the reinsurer $(J - E(K) - B)$, on the covariance of the expected number of primary insurance company claims in a year with the market rate of return and with the squared market rate of return $\{Cov(n, r_m)$ and $Cov(n, r_m^2)\}$, on the average amount paid by the reinsurer per claim faced by the primary insurance company α_k, on the probability that shareholders let the insurer go bankrupt Π and on the market parameters p_β, p_ξ, μ_{m2}, μ_{m3}, and r_f. Disregarding systematic risk and co-skewness and assuming a perfectly solvent reinsurer implies that the value of the stakes of the reinsurer increases with a reduction in the probability of ruin of the primary insurance company. This can generally be brought about by additional reinsurance cover, which moreover creates a higher absolute mark-up-amount. In principle the *sakes* of the reinsurer will be limited by the **amount** of value available in the primary insurer and by the fact **that the** owners will not be inclined to cede too much The owners will **not** provide the reinsurer with additional **value** *at* the *expense* of **their** own.

3. The value of the interests of the employees, the insurance agents and the ordinary debtholders

The effects of **ruin** to the shareholders, **the** reinsurer and all the **stakeholders together** were discussed in the preceding **(sub)sections.** By introducing additional assumptions, more specific results will be **found.** We now assume that **the providers** of the **factors** of production, the insurance agents and the *ordinary* **debtholders** receive their **income** directly at the beginning of the **period.** If we hold the *end of* **period** value *of* income **for** these interest groups at C, B_p and $r_o.D_o$ respectively, these groups in fact receive the **discounted** cash flows at the **beginning** of **the** period. **Because** the cash flows are **free** of systematic risk, the present value of these cash flows are $C/(1+r_f)$, $B_p/(1+r_f)$ **and** $r_o.D_o/(1+r_f)$. To the **providers** of the **factors** of production and to the insurance agents no **income loss will** result in **the ruination** period. Because **these** groups will **not** receive any future income, the value *of* that income **will be zero** at **the** end of the ruination period. We then find for the ***providers of the factors of production (employees):***

$$
\begin{array}{lll}
(37) & V^f_e = C + V_e & \text{for } Y \geq -A^* \\
 & V^f_e = C & \text{for } Y < -A^*
\end{array}
$$

where:
V^f_e = the value of the **cash** flows to **the** employees at the end of **the** period
V_e = the value to that **group** at **the** beginning of the **period**
C = the *costs* of the use of labour **and** physical capital to the primary **insurance** company

After discounting with the riskless rate due *to* the **certainty** of **the** cash flows, we find:

$$
\begin{aligned}
(38)\quad V_e &= \{(C + V_e).(1 - \Pi) + \Pi.C\}/(1+r_f) \\
&= C/(\Pi + r_f)
\end{aligned}
$$

The *same* reasoning gives the value for the ***insurance* agents:**

$$
(39)\quad V_b = B_p/(\Pi + r_f)
$$

where:
V_b = the value to the **insurance agents** *at* the **beginning** of the **period**
B_p = the commission received by the agents

Equations 38 and 39 show that a smaller probability of **ruin** will increase the value of the stake of the employees as well as the value of that of the insurance agents. As **reinsurance does** not affect the cash flows to these interest **groups, additional cover** will in general **improve** their stakes.

The value of the stake of the *ordinary debtholders* can also be calculated in the same manner if we assume that the value of the remaining ordinary debt is zero after ruin. Because of priority arrangements in most countries, a refund to debtholders will only take place if the clients of the primary insurer are completely remunerated. If there are bankruptcy costs, these will mostly also be subtracted from the remaining stake of the ordinary debtholders in the event of ruin. It is therefore assumed that after ruin the ordinary debtholders are not repaid their investments. The value of the interests of the ordinary debtholders V_o can then be calculated in the same manner as that of the employees and the insurance agents. It is:

$$(40)\quad V_o = r_o.D_o/(\Pi + r_f)$$

where:
- V_o = the value of the stake of the ordinary debtholders at the beginning of a period
- r_o = the interest rate to be paid by the primary insurance company on ordinary debt
- D_o = the amount of debt invested in the primary insurance company by the ordinary debtholders.

If the interest rate used by the debtholders incorporates the losses after ruin completely (i.e. if $r_o = \Pi + r_f$), the value of the stake of the debtholders derived from the expected future cash flows exactly equals the amount invested by the ordinary debtholders (i.e. $V_o = D_o$): reinsurance will not change the value to the ordinary debtholders if its impact on the probability of ruin will be accounted for in the interest rate used by the debtholders. For a given exogeneously determined rate of interest used by the debtholders, however, a reduction in the probability of ruin increases the value of the stake of the debtholders.

4. The value of the interests of the clients

The value of the interests of the clients can now be calculated. It is the value of the interests of all the stakeholders minus the value of the stakes of the shareholders, of the reinsurer, of the providers of the production factors, of the agents and of the providers of ordinary debt. We thus find by using the definition of $E(Y^*)$ in equation 4 and equations 1, 10, 18, 19, 31, 36, 38, 39 and 40 the present value of the stakes of the clients V_g:

$$\begin{aligned}(41)\quad V_g = {} & \{I.(1 - \Pi + \Pi.\lambda_I)\}/\{r_f + \Pi.(1 - \lambda_V)\} \\ & - E(Y^*)/(\Pi + r_f) \\ & - \{(p_\beta/\mu_{m2}).\alpha_z.Cov(n,r_m) + (p_\xi/\mu_{m3}).\alpha_z.Cov(n,r_m^2)\}/(\Pi + r_f) \\ & - \{J - E(K) - B\}/(\Pi + r_f) \\ & - \{(p_\beta/\mu_{m2}).\alpha_k.Cov(n,r_m) + (p_\xi/\mu_{m3}).\alpha_k.Cov(n,r_m^2)\}/(\Pi + r_f) \\ & - C/(\Pi + r_f) \\ & - B_p/(\Pi + r_f) \\ & - r_o.D_o/(\Pi + r_f)\end{aligned}$$

The sum of the expected amount of one claim retained by the primary insurer α_z added to the expected amount regained by reinsurance cover per claim α_k equals the gross expected amount of a claim of the clients α_s (i.e. $\alpha_s = \alpha_z + \alpha_k$). Therefore, equatian 41 can be rewritten as:

$$
\begin{aligned}
(42)\quad V_g = {} & \{I.(1 - \Pi + \Pi.\lambda_I)\}/\{r_f + \Pi.(1 - \lambda_V)\} \\
& - \{E(Y^*) + J - E(K) - B + C + B_p + r_o.D_o\}/(\Pi + r_f) \\
& - \{(p_\beta/\mu_{m2}).\alpha_s.\mathrm{Cov}(n,r_m)\}/(\Pi + r_f) \\
& - \{(p_\xi/\mu_{m3}).\alpha_s.\mathrm{Cov}(n,r_m^2)\}/(\Pi + r_f)
\end{aligned}
$$

Explicitly writing $E(Y^*)$ yields by using equation 12:

$$
\begin{aligned}
(43)\quad V_g = {} & \{I.(1 - \Pi + \Pi.\lambda_I)\}/\{r_f + \Pi.(1 - \lambda_V)\} \\
& - \{P - E(S) + I + \Pi.E(UC)\}/(\Pi + r_f) \\
& - \{(p_\beta/\mu_{m2}).\alpha_s.\mathrm{Cov}(n,r_m)\}/(\Pi + r_f) \\
& - \{(p_\xi/\mu_{m3}).\alpha_s.\mathrm{Cov}(n,r_m^2)\}/(\Pi + r_f)
\end{aligned}
$$

where:

where:	V_g	=	the present value which clients derive from their transactions with the primary insurance company
	I	=	the investment income
	Π	=	the probability of ruin
	λ_I	=	the remaining fraction of investment income with bankruptcy
	λ_V	=	the remaining fraction of firm value with bankruptcy
	r_f	=	the risk free rate
	P	=	the gross premiums paid by the clients
	S	=	the stochastic total claim amount received by the clients
	$E(UC)$	=	the expected amount of unindemnified claims with ruin
	p_β	=	the price of systematic risk
	p_ξ	=	the price of co-skewness
	μ_{m2}	=	the second central moment of the market rate of return
	μ_{m3}	=	the third central moment of a market rate of return
	α_s	=	the gross expected amount of a claim of the clients
	n	=	the stochastic central density parameter of the Poisson distribution which indicates the expected number of claims in one year
	r_m	=	the stochastic rate of return on the market portfolio

Equation 43 simplifies if we take λ_V to equal 0 and at the same time λ_I to equal 1:

$$
\begin{aligned}
(44)\quad V_g = {} & \{- P + E(S)\}/(\Pi + r_f) \\
& - \{\Pi.E(UC)\}/(\Pi + r_f) \\
& - \{(p_\beta/\mu_{m2}).\alpha_s.\mathrm{Cov}(n,r_m)\}/(\Pi + r_f) \\
& - \{(p_\xi/\mu_{m3}).\alpha_s.\mathrm{Cov}(n,r_m^2)\}/(\Pi + r_f)
\end{aligned}
$$

The *first term* of equation *44* denotes the expected cash flow during normal operations $\{-P + E(S)\}$ divided by the summation of the risk free rate and the probability of ruin. Most of the time premiums will be higher than the expected total claim amount. If this is the case, the first term shows that a reduction in the probability of ruin can have a negative impact on the clients. The negative cash flow is then expected to continue during a longer period of time. Clients will then - solely according to the first term - not be interested in additional reinsurance cover. Premiums can also be smaller than the expected claims. This is the case if premiums are reduced by implicit interest payments based on high amounts of premium and loss reserves. An improvement of solvency caused by reinsurance will then - according to the first term - be positively evaluated by the clients.

The ***second term*** of equation ***44*** indicates that the value of the stakes of the clients increases if the expected amount of unindemnified claims $E(UC)$ as well as the probability of ruin Π diminish. This can be seen if we use the equivalent term $\{-E(UC)/(1 + r_f/\Pi)\}$. The numerator $-E(UC)$ will in general become less negative if additional reinsurance cover is obtained. Because reinsurance is likely to reduce the probability of ruin, the denominator will increase. The resulting reduction of the absolute value of the second term is probably the most important reason f a solvency regulations. If such regulation aims at a maximisation of the stakes of the clients, it is not necessarily the best regulation, because it should then also take the first, third and fourth terms into account.

The third *term* shows that *the* present value of the stakes of the clients increases if there is a negative relation between the expected number of claims in *one* year and the market rate of return : systematic risk is not appreciated by the clients either. They prefer the payment of claims in a bear-market above the remuneration of losses in a bull-market. If the price of co-skewness is negative in the ***fowth term,*** the value of the clients' interests increases if the covariance of the expected number of claims in *one* year with the squared market rate of return is positive.

In comparison with equation 44, equation 43 has a different value of λ_v. If there is some value of the company left with bankruptcy ($\lambda_v > 0$), ruin does not affect the clients as seriously as indicated in equation 44. The ruin reducing effect of reinsurance cover - if it affected V_g positively in equation 44 - will then be less valuable. If however, not only $\lambda_v = 0$ (as it is in equation 44), but also $\lambda_I < 1$ then reinsurance will be more interesting to the clients because the loss with bankruptcy is higher.

Reinsurance cover *can* only influence the probability of ruin and the expected amount of unindemnified claims. But it can not change the gross expected amount of a claim *of* the clients (see equations 43 and 44). Whether the impact of reinsurance is positive for the clients cannot be indicated without further knowledge of the relevant cash flows and parameters. Even for the clients no simple unidirectional answer *on* the usefulness of reinsurance cover *can* be given.

4. REINSURANCE AND THE VALUE TO THE MEMBERS OF A MUTUAL INSURANCE COMPANY

Within the same framework, the value of a mutual insurance company to its members can be calculated. In principle three groups of mutual insurance companies exist: 1) mutuals whose members are not liable to any of the occurring losses, 2) mutuals where the members are forced by regulations to replenish losses up to a certain amount and 3) mutuals in which the members are completely liable to all originating losses.

We only consider mutual insurance companies, where the members are not completely liable to the losses, i.e. groups 1 and 2. The members are the owners of a mutual insurance company and in that respect they are - by assumption - in the same situation as the shareholders who use their right of limited liability. This implies that we disregard efficiency enhancing effects of mutualization [Mayers and Smith, 1986]: the costs of the potential conflict between the interests of the shareholders and the clients are discarded. This is not to say that the cash flows in mutuals equal those of stock insurance companies [Spiller, 1972], but only that they do not differ in formulae form.

The members of a mutual do not necessarily use the same threshold as the shareholders. The amount at which the majority of the members of a mutual stops to remunerate the claims of their less lucky fellow members may differ from that of the shareholders. Because we use a general, exogeneous ruining threshold A*, this does not affect the following formulae. It is also irrelevant to the formulae whether the members of the mutual are forced to apply at least the threshold A* (group 2 mutuals) or not (group 1 mutuals).

Because members are at the same time clients of the primary insurance canpany, they will also receive the value of the stakes of the clients. Therefore the value to the members of a mutual is found by adding the value of the stakes of the shareholders according to equation 10 (by using a ruining threshold A*) and the value of the stakes of the clients according to equation 43. This yields:

$$
\begin{aligned}
(45)\quad V_{me} = {} & \{E(Y^*) - (p_\beta/\mu_{m2}).\mathrm{Cov}(Y^*,r_m) - (p_\xi/\mu_{m3}).\mathrm{Cov}(Y^*,r_m{}^2)\}/(\Pi + r_f) \\
& + \{I.(1 - \Pi + \Pi.\lambda_I)\}/\{r_f + \Pi.(1 - \lambda_V)\} \\
& - \{P - E(S) + I + \Pi.E(UC)\}/(\Pi + r_f) \\
& - \{(p_\beta/\mu_{m2}).\alpha_s.\mathrm{Cov}(n,r_m)\}/(\Pi + r_f) \\
& - \{(p_\xi/\mu_{m3}).\alpha_s.\mathrm{Cov}(n,r_m{}^2)\}/(\Pi + r_f)
\end{aligned}
$$

This equation simplifies according to the definition of the cash flows to the shareholders presented in equation 1 in:

$$
\begin{aligned}
(46)\quad V_{me} = {} & \{I.(1 - \Pi + \Pi.\lambda_I)\}/\{r_f + \Pi.(1 - \lambda_V)\} \\
& \{- B_p - C - J + E(K) + B - r_o.D_o\}/(\Pi + r_f) \\
& - \{(p_\beta/\mu_{m2}).\alpha_k.\mathrm{Cov}(n,r_m)\}/(\Pi + r_f) \\
& - \{(p_\xi/\mu_{m3}).\alpha_k.\mathrm{Cov}(n,r_m{}^2)\}/(\Pi + r_f)
\end{aligned}
$$

where:

- V_{me} = the present value of the cash flows to the owners of a mutual insurance company
- I = the investment income
- Π = the probability of ruin
- λ_I = the remaining fraction of investment income with bankruptcy
- λ_V = the remaining fraction of firm value with bankruptcy
- r_f = the risk free rate
- B_p = the commission paid to the insurance agents
- C = the costs of the production process
- J = the reinsurance premiums
- K = the claims paid by the reinsurer
- B = the reinsurance commission received
- p_β = the price of systematic risk
- p_ξ = the price of co-skewness
- μ_{m2} = the second central moment of the market rate of return
- μ_{m3} = the third central moment of the market rate of return
- α_k = the average amount received from the reinsurer per claim of the clients
- n = the stochastic central density parameter of the Poisson distribution, which indicates the expected number of claims in one year
- r_m = the stochastic rate of return on the market portfolio

The cash flows to the members of a mutual insurance company during normal operations will be indicated by M. M equals by definition the income to the owners summed with the income of the clients:

$$\text{(47)}\quad \begin{aligned} M &= Y + S - P \\ &= I - B_p - C - J + K + B - r_o.D_o \end{aligned}$$

It may be noted that the limited liability effects for the totality of the members of a mutual insurance company disappear. The positive impact of limited liability to the shareholders is compensated by the negative impact to the clients of exactly the same magnitude. We find that $E(M) = I - B_p - C - J + E(K) + B - r_o.D_o$. Therefore, if $\lambda_I = 1$ and $\lambda_V = 0$, equation 46 simplifies in:

$$\text{(48)}\quad \begin{aligned} V_{me} &= \{I - B_p - C - J + E(K) + B - r_o.D_o\}/(\Pi + r_f) \\ &\quad - \{(p_\beta/\mu_{m2}).\alpha_k.Cov(n,r_m)\}/(\Pi + r_f) \\ &\quad - \{(p_\xi/\mu_{m3}).\alpha_k.Cov(n,r_m^2)\}/(\Pi + r_f) \\ &= E(M)/(\Pi + r_f) \\ &\quad - \{(p_\beta/\mu_{m2}).\alpha_k.Cov(n,r_m)\}/(\Pi + r_f) \\ &\quad - \{(p_\xi/\mu_{m3}).\alpha_k.Cov(n,r_m^2)\}/(\Pi + r_f) \end{aligned}$$

We find in equation 48 that the value of a mutual is governed by the amount of investment income minus the amounts to be paid to the employees, to the insurance agents, to the reinsurer and to the providers of ordinary debt. This implies that in the absence of reinsurance ($\alpha_k = 0$) systematic risk becomes irrelevant to the members of a mutual insurance company. Disregarding systematic risk we find that, with a positive cash flow to the members of a mutual insurance company, a reduction in the probability of ruin increases the value to the members. Reinsurance should then be sought by the members of a mutual. If the expected cash inflow to the members of the mutual insurance company is, however, negative, reinsurance reduces the value of the stakes of the members of a mutual. In comparison with equation 48 we see in equation 46 that reinsurance becomes more interesting to the members if more value is lost with bankruptcy ($\lambda_I < 1$). Reinsurance cover is, however, less desirable than indicated by equation 48 if less value is lost in the ruination period (i.e. if $\lambda_V > 0$).

If there is a negative relation between the number of yearly expected claims and the rate of return on the market portfolio, an increase in reinsurance cover (and α_k) will improve the value of the mutual insurance company to its members. The systematic risk of the members of the mutual is in part transferred to the reinsurer. A positive relationship between the number of claims and the squared market rate of return will (for a negative ρ_ξ) increase the value of reinsurance cover to the members.

5. CONCLUSIONS

In this paper we addressed the integration of actuarial ideas within the theory of finance. It was shown that the concepts of the normal power approximation and of structure variation can usefully be applied in evaluating the stakes of the interest groups of a primary insurance company. Our analysis is based on the following assumptions:

1) The price of insurance and reinsurance cover is given. Insurance and reinsurance premiums are not affected by the solvency considerations nor by systematic risk or co-skewness in the claims.

2) The shareholders of the insurance company expect a profit.

3) The expected end of period value of the primary insurance company is discounted with a rate of return that incorporates systematic risk and co-skewness.

4) The shareholders may unexpectedly face a huge amount of claims, which makes it interesting for them to use their rights of limited liability at an exogeneous ruining threshold A^*.

5) In the event of ruin, the shareholders will not be able to reap any of the future firm value.

This way of reasoning resulted in equation 10, which gives the value of the primary insurance company to its shareholders. With additional assumptions, we were also able to derive the value of the stakes of the members of a mutual insurance company (equation 46). Having found these equations, we can find the impact of reinsurance on the value of both stock insurance companies and mutual insurance companies.

Regarding thecomplexity of the world, we recognise that we made a series of simplifying and far-reaching assumptions. Despite these assumptions, the impact of reinsurance on the present value of an insurance company to its owners is not unidirectional. The final sign of the impact of reinsurance on the value of stock insurers (equation 10) can only be found if all the characteristics of the cash flows to the owners, both covariances, the risk free rate, the prices of systematic risk and co-skewness and the moments of the market portfolio are available. The final impact of reinsurance on the value to the members of a mutual (equation 46) demands, in addition, knowledge of bankruptcy costs. Our analysis therefore does not give simple answers. Because of the difficulty of obtaining all the relevant information, the desirability of reinsurance cover will not be easy to assess in practice. But we think that our approach shows the features which should at least be addressed in evaluating reinsurance cover. In the meantime we hope to have shown how actuarial concepts and financial theory can be integrated.

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LIST OF SYMBOLS USED

The list of symbols used in this study are given below. In the main text many symbols are written in italics. These italic symbols refer to stochastic variables. Italics will not be used for stochastic variables in suffixes. It is then in general evident from the context whether or not it is a stochast. Capitals indicate amounts of money and can be stock or flow variables.

A*	=	a ruining threshold used by the owners of a company
b	=	a suffix indicating the insurance agents
B	=	reinsurance commission received
B_p	=	commission paid to insurance agencies
C	=	costs of the use of labour and physical capita1 (operation costs)
D	=	debt
e	=	a suffix which denotes the employees
E	=	expectation operator
f	=	a (density) function or a suffix denoting the risk free rate of return or a superfix indicating the end of the. period
G	=	gross insurance results
h	=	a suffix indicating the reinsurer
H	=	reinsurance results
I	=	investment income
J	=	reinsurance premiums
k	=	a suffix denoting the reinsured amount per claim
K	=	claims paid by reinsurers
m	=	a suffix denoting the market portfolio
me	=	a suffix denoting the members of a mutual insurance company
M	=	the cash flow to the members of a mutual insurance company
n	=	number of claims
o	=	a suffix indicating the ordinary debtholders
P	=	gross premiums
q	=	structure variation variable
Q	=	assets of a company
r	=	rate of return
s	=	a suffix denoting the gross amount per claim
S	=	gross claim amount
UC	=	unindemnified claims
V	=	value of the firm
y	=	a suffix indicating the shareholders
Y	=	cash flow to the shareholders under complete liability
Y*	=	cash flow to the shareholders under limited liability
z	=	a suffix denoting the retained amount per claim
Z	=	after reinsurance (net) claim amount
α	=	a raw moment of a variable (a moment about zero)
β	=	a measure of systematic risk
γ	=	skewness of a variable

λ = a parameter indicating the relative amount of value or investment income left after bankruptcy
μ = a central moment of a variable
ξ = a measure of co-skewness risk
Π = the probability of ruin
a = standard deviation of a variable
4 = a solvency measure without correction for skewness
Ω = a solvency measure with correction for skewness