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THE PRICE OF RISK EMPIRICALLY DETERMINED BY THE CAPITAL MARKET LINE

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DETERMINATION EMPIRIQUE DU PRIX DU RISQUE PAR LA "LIGNE DU MARCHE FINANCIER"

DÉTERMINATION EMPIRIQUE DU PRM DU RISQUE PAR LA "LIGNE DU MARCHÉ FINANCIER"

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RESUME

Cet article porte sur la structure des préférences des investisseurs, en en matière de rentabilitk et de risque. Nous avons d'abord estimé la fonction d'utilité cardinale du patrimoine investi pour un certain nombre de clubs d'étude de placements : il a été possible de deriver de cette fonction d'utilité lognormale l'aversion relative à l'égard du risque (RRA : Relative Risk Aversion), qui semble varier d'un club à l'autre, selon le montant du patrimoine investi, la taille du club, l'âge moyen de ses membres et le pourcentage de membres de sexe masculin. La RRA est d'une grande importance car, dans un contexte de variance moyenne, il détermine presque complètement les courbes d'indifférence, et de ce fait l'attitude à l'égard de la rentabilité et du risque d'un investisseur. En utilisant l'équivalent de certitude (CE : Certainty Equivalent) du portefeuille actuel, qui peut être déterminé en posant une question type aux clubs d'étude de placements, on a mesuré un niveau de référence stratégique : la "ligne du marché financier" (CML : Capital Market Line). Il apparaît que cette CML varie d'un club à l'autre, selon la RRA et le CE. Enfin, la "CML" et donc le "prix du marché" du risque ont été mesurés en prenant une moyenne pondérée des CML individuelles.

La méthode que nous utilisons pour obtenir le prix du risque diffère des méthodes plus traditionnelles, qui travaillent sur des données historiques. Friend et Blume (1975), par exemple, évaluent d'abord le prix du march6 du risque - qui est défini de façon légèrement différente de notre mesure - et en concluent que la RRA est presque constante, approximativement égale à 2. Pour notre part, nous estimons d'abord la fonction d'utilité du patrimoine et la RRA associée, en moyenne approximativement égale à 0,76, qui varie d'un club d'étude de placements à l'autre. Nous en concluons que la CML et le prix du risque varient entre les investisseurs.

Bien que le risque soit en fait un concept à plusieurs variables, nous considérons uniquement la variance des gains comme mesure du risque. Notre analyse est fondée sur l'hypothèse d'un comportement optimal de l'investissement, d'expectations et préférence données, ainsi que sur la forme quadratique que nous utilisons comme approximation des courbes d'indifférence dans un univers à variance moyenne. On pourrait se demander si l'hypothèse de rationalité demeure appropriée, lorsqu'aucune opinion raisonnée n'est formulée en ce qui concerne les expectations futures en matière de rentabilitk et de risque. Malheureusement, dans l'étude sur laquelle nous fondons ces travaux, aucune question n'a été posée concernant la perception subjective du taux de rentabilitk sans risque. On peut supposer qu'elle varie d'un club d'étude de placements à l'autre, ce qui implique que notre hypothèse du rf égal à 5% est réfutable. Notre conclusion concernant les expectations hétérogènes dépend, bien sûr, des diverses hypothèses que nous avons faites.

Il serait intéressant d'appliquer cette procédure relativement simple, en complement de méthodes plus classiques, à une étude qui porterait sur des categories d'investisseurs plus nombreuses que celles limitées aux clubs d'investissement.

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BY NICO L. VAN DER SAR
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1 - INTRODUCTION

Individual investment decision making can be seen as the outcome of the confrontation between one's expectations and one's preferences, the restrictions given. Our information or beliefs determine the probabilities of the possible outcomes of our decisions and our wants or desires determine the values or utilities of the possible outcomes. Often, it is assumed that all investors have homogeneous expectations of the distributions of returns. In the Capital Asset Pricing Model (CAPM) (Treynor (1961), Sharpe (1963), Lintner (1965), and Mossin (1966) it is assumed that all investors' estimates of return and risk are the same. Next to expectations, preferences play an important part in explaining people's behavior on stock markets. Each investor allocates his available funds, that is wealth to be invested, over the available assets with a varying degree of risk. In general the investor's optimal financial position is described by the solution of the problem how to maximize the (subjective) expected utility of wealth. We assume that an individual's preferences with respect to invested wealth are stored in a cardinal utility function of wealth U (.). In contrast with the case of expectations, most financial models do not presuppose homogeneity with respect to preferences but leave room to variation among the individuals, implying that utility functions are specific to individuals. In most financial models people are assumed to be risk averse meaning a strictly concave wealth utility function where the degree of concavity is indicative of an individual's degree of risk aversion . According to Modern Portfolio Theory a rational investor will choose his optimal portfolio along the Capital Market Line (CML). The choice will be in agreement with his (return, risk) - preferences. The tangency point of the investor's set of indifference curves to the CML corresponds with his optimal portfolio.

In this article we shall follow a less common line of approach. Our starting point will be the investor's preferences with respect to return and risk. On the basis of theoretical and empirical arguments we will apply one single type of wealth utility function U (.). We will use the so - called Wealth Evaluation Question (WEQ) in which respondents are asked to state levels of invested wealth that they feel are VERY SMALL, SMALL, . . . , VERY BIG to estimate and explain U(.) in terms of individual characteristics. Hence, it is possible to find the set of indifference curves for every investor. It will be shown that Pratt's (1964) relative risk aversion (rra), which is determined by the path of the utility curve, or to be more precise by its degree of curvature, almost completely pins down the investor's net of indifference curves and therewith his (return, risk) - attitude. Under the assumption of optimal investment decision making the relevant indifference curve can be found by asking the respondents the certainty equivalent (ce) of their current portfolio. The individual CML is the straight line drawn from the risk - free rate of return tangent to the indifferencecurve with the ce of the optimal current portfolio on it. We have estimated this CML f a every investor. Next, the "market price" of risk has been estimated by aggregating the individual CMLs.

The purpose of this paper is, first, to determine empirically if and how preferences with respect to stock returns vary among investors, and, second, to draw some conclusions with respect to the "individual price" and the "market price" of risk. The investor group on which we shall focus consists of a sample drawn from the members of the Dutch Central Union of Investment Study Clubs (NCVB). Section 2 deals with our model. In Section 3 we discuss the data and present our empirical results. Finally, Section 4 ends with some concluding remark.

2 - MODEL

First, we focus on the utility function of wealth and the corresponding estimation procedure. For the measurement of people's attitudes towards amounts of money we use the so - called Evaluation Question Approach (see Van der Sar and Van Praag (1987)). A set of attitude questions is offered to the respondent who is asked to associate an amount of money, which according to him, fits in best with each qualification. In 1987 Koolstra used the so - called Wealth Evaluation Question (WEQ) to study people's preferences with respect to amounts of invested wealth. This WEQ that has been supplied to Dutch investment study clubs runs as follows

"For a club like ours, in our circumstances, we consider an amount of wealth to be

VERY SMALL	if it is about ..... Dfl
SMALL	if it is about ..... Dfl
NEITHER BIG, NOR SMALL	if it is about ..... Dfl
BIG	if it is about ..... Dfl
VERY BIG	if it is about ..... Dfl".

The response of investment study club n to the WEQ is a vector with five amounts of wealth, to be denoted by (w_{1n}, ..., w_{5n}). These can be seen as the respondent's expression of his wealth judgments and in general these will vary among the clubs. We assume that an investment study club evaluates different amounts of invested wealth w by a lognormal distribution function

$$U(w) = N(\ln w; \tau, \phi) = \Lambda(w; \tau, \phi).$$

Where N (. ; Υ, ϕ) is normal distribution function with mean Υ and variance ϕ^2, where Λ (. ; Υ, ϕ) is the lognormal distribution function with median Υ and log - variance ϕ^2.

Since the utility function of investment wealth is expected to vary among the clubs we use a subscript, reading in case of investment study club n

$$U_n(w) = N\left(\frac{\ln w - \tau_n}{\phi_n}; 0,1\right).$$

Adopting the equal interval hypothesis, meaning that the verbal qualifications of the WEQ are equally spaced in the [0,1] - interval, yields

$$\frac{\ln w_{in} - \tau_n}{\phi_n} = u_i \qquad (i = 1, \ldots\ldots, 5)$$

where $u_1, \ldots, u_5$ equal the quantiles 1/10, 3/10, 5/10, 7/10 and 9/10 of the standard normal distribution, viz.

$$N(u_i;\ 0,1) = \frac{i - \frac{1}{2}}{5} \qquad (i = 1, \ldots, 5).$$

The plausibility of specifying the utility function of wealth U (.) by a lognormal distribution function and the adoption of the equal interval hypothesis rests on arguments similar to the ones used in research on utility of income (cf. among others Van Praag (1968), and Kapteyn and Wansbeek (1985) for a review of research on the so - called individual welfare function of income). In view of the foregoing we obtain

$$\ln w_{in} = \Upsilon_n + \phi_n u_i \qquad (i = 1, \ldots, 5).$$

The study club's answers will not satisfy this equation exactly. Adding an error term, which is assumed to be identically, independently distributed we can obtain estimates of Υ_n and ϕ_n by means of Ordinary Least Squares (OLS). Subsequently, it will be investigated how Υ_n and ϕ_n, and therewith U_n (.) vary among the investment study clubs (see section 3).

Now, we put our mind to the relationship between the utility function of wealth and the indifference curves in the mean - variance world. Let r denote the rate of return on the amount of wealth w_n invested by study club n, then

$$r = \frac{w - w_n}{w_n},$$

where w_n stands for the uncertain end - of - period wealth. The value of the utility function of wealth can be expressed by a Taylor's series expansion around w_n, yielding

$$U_n(w) = U_n(w_n) + (w - w_n)U_n'(w_n) + 0,5(w - w_n)^2 U_n''(w_n) + \ldots$$

In view of this, it follows for the expected utility EU that

$$EU \approx U_n(w_n) + w_n U_n'(w_n)E(r) + 0,5U_n''(w_n)\{E(r)^2 + \sigma^2(r)\}$$

where E(r) denotes the expected rate of return and $\sigma^2(r)$ the variance of returns. This leaves us with circles, each one with its center at

$E(r) = - U'_n(w_n) / w_n U''_n(w_n)$ (in the sequel to be denoted by r^*_n) and $\sigma(r) = 0$, see figure 1.

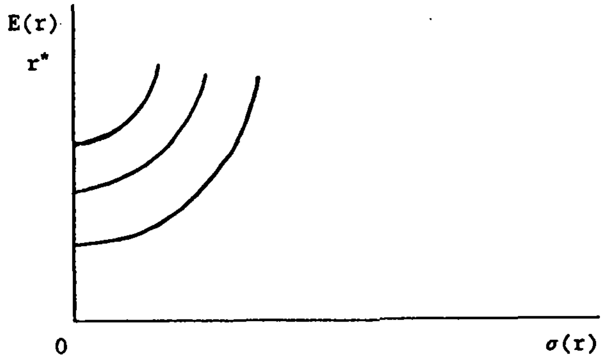

Figure 1 A net of indifference curves.

Under the assumption that the indifference curves can be approximated by (segments of) circles over some relevant range of rate of return, we have that the (return, risk) - attitude of any single investor can be completely described by only one number r* being the reciproke of the investor's relative risk aversion rra . This measure which has been introduced by Pratt in 1964 (see also Arrow (1970)) is indicative of an individual' s propensity of being risk averse if the bets are measured not in absolute terms but in proportion to wealth.

With help of figure 2 it can easily be seen that the greater rra, the smaller r* and the more close the tangency portfolio is to the riskless asset along the CML, with risk - free rate of return r_f, implying that more is held of the risk - free asset and less of the risky market portfolio M. This is in agreement with the meaning of rra as a (local) relative risk aversion measure.

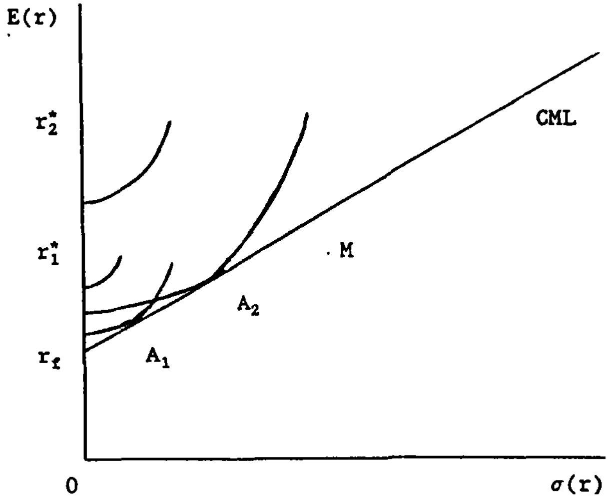

Figure 2 The risk averse investor n chooses his optimal portfolio in point A_n

Determining the CML is not a matter of course. To measure the mean - variance efficient frontier one has to have knowledge about the way the stock returns are expected to move to one and other. Then the CML can be found by drawing the straight line from the risk - free rate of return tangent to the mean - variance efficient frontier. However, in general the covariances will be unknown since it concerns future expectations. That's why, in practice, often historical data are used, assuming that the distribution of returns doesn't change over time.

With our model it is possible to measure the CML in another way. Assume that our investor's objective is to maximize the utility of wealth, and that his current portfolio is the optimal one, his expectations and preferences given. When supplying the WEQ to the investor, it is possible to assess his utility function of wealth U (.). Using this result, the investor's ra, r* and consequently the shape of his net of indifference curves can be estimated. If we know one single point of the indifference curve with the investor's current portfolio on it, then it is possible to determine this relevant indifference curve. Subsequently, the CML can be measured by drawing the straight line from the risk - free rate of return tangent to this indifference curve.

In figure 3 it can be seen that the riskless asset ce, viz. the certainty equivalent of the

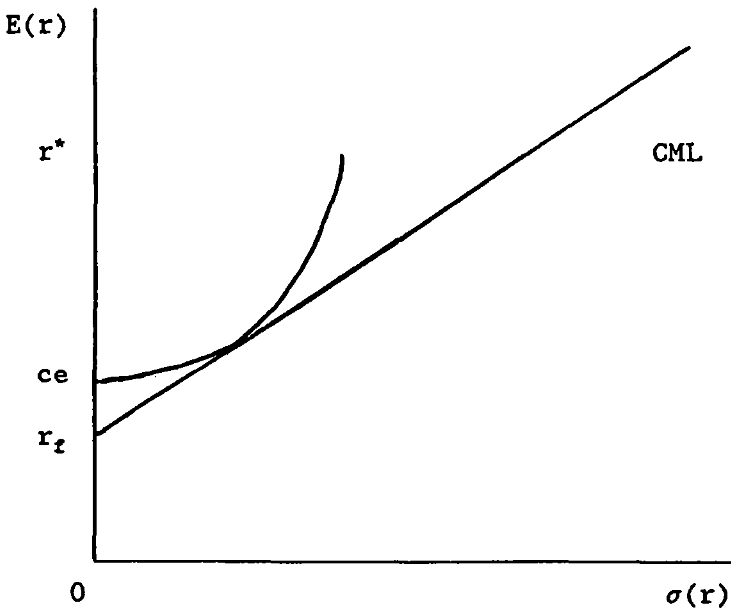

Figure 3 The CML is the straight line emerging from r_f tangent the indifference curve with ce on it.

This point can be found by offering the respondents a typical question that runs as

"What would be the minimum rate of return on a bank account, government bond etc., such that you would not invest your money in stocks but put it on a bank account, buy government bonds etc. ? . . . . . %".

Henceforth, this will be called the Certainty Equivalent Question (CEQ). It remains to be seen, of course, whether our procedure yields the same straight line for every respondent, and whether in this context speaking of " the CML" still is appropriate.

3 - EMPIRICAL RESULTS

The data upon which our analysis is based were collected as part of a larger effort to gain some insight on the one hand into the investors' future expectations with respect to the returns of financial assets and on the other hand into the preference structure of the investors with respect to rate of return and risk In April 1987, a questionnaire created by Koolstra was sent to members of the Dutch Central Union of Investment Study Clubs (NCVB) in the western part of the Netherlands, viz. the provinces of North Holland, South Holland, North Brabant and Utrecht A total of 63 responses appeared to be amenable to analysis. The response rate of 23% is consistent with those of other mail surveys which are typically in the 20- 30 percent range. Although there is no reason to doubt the sample's representativeness, the possibility of (partial) non - response bias cannot be ruled out Our special interest is in the questions relevant to analyzing the investment study clubs' preferences like the WEQ and the CEQ (cf. section 2), and the questions on the clubs actual circumstances like, e.g., the amount of invested wealth (on average about 15, 400 Dfl), the mean age (approximately 39 years), the size of the club (ca 12 members per club), and the percentage of men being club member (about 74%).

Applying the estimation procedure for the utility function of invested wealth $U_n(.)$, that has been described in section 2, yields estimates of Υ_n and ϕ_n with n running over all clubs. To estimate the variation among the investment study clubs, two regressions with Υ_n and ϕ_n depending on the amount of wealth invested by the club, the club size, the club members' mean age and the percentage of men being members of the club are run with OLS. The empirical results are given in Table 1.

Table 1. Estimation results for 40 investment study clubs, with standard deviations in parentheses[1].

	Υ	ϕ
constant	- 0.122 (2.315)	1.359 (1.145)
ln (wealth)	0.482 (0.132)	- 0.066 (0.065)
ln (size of club)	- 0.703 (0.327)	- 0.330 (0.152)
ln (age of members)	1.624 (0.675)	0.377 (0.334)
percentage of men	1.396 (0.430)	0.111 (0.213)
$\bar{R}^2$	0.447	0.051

1 Due to partial non-response the number of questionnaires used here is less than the 63 that were available.

We take an interest in the meaning of these results with respect to the risk attitude of the investment study clubs. A club's utility function is S - shaped consisting of an initial convex segment followed by a terminal concave one. It appears that most clubs evaluate their own amount of invested wealth generally as somewhat less than 1/2. Since the inflection point which corresponds with the utility level N ($\Upsilon - \phi^2$; Υ, ϕ) is clearly below the $^1/_2$ - level we may say that in general the club's position falls within the concave segment, a put in other words the club's relative risk aversion rra generally has a positive value. Making the substitutions for Υ_n and ϕ_n in $rra_n = (\ln w_n - \Upsilon_n) / \phi_n^2 + 1$ it is straight forward to put rra_n in terms of club - specific characteristics.

Among economists there is no consensus about whether realistic risk aversion measures should increase, remain constant, or decrease as people grow wealthier. Arrow (1970) suggests an increasing rra_n using both theoretical and empirical arguments. However, the results of Friend and Blume (1975) are consistent with a constant rra_n equal to 2.0, and Cohn, Lewellen, Lease and Schlarbaum (1975) provide evidence of a decreasing rra_n. Our study empirically supports the hypothesis of an increasing relative risk aversion since rra_n appears to vary positively with the amount of invested wealth. The club's mean age has a negative effect on rra_n which shows that the older a club gets on average, the more it is inclined to take risks. The rra_n is negatively correlated with the club's percentage of men which is indicative of men being less risk averse than women. The rra_n appears to be positively affected by the size of the study club implying that group pressure strengthens risk aversion, viz. groups shift toward greater caution. This fact can be rationalized by the assumption that caution sometimes is a value. Vinokur (1971) proposed an explanation of both the phenomena of risky and cautious shift emphasizing the rationality of group discussion (cf. also Steiner (1982)).

Together with the investment study club's risk attitude we measured the net of indifference curves being segments of concentric circles. The indifference curve relevant to us is the one with the current (optimal) portfolio on it. It is the segment of the circle emerging from the certainty equivalent ce. With help of the CEQ we found ce for every investment study club. It appears to vary among the clubs but regressing it on club - specific characteristics yielded very poor estimation results. We take an interest in the CML emerging from the risk - free portfolio being drawn through point A corresponding with the investment study club's optimal portfolio.
For club n, both $A_n = (E_A(r_n), \sigma_A(r_n))$ and $(ce_n, 0)$ are lying on the indifference curve tangent to the CML.
Then it follows that

$$(E_A(r_n) - r_n^*)^2 + \sigma_A^2(r_n) = (ce_n - r_n^*)^2.$$

The slope of the CML is equal to the marginal (subjective) rate of substitution of the expected rate of return with respect to the standard deviation $[dE(r_n) / d\sigma(r_n)]_{A_n}$ in point A_n. In view of this we obtain

$$\frac{E_A(r_n) - r_f}{\sigma_A(r_n)} = \frac{\sigma_A(r_n)}{r_n^* - E_A(r_n)}$$

From these two equations we can derive the point $A_n = (E_A(r_n), \sigma_A(r_n))$ and as a consequence the CML running through A_n en $(r_f, 0)$ can be measured. We've chosen the risk - free rate of return equal to 5%. The CML that can be faund with help of the two foregoing equations varies among the investment study clubs, depending on rra (= $1/r^*$) and ce. That's why we will call it the " individual CML". Our empirical results provide evidence that investment study clubs have heterogeneous expectations of the joint distributions of returns. The price of risk varies among the clubs depending on the degree of relative risk aversion rra, which is fairly well explained by club - specific characteristics, and the certainty equivalent ce, which varies randomly.

Finally, we measured the "market price" of risk by aggregating the individual CMLs, weighted by the relative amount of invested wealth. The mathematical expression of the "average CML" is

$E(r) = 0.05 + 0.322\ \sigma(r)^2.$

4 - SUMMARY AND DISCUSSION

The foregoing has focused on the preference structure of investors with respect to return and risk. First, we estimated the cardinal utility function of invested wealth for a number of investment study clubs. From this lognormal utility function it was possible to derive the relative risk aversion rra which appears to vary among the clubs, depending on the amount of invested wealth, the club size, the mean age of the club members and the percentage of men being a club member. The rra is of great importance because, as it is proven here, in the mean - variance world it almost completely pins down the indifference curves and therewith the (return, risk) - attitude of an investor. Using the certainty equivalent ce of the current portfolio, that could be found by offering the investment study clubs a typical question, the Capital Market Line (CML) has been measured. This CML appears to vary among the clubs, depending on rra and ce. Finally, "the CML" and therewith "the market price" of risk has been measured by taking a weighted average of the individual CMLs.

The method we use to come to the price of risk differs from the more traditional one that works with historical data. Friend and Blume (1975), e.g., first assess the market price of risk (which is defined slightly different from our measure) and, subsequently, conclude that rra is almost constant, being approximately equal to 2.0. However, we, first, estimate the utility function of wealth and the associated rra, on average being approximately equal to 0.76, which varies among the investment study clubs. Subsequently, we conclude that the CML and the price of risk vary among the investors.

Although, risk actually is a multivariate concept we only consider the variance of returns as a risk measure. Our analysis is based on the assumption of optimal investment behavior, expectations and preferences given, and on the quadratic form we apply to approximate the indifference curves in the mean - variance world.

2) *Due to partial non - response going with the CEQ our estimation results apply to only 29 investment study clubs, which is a lower number than the 40 for which Υ and ϕ have been estimated (see table l).*

One could query whether rationality still is appropriate, if there is no sound judgment at all of future expectations with respect to return and risk. Unfortunately, in the survey we use, there were no questions asked relating to the subjective perception of the risk - free rate of return. It possibly varies among the investment study clubs implying that our assumption of a constant r_f, equalling 5%, is challengeable. Our conclusion of heterogeneous expectations depends, of course, on the various assumptions we made.
It would be interesting to implement our relatively simple procedure in a survey with more investor - categories than merely investment study clubs, next to the more traditional methods.

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