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FORWARD TRADING AND EXCHANGE RATES VARIABILITY

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RESUME

Cet article étudie, dans le cadre dun modèle d'équilibre partiel, l'influence d'un marché d'opérations à terme sur la volatilité des taux de change au comptant, question qui n'a pas encore fait l'objet d'une analyse exhaustive dans l'ensemble des travaux théoriques sur les opérations à terme. Deux versions du modèle sont présentées : des conclusions tranchées en faveur d'un effet stabilisateur des opérations à terme, découlent du modèle, dans lequel est incorporée la perturbation différentielle dûe au taux d'intérêt.

* Cet article fait partie d'une recherche sur les opérations à terme, entreprise durant la thèse préparée par l'auteur auprès de l'Université de Warwick et poursuivie dans le cadre de la préparation de sa thèse auprès de l'Université de Bologne.

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ABSTRACT

The paper investigates, in a partial equilibrium model, the influence of a forward exchange market on spot exchange rates volatility, an issue which has not been exhaustively analyzed in the theoretical literature on futures. Two versions of the model are given : clear - cut conclusions in favour of a stabilizing effect of forward trading emerges in the model where the interest rate differential disturbance is incorporated.

INTRODUCTION

The successful opening of financial futures markets during this decade, has led futures theorists to analyze the role of financial futures in equilibrium models. Yet, as far as we know, most of the literature investigates the issue in a CAPM (Capital Asset Pricing Model) context. For reasons we have already expounded in other works (Torricelli (1988) and (1989)), we believe that the true interrelations between the futures and the underlying spot market are better captured in the HPA (Hedging Pressure Approach) framework. In this strand of literature the only paper on noncommodity futures that we have come across, is Kawai's (1984), where the author analyzes the influence of foreign exchange futures on the volatility of the spot exchange rate. In the above - cited papers we have performed a similar kind of analysis for commodity futures getting to well - defined conclusions on a purely analytical basis.

In this chapter we shift our focus to a foreign exchange market in order to investigate the effect of the inception of a forward[1] market on the spot rate variability. Our aim is that of drawing conclusions from the analytic solutions of the model without resorting to empirical methods. In fact in Kawai's paper non - linearities in the coefficients prevent him from finding conclusions without imposing more specific assumptions. Furthermore such conclusions are not analytically expounded and not much intuition is provided.

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(1) *"Forward is used here instead of "futures" since it is more appropriate in the foreign exchange care. We recall that forward and futures market differs essentially in institutional aspects, but the economic function of the two type of contract is the same.*

That is why in what follows we will try to replicate and reinterpret the results obtained for commodity futures in the context of a foreign exchange market (Torricelli (1988) and (1989)). To this end, namely to obtain analytic conclusions on the spot exchange rate volatility in the presence of futures, we set up a two - date - one - period model which has analogies to the one presented in the abovementioned work but still has peculiar features that need to be described and commented upon

The plan of this paper is the following : we will first look at the model without forward trading in #1. In #2 we will extend the model to forward trading. #3 focuses on the comparison of spot rate volatility in the forward market and no forward market case. In the last section we perform the same comparison in a modified model which offers an interesting interpretation of the arbitrage equation,
Before expounding the characteristics of the model in the next section, we want here to stress that it is RE model since we do not want to inestigate the issue of the informativeness of the futures price.

1 - A MODEL OF A FOREIGN EXCHANGE MARKET

To keep our two - date - one period model as simple as possible, we follow Kawai (1984) , and with him most of the traditional literature on forward exchange, in assuming the fdlowing :

i. income, prices and interest rates are exogenous ;

ii. foreign exchange rates are the only endogenous variables ;

iii. the domestic currency is the monetary habitat of the domestic agents ;

iv. the representative agents in the model are two: an export - import trader and an interest arbitrageur ;

v. foreign exchange transactions result from the optimizing behaviour of the interest arbitrageurs and the given behavioral function of the export-import traders.

The latter two assumptions need some comments. The export - import traders are meant to be spot commercial traders, i.e. they are assumed to take their export - import decisions at the time when they face actual market exchange rates. In other words they are not subject to exchange risk and their excess supply of foreign exchange can be assumed to have the following form :

(1) $T_t = br_t + u_t$

with $b>0$; $t=1,2$; $u_t \sim N(0,\sigma_u)$ for any t;
and where :

T_t is the excess supply of foreign exchange of the spot commercial trader,

r_t is the actual level (a logarithm of level) of period t spot exchange rate defined as price of foreign in teams of domestic currency,

u_t is a trade disturbance summarizing various non - systematic factors influencing T_t.

All the variables are expressed in terms of deviations from an equilibrium level which can be thought of as the equilibrium characterizing the model if there were no disturbances. Hence constant terms are omitted from (1) and the solutions.

If compared with Kawai, our model neglects the existence of non-spot commercial traders, i.e. those traders who face a technological lag between the time of deciding on trade commitments and the time of receiving or paying foreign currency. Our assumption of non - existence of non-spot commercial traders has to be seen as defining a limiting case and can be justified either on the basis of infinitly large non - spot trading costs or of an infinitly large risk-aversion on behalf of the non - spot trader. It is taken in our model because we want to focus on the impact of forward trading on interest arbitrage decisions which can be read in a somewhat parallel way to the storage decision for commodity goods. As standard in the literature on futures we derive this sort of decisions from the maximizing behaviour of the interest arbitrageurs under exchange rate uncertainty : interest arbitrageurs are agents who invest their wealth in domestic and foreign assets in order to look f a higher return at a smaller risk[2]. Yet, in this context, the "inventory behaviour" is governed only by a speculative term : the expected spot rate differential[3].

In order to make these considerations more clear, we need to go through the maximization problem of the representative interest arbitrageur. Before this, we still want to assume the following :

- the n (with n large) interest arbitrageurs behave competitively,
- the discount factor is one[4].

Moreover the timing of spot (and, in the next section, forward) decisions is the following : at a point in time immediately after 1, $1+\varepsilon$, uncertainty about first period excess supply of foreign exchange is resolved $u_1 = u_1^*$, together with the uncertainty about interest arbitrage demand, $v_1 = v_1^*$. At this point in time, still characterized by uncertainty with respect to the period 2 trade balance equation, spot trading and interest arbitrage trading take place. Therefore the interest arbitrageurs take decisions conditional on the realization of u_1 and v_1. At time 2, only spot trading takes place. It is like saying that individual agents' decisions are taken at time $1+\varepsilon$, but the functioning of the market is analyzed at time 1 so that we can observe how much randomness is carried forward to date 2 and through which channels.

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2 This denomination is kept even when foreign and domestic interest rates are assumed to be zero.

3 Alternatively, this simple model can be seen as analyzing the behaviour of a single agent whose trade decisions are separate from the interest arbitrage ones and where the market environment is given by eq. (1).

4 This assumption is taken for the sake of algebraic simplicity. Still, the content of the result would not be substantially changed with a non - unitary discount factor. This hypothesis is taken in Kawai (1984) as well and is equivalent to that the domestic and foreign interest rates are identical and equal to zero.

We have thought of this time structure as a two-period analysis equivalent to a multiperiodal one as is presented, e.g, in Kawai (1984) where decisions at time t are taken after the realization of the disturbances characterizing time t.

Following the standard assumption of the literature on futures, we can state the risk - averse interest arbitrageur's maximization problem in the mean - variance approach :

$$(2) \quad \max_{k_1^*} \{E(\pi/v_1^*,u_1^*) - \frac{1}{2} \tau Var(\pi/v_1^*,u_1^*)\}$$

where :

$$\pi = r_2 k_1^* - r_1 k_1^* - \frac{1}{2} h(k_1^*+v_1^*)^2$$

$$E(\pi/v_1^*,u_1^*) = E(r_2/v_1^*,u_1^*)k_1^* - r_1^* k_1^* - \frac{1}{2} h(k_1^*+v_1^*)^2$$

$$Var(\pi/v_1^*,u_1^*) = k_1^{*2} Var(r_2/v_1^*,u_1^*)$$

and τ is the risk - aversion coefficient.

In plain words, the interest arbitrageur faces a quadratic cost of keeping foreign exchange. More precisely, since the portfolio management cost of domestic currency is much lower than the cost of managing and handling foreign currency, the former is for simplicity assumed to be zero. We will interpret the term v_1^* as a shift in the managing costs. Prior to its realization such a term is a normally distributed random variable of the type :

$$v_1 \sim N(0,\sigma_v^2)$$

and is independently distributed from u_1 and u p

At this stage we want to stress another difference between our model and that of Kawai : the difference concerns the interpretation of the disturbance in the interest arbitrage demand. In fact Kawai, who does not provide in the abovecited paper the maximization problem underlying the excess stock demand, when interpreting the disturbance term writes :

- The disturbance vt affects the relative yield on domestic and foreign assets and may represent either the cost of managing foreign assets (when st is the level of a spot exchange rate) or the differential between the domestic and foreign interest rate (when st is the logarithmic spot rate) ; vt is henceforth referred to as the "interest - rate - differential disturbance". -

In our opinion, the interpretation of v_t as an interest rate differential disturbance is appropriate only as an ex - post interpretation of the disturbance term in the interest arbitrage excess stock demand ((3) in Kawai (1984) or (3) in the present model) providing that exchange rates are measured in logarithms. Yet, we think that this interpretation is not logically justifiable in the underlying maximization problem where the interest rate differential disturbance appears as an additive term to the stock level. Hence a different maximization problem should be set up. In our opinion, if one wants to incorporate interest rate differential considerations in a model like Kawai's or ours, one should include an interest rate differential disturbance in the revenue term in the profit function. Still, setting the problem in such a way, would lead to an interest arbitrage demand with different features from the one obtained in Kawai's paper. Therefore, we will stick to Kawai's framework, but, in this section, we will unequivocally interpret the disturbance v_1 as a shift in the managing costs of foreign currency.

Going back to maximization problem (2), from the FOC, we have the interest arbitrage demand :

$$k_1^* = \frac{E(r_2/v_1^*, u_1^*) - r_1 - hv_1^*}{h'} \qquad (3)$$

where $h' \equiv h + \tau \,\mathrm{Var}\,(r_2 / v_1^*, u_1^*)$ and h' can be interpreted as the rate of change in the marginal cost of keeping foreign exchange.

(3) has exactly the same form as the interest arbitrage demand obtained in Kawai : the optimal stock of foreign exchange for interest arbitrage motives, depends positively on the anticipated capital gains adjusted for the abovementioned disturbance and inversely on the rate of change in the marginal cost of stocking. The latter takes into account the conditional variance of period 2 spot rate as a measure of exchange risk which is weighted with the risk - aversion coefficient.

Now, the next step is that of working out the equilibrium solutions far prices. In our simple two - period model of only spot trading, equilibrium over the two periods is given by the following conditions :

$$\begin{aligned} T_1^* &= K_1^* \\ T_2 + K_1^* &= 0 \end{aligned} \qquad (4)$$

where $K_1^* = nk_1^*$.

Substituting (1) and (3) into (4) and solving f a the rates we have :

$$E(r_2/v_1^*, u_1^*) = -\frac{n}{b(h'b+2n)}u_1^* + \frac{hn}{h'b+2n}v_1^* \qquad (5)$$

$$r_1^* = -\frac{h'b+n}{b(h'b+2n)}u_1^* - \frac{hn}{h'b+2n}v_1^* \qquad (6)$$

$$r_2^* = -\frac{u_2}{b} - \frac{n}{b(h'b+2n)}u_1^* + \frac{hn}{h'b+2n}v_1^* \qquad (7)$$

where $h' = h + \tau Var(r_2/v_1^*, u_1^*)$. Hence we can work out the RE conditional variance for r_2:

$$(8) \qquad Var(r_2/v_1^*, u_1^*) = \frac{1}{b^2}\sigma_u^2$$

2 - A MODEL OF FOREIGN EXCHANGE MARKET WITH FORWARD TRADING

In this section we will show how the results of the previous section are modified when a forward market operates, i.e. when the interest arbitrageurs can enter a forward contract in period 1 to deliver (or receive delivery of) a stipulated amount of foreign exchange in period 2 at a known forward exchange rate, f.

The framework that we will use for the investigation of this issue is the one of #1 with a few more qualifications. First of all there is one more type of agent in the model which operates only in the forward market without committing himself to commercial trade or interest arbitrage : a representative pure speculator, whom we assume to be risk - neutral in order to abstract from considerations on the market bias (difference between the futures and the expected spot price)[5]. In fact it is a well known results in the futures literature that a risk - neutral speculator would be always willing to accept any level of futures offered by the hedger thus setting the bias to zero. Secondly we must add forward trading in the time axis of trading : at the time $1 + \varepsilon$ the interest arbitrageurs take both spot an forward decisions and trade spot with commercial traders and forward with pure speculators. At time 2 only spot trading take place and the forward contract matures.

Since the commercial traders do not take part in the forward market, we now look at the modification of the interest arbitrageurs' optimal decisions:

$$(9) \qquad \max_{k_1^{*f}, F} \{E(\pi/v_1^*, u_1^*) - \frac{1}{2}\tau Var(\pi/v_1^*, u_1^*)\}$$

$$\pi = r_2^f k_1^{*f} - r_1^{*f} k_1^{*f} - \frac{1}{2} h(k_1^{*f} + v_1^*)^2 + (f - r_2^f)F$$

where exchange rates (r_1^{*f} and r_2^f) and the stock of foreign exchange (k_1^{*f}) are now denoted with the superscript f since they are referred to the existence of a forward market and f is the forward exchange rate.

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[5] Even if this issue is of great interest, we do not want to investigate it here since our main concern is the effect of forward trading on spot rates via a modification of interest arbitrage decisions.

From the first order conditions we get the optimal stock and forward decisions of the interest arbitrageur :

$$k_1{}^{*f} = \frac{f - r_1{}^{*f}}{h} - v_1{}^{*} \tag{10}$$

$$F = k_1{}^{*f} + \frac{f - E(r_2{}^{f}/v_1{}^{*}, u_1{}^{*})}{War(r_2{}^{f}/v_1{}^{*}, u_1{}^{*})} \tag{11}$$

As one would expect, the interest arbitrage demand differs from the one in the no-forward case for two reasons. The expected future spot rate is here replaced with the forward rate and the level of demand is now inversely proportional only to the rate of change in the marginal cost of stocks and not anymore to the period 2 rate variance. In fact, in the presence of a forward market, the interest arbitrageurs hedge their whole spot demand in the forward market(see eq. (11)) and then they speculate on the differential between the forward rate and the expected period 2 spot rate.

At this stage, we should work out the optimal forward decisions of the pure speculators. Yet, since we have assumed risk - neutrality on behalf of speculators, we just recall here that this assumption leads in equilibrium to a forward rate equal to the expected value of period 2 spot rate (i.e. no bias).

Hence we can now look at the following equilibrium conditions :

$$T_1{}^{*} = K_1{}^{*f}$$

$$T_2 + K_1{}^{*f} = 0 \tag{12}$$

$$f = E(r_2{}^{f}/v_1{}^{*}, u_1{}^{*})$$

where $K_1{}^{*f} = nk_1{}^{*f}$.

Substituting (10) into (12) and following the same procedure as in #1, we have the following solutions :

$$r_1{}^{*f} = -\frac{hb+n}{b(hb+2n)}u_1{}^{*} - \frac{hn}{hb+2n}v_1{}^{*} \tag{13}$$

$$r_2{}^{f} = -\frac{u_2}{b} - \frac{n}{b(hb+2n)}u_1{}^{*} + \frac{hn}{hb+2n}v_1{}^{*} \tag{14}$$

$$E(r_2{}^{f}/v_1{}^{*}, u_1{}^{*}) = -\frac{n}{b(hb+2n)}u_1{}^{*} + \frac{hn}{hb+2n}v_1{}^{*} \tag{15}$$

Having solved for the price system in both the forward and no - forward case, we can now tackle the issue of spot rate volatility in connection with the inception of a forward market : this is the *object* of the next section

3 - FORWARD EXCHANGE AND SPOT RATE VOLATILITY

In this section we assess analytically the effect of forward trading on spot rate volatility and we will try to provide an intuitive interpretation of the results *obtained.*

To this end we first work out the unconditional (i.e. prior to the realization of u_1 and v_1) variances[6] of the spot rates in the no - forward and forward model respectively.

From (6) and (7) we have the following expressions for the period 1 and period 2 spot rates in the no - forward case :

$$(16) \qquad Var(r_1) = \left[\frac{h'b+n}{b(h'b+2n)}\right]^2 \sigma_u^2 + \left[\frac{hn}{h'b+2n}\right]^2 \sigma_v^2$$

$$(17) \qquad Var(r_2) = \frac{1}{b^2}\sigma_u^2 + \left[\frac{n}{b(h'b+2n)}\right]^2 \sigma_u^2 + \left[\frac{hn}{h'b+2n}\right]^2 \sigma_v^2$$

with $h'=h+\tau Var(r_2/v_1^*, u_1^*)$

From (13) and (14), we get the corresponding expressions in the forward case :

$$(18) \qquad Var(r_1^f) = \left[\frac{hb+n}{b(hb+2n)}\right]^2 \sigma_u^2 + \left[\frac{hn}{hb+2n}\right]^2 \sigma_v^2$$

$$(19) \qquad Var(r_2^f) = \frac{1}{b^2}\sigma_u^2 + \left[\frac{n}{b(hb+2n)}\right]^2 \sigma_u^2 + \left[\frac{hn}{hb+2n}\right]^2 \sigma_v^2$$

We start with the comparison of *spot* rate variances in period 1 since in *our* two - date - one - period model only period 1 is characterized by simultaneity of spot and forward decisions.

From the comparison of (16) and (18) we can say that it is not possible to draw unequivocal conclusions without making further assumptions at least on the relative size

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6 As a measure of exchange rates' volatility we take the spot rate variances. Furthermore we need to consider unconditional variances since conditional ones would account only for period 2 randomness and hence would be the same in the no -forward and forward case.

of the trade and interest arbitrage equation disturbance. In fact, the first term on the RHS is smaller in the forward case, while the second term is bigger. In other words, in the presence of forward exchanges, the spot rate variance in period 1 is less sensitive to the trade disturbance but more sensitive to the interest arbitrage equation disturbance.

The interpretation of this statement can be given more easily if we look at each of the two subcases separately, i.e. only trade disturbance or only managing costs disturbance. We start with the former which can be read in a parallel way to the model presented in Torricelli (1989) where a random disturbance characterizes only the supply.

If we set $\sigma_v^2 = 0$, we have unequivocally that:

$$Var(r_1^f) < Var(r_1).$$

Again, as in the commodity case presented in the work just mentioned, this can be intuitively explained referring to the determinants of the spot rate volatility. In particular we know that it is determined by the sum of the excess supply variance, the stock variance and the negative covariance between excess supply and stock of foreign exchange kept for arbitrage motives. This negative covariance is bigger in the forward case due to the smaller rate of change in the marginal cost of stocking ($h < h'$) which increases the sensitivity of the interest arbitrage demand to the excess supply disturbance. Hence we can say that the negative covariance between the stock of foreign exchange and the excess supply disturbance, offsets the sum of the supply and stock variance more heavily in the forward case than in the pure spot case.

The reverse situation, i.e $\sigma_u^2 = 0$, lends itself to a more straightforward interpretation: foreign exchange stock's variance is bigger in the forward case due to a smaller value of the rate of change in the marginal cost of stocking which increases the sensitivity of the interest arbitrage demand to the managing costs disturbance, hence period 1 spot rate variance is bigger in the forward case.

A definitive conclusion on the intermediate case, i.e. $\sigma_u^2 > 0$ and $\sigma_v^2 > 0$, is not so immediate even if we could be tempted to conclude that forward trading has a stabilizing effect on period 1 spot rate only if there is some sort of dominance of the trade disturbance, which would be analogous to Kawai's conclusion. But what sort of dominance do we have in mind ? The above - stated tentative conclusion definitely needs mare qualifications. To the end of providing an answer to the question raised, we write the following inequality and we will try to determine whether and when it is true:

$$(20) \quad Var(r_1) > Var(r_1^f)$$

which, after having substituted respectively (16) and (18) for the two variances, can be rewritten as:

$$(20') \quad \sigma_v^2 > \frac{\left[\dfrac{n+hb}{b(hb+2n)}\right]^2 - \left[\dfrac{n+h'b}{b(h'b+2n)}\right]^2}{\left[\dfrac{hn}{h'b+2n}\right]^2 - \left[\dfrac{hn}{hb+2n}\right]^2} \sigma_u^2$$

where it must be kept in mind that $h' = h + (\tau / b2)\, a12$ and hence the ratio on the RHS is itself a function of su2.

Referring to the axes a12 and σv2, it turns out that there are regions for which the inequality (20') is true and regions for which it is false. To this end we start with noticing that the term on the RHS of (20') is a function of σu2, more precisely, the first term on the denominator is a monotonically increasing function of σu2. As for the second term on the numerator (taken with its negative sign), we can prove, working out the first derivative, that it is monotonically increasing only if :

$b>2$

In such a case, the whole ratio on the RHS is monotonically increasing in σu2 and hence it will be possible to find two regions : each of them characterizes a situation of stabilizing or destabilizing influence of futures trading. The characterization of each region is in term of the disturbances on the trade and interest arbitrage equation : if the interest arbitrage equation disturbance is bigger than the trade disturbance in the sense specified by (20') then futures trading has a stabilizing effect and vice versa. Still, this conclusion holds providing that the slope of the trade balance is bigger than 2. Where this not true, conclusions might be reversed a it might happen that the plane is divided in more than two regions each characterizing a stabilizing or destabilizing influence of futures trading.

Summing up the discussion so far, we can say that in the presence of both the trade and interest arbitrage disturbance, conclusions on the effect of futures trading on spot prices, are dependent on the characteristics of the trade balance equation and are ultimately referring to a sat of dominance which is defined in terms of the (20').

This in some way contrasts with Kawai's (1984) contention that "the introduction of forward trading stabilizes spot exchange rates in the short run if the trade disturbance is the predominant random element in the foreign exchange market relative to the disturbance in the interest rate differential, and vice versa". We will return to this in the last part of this section.

As far as period 2 spot rate variance is concerned, the relevant terms for the comparison are the second and the third one on the RHS of (16) and (18). Since both of them increase in the forward case, we can conclude that independently of the relative size of the two disturbances, period 2 spot rate variability is increased in the presence of forward trading ; in fact the sensitivity to both the trade and interest arbitrage equation disturbance is increased in such a case due to a smaller value of the rate of change in the marginal cost of stocking.

As we have already stressed, the results we have obtained for period 1 spot rate volatility can be usefully compared with those presented in Kawai (1984), in the special case characterized by the absence of non - spot commercial trading. Kawai too ends up by saying that a definitive conclusion depends on the relative size of the two disturbances characterizing the model. Yet, such a conclusion is not made precise in Kawai's paper and analytically argued. In our model instead, we have been able to obtain a precise condition on the relative size of the disturbances referring to the parameters of the model : among those, the relevant one is the slope of the trade balance whose magnitude ultimately determines the conclusion on spot rate variability. Still, the parallel between our work and Kawai's one has to be done bearing in mind the specific interpretation given to the interest arbitrage equation disturbance. In fact, we think that as it has been incorporated in our model, v_1 must be thought of as a random shift in the managing costs of the foreign currency. An interpretation in terms of an interest rate differential disturbance, should be more cautious and would definitely require reading the model in terms of logarithms.

4 A DIFFERENT MODEL OF A FOREIGN EXCHANGE MARKET : SOME MORE CONSIDERATIONS ON THE SPOT RATE VOLATILITY

As we have already stressed in the previous sections of this paper, one of the differences between our model and Kawai's (1984), is to be found in the interpretation of the disturbance affecting the interest arbitrage demand. In our opinion the interpretation of the latter as an interest rate differential disturbance even if interpretationally acceptable when reading the interest arbitrage demand in logarithmic terms, is not consistent with the underlying maximization problems ((2) and (9)). In a mathematical appendix to his work, Kawai writes that in the case of non - zero interest rates, one may simply interpret the spot and forward exchange rates as the natural logarithms of their level and the disturbance term as the interest rate differential, while the excess supply and the stock of foreign exchange would still be measured in foreign currency units.

In our opinion this interpretation is sensible only when taking the arbitrage demands ((3) and (10)) as exogeneous, but still does not give support to the consideration, in the underlying maximization problem, of an interest rate differential disturbance in the cost function as an additive term to the stock level. Instead, as we have anticipated in #1, we think that if one wants to incorporate interest rate differential considerations in a model like Kawai's or ours, one should include an interest rate differential disturbance in the revenue term in the profit function. To this end, in the present section we set up a model which is only slightly different from the one used so far, where the difference is in the interpretation of the disturbance affecting the interest arbitrage demand.

As usual, we start off with the no - forward case, to proceed then to the forward case.

The time pattern of tradings and of decisions is the same described in #l. As f a the variables they are defined as in #1 and #2 except f a the disturbance v_1. Since we want the disturbance affecting the arbitrage demand to be an interest rate differential disturbance, we will use the subscipt 2 in order to, more realistically, assume decisions at date $1 + \varepsilon$ governed by the interest rate differential prevailing at date 2.

Hence we define :

$v_2 = i_f - i_d$

where

i_f = foreign interest rate

i_d = domestic interest rate

and we assume

$$v_2 \sim N(0, \sigma_v^2)$$

and v2 to be independent of u1 and u2.

Under the above - stated assumptions, we can restate maximization problem (3.2) as follows:

$$(2') \quad \max_{k_1^*} \{E(\pi/u_1^*) - \frac{1}{2}\tau Var(\pi/u_1^*)\}$$

where:

$$\pi = [(r_2+v_2) - r_1] k_1^* - \frac{1}{2} h k_1^{*2}$$

$$E(p/u1^*) = [E(r_2/u_1^*) - r_1^*]k_1^* - \frac{1}{2}h k_1^{*2}$$

$$Var(p/u_1^*) = k_1^{*2} Var(r_2+v_2/u_1^*)$$

which under the assumptions on v_2 becomes:

$$Var(p/u_1^*) = k_1^{*2} [Var(r_2/u_1^*)+\sigma_v^2]$$

From the FOC we get the following optimal interest arbitrage demand:

$$(3') \quad k_1^* = \frac{E(r_2/u_1^*) - r_1}{h'}$$

where we now define $h' \equiv h + \tau\, Var(r_2 + v_2/u_1{}^*)$.

If compared with (3), it is immediate to realize that the interest rate differential disturbance affects the interest arbitrage demand only via its variance. M a e precisely, the presence of this type of disturbance makes the rate of change in the marginal cost of stocking bigger since, as shown in the equilibrium solutions, the conditional variance of the spot rate at date 2 is the same as in (3).

On the other hand, the disturbance does not appear in an additive fashion in (3) and this because we have characterized it as a noise with zero mean We shall now work out the equilibrium solutions for the no - forward case. The equilibrium conditions are the same as in (4). Substituting (3') and (1) into (4), and following the RE solutions procedure we have already used, we have:

$$(5') \quad E(r_2/u_1^*) = - \frac{n}{b(h'b+2n)} u_1^*$$

$$(6') \quad r_1^* = - \frac{h'b+n}{b(h'b+2n)} u_1^*$$

$$(7') \quad r_2^* = - \frac{u_2}{b} - \frac{n}{b(h'b+2n)} u_1^*$$

$$(8') \quad Var(r_2/u_1^*) = \frac{1}{b^2} \sigma_u^2$$

where we recall that $h' = h + \tau\, Var(r_2+v_2/u_1^*)$.

We now look at the forward case. Under the assumptions of this section, we have that the maximization problem for the interest arbitrageurs is :

$$(9') \qquad \max_{k_1^{*f}, F} \quad \{E(\pi/u_1^*) - \frac{1}{2}\tau Var(\pi/u_1^*)\}$$

$$\pi = [(r_2^f+v_2)-r_1^{*f}]k_1^{*f} - \frac{1}{2} h\, k_1^{*f2} + (f-r_2^f-v_2)F$$

From the FOC we have the following interest arbitrage demand for the forward case :

$$(10') \qquad k_1^{*f} = \frac{f - r_1^{*f}}{h}$$

In the forward case, as noticed in #2, the interest arbitrage demand depends only on the rate of change in the marginal cost of stocking and does not imply variance considerations since the interest arbitrageur can hedge by means of forward contracts. Therefore the only difference between (10) and (10') is the absence of the additive disturbance term in the latter. The equilibrium conditions in this case are the same as in (12). By substitution of (1) and (10') into (12) and using the standard RE solution procedure, we have the following equilibrium solutions :

$$(13') \qquad r_1^{*f} = -\frac{hb+n}{b(hb+2n)}u_1^*$$

$$(14') \qquad r_2^f = -\frac{u_2}{b} - \frac{n}{b(hb+2n)}u_1^*$$

$$(15') \qquad E(r_2^f/v_1^*, u_1^*) = -\frac{n}{b(hb+2n)}u_1^*$$

We are now able to make a comparison between the spot rate volatility in the no-forward and forward case using the same sort of analysis as in #3. We start by working out the unconditional variances of the spot rates in period 1. We have :

$$(16') \qquad Var(r_1) = [\frac{h'b+n}{b(h'b+2n)}]^2\sigma_u^2$$

$$(18') \qquad Var(r_1^f) = [\frac{hb+n}{b(hb+2n)}]^2\sigma_u^2$$

Since the coefficients on the RHS of (16') is bigger than the one in (18'), we can definitely conclude that the period 2 spot rate variance is smaller in the presence of a forward exchange market. This because the hedging opportunity offered by a forward market makes the rate of change in the marginal cost of stocking foreign currency smaller ($h < h'$), which in turn implies a smaller sensitivity of the spot rate to the trade balance disturbance in the forward case.

As for period 5 the unconditional variances for the spot rate are the following:

$$(17') \qquad Var(r_2) = \frac{1}{b^2}\sigma_u^2 + \left[\frac{n}{b(h'b+2n)}\right]^2 \sigma_u^2$$

$$(19') \qquad Var(r_2^f) = \frac{1}{b^2}\sigma_u^2 + \left[\frac{n}{b(hb+2n)}\right]^2 \sigma_u^2$$

The comparison is straightforward: period 2 spot rate variance is bigger in the forward case. Here the smaller rate of change in the marginal cost of stocking foreign exchange plays a role in the opposite direction than in period 1.

Hence the results we have obtained in this section are alike to those obtained in the commodity case (Torricelli (1989) : the presence of a forward market, via a modification of the storage behavior, contributes to the stability of the spot market. And this contrary to what sometimes believed on the basis of a misplaced emphasis on a negative speculative characterization of the futures and forward markets. It is interesting to notice however that, assuming an interest rate differential disturbance instead of a disturbance in the cost of stocking, we are able to draw conclusions which are both determinate and clear cut. Again, if compared with Kawai's, our results are neater even when giving the more interesting interpretation of the disturbance v in terms of an interest rate differential.

CONCLUSIONS

As a concluding comment to this paper, we will say that it was meant to show how the analysis that we have carried out in the commodity case (Torricelli (1989)) can be usefully extended to a different context. The complexity of the latter, a foreign exchange market, has required the assumption of simplifying hypotheses which are justified on the basis of the restricted focus of this paper: once again the interaction of forward decisions with decisions about the carry - wer from one period to another which in turn affect the equilibrium in the spot market.

Morevover our results can be usefully compared with those obtained in Kawai (1984) which, as far as we know, is the only paper that analyzes the effect of foreign currency futures on spot exchange rates in the Hedging Pressure Approach framework.

Summing up the results that stem from our simple two - period - one - date partial equilibrium model, we can state that when assessing the influence of a forward - futures market on the volatility of the underlying spot foreign currency market, we hive found theoretical presumption in favour of a stabilizing influence of forward trading.

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