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THEORY AND PRACTICE IN STOCK INDEX PORTFOLIO INSURANCE ON THE ITALIAN MARKET : SOME REFLEXIONS

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THÉORIE ET PRATIQUE DE L'AMÉLIORATION DU RENDEMENT D'UN PORTEFEUILLE FONDÉE SUR UN INDICE BOURSIER DU MARCHÉ ITALIEN : QUELQUES RÉFLEXIONS

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RESUME

Cet article concerne l'amélioration du rendement d'un portefeuille fondée sur un indice boursier, plus précisément le Comit, un indice du march6 italien ti large base.

En l'absence d'un marché des indices boursiers, ainsi que d'options sur ces actifs, il n'est pas possible de réaliser en Italie une amélioration directe du rendement d'un portefeuille : pour atteindre cet objectif, il faut recourir à une méthode "synthétique", fondée sur les principes de la méthode, maintenant classique, proposée dans l'article d'avant - garde de Brennan et Schwartz (1976).

La théorie, fondée sur des résultats fondamentaux de la théorie de la fixation des prix d'options, suggère précisément une stratégie de révision continue d'une combinaison d'actifs non risqués et de l'indice de référence, qui est capable de reproduire à maturité le portefeuille concerné, quelle que soit l'évolution stochastique du portefeuille de référence.

La théorie repose sur des hypothèses non réalistes, telles que : la possibilité d'opérer des transactions en permanence, sans coût transactionnnel ni taxe affectant le portefeuille de référence, et le comportement non stochastique de la volatilité de l'indice et de la structure des termes des taux d'intérêt.

Mis à part le risque du taux d'intérêt - qui fait l'objet d'un traitement spécial fondé sur les théories dites d'immunisation -, la pratique pourrait s'écarter de la théorie du fait de : (1) la nécessité de négocier uniquement un petit nombre d'actions, en suivant des stratégies d'ajustement discrètes, pour Cviter de forts coûts transactionnels ; et (2) d'erreurs défavorables dans l'estimation de la volatilité de l'indice de référence.

Intuitivement, une stratégie bonne candidate pour la pratique réelle, serait de sélectionner un portefeuille approprié, constitué d'un certain nombre de titres facilement négociables, restreint mais en même temps capable de se comporter comme un représentant de l'indice de référence, et de ne revoir les proportions entre ce représentant et l'actif sans risque que lorsque la différence entre la composition actuelle et la composition théoriquement optimale, dépasse une limite fixée. (Du fait de la disponibilité hebdomadaire de données sur le marché boursier de Milan, des révisions d'une fréquence au plus hebdomadaire ont été envisagées).

En particularisant cette intuition, et en se référant à plusieurs périodes de durées différentes et caractérisées par divers comportements du march6 boursier - haussier, baissier, ferme, fluctuant, ... -, au cours d'une période de 169 semaines du ler janvier 1986 au 20 mars 1989, des calculs numériques ont été effectués, avec un portefeuille représentant constitué de 48 valeurs italiennes de père de famille - qui s'avèrent un très bon représentant de l'indice Comit -, et pour trois valeurs de la limite (3%, 2%,0%) ou

avec révision hebdomadaire inconditionnelle, en supposant les coûts transactionnels proportionnels au volume des transactions entraînées par la recalibration. Deux valeurs différentes de la volatilité, estimées a priori, ont été utilisées : 20% et 30% - la dernière étant très proche des valeurs réelles dominantes au cours de toutes les périodes testées.

La valeur finale (coûts transactionnels globaux bruts et nets) des différentes stratégies d'investissement pour les deux valeurs de la volatilité ont été calculées pour chaque période testée.

Enfin, des indices d'efficacité bruts et nets des coûts transactionnels ont été dérivés en calculant la différence (nette ou brute) entre ces valeurs finales et la valeur à maturité du portefeuille dont on cherche à améliorer le rendement (max du comit et de la garantie) ou du représentant de l'indice de référence dont on veut améliorer le rendement (max de ce représentant et de la garantie).

La dernière partie de l'article présente des commentaires sur les données obtenues par ce "test d'efficacité" : on montre que quel que soit le choix de la limite, l'incidence des coûts transactionnels peut être signficative, notamment s'ils se combinent avec des erreurs défavorables dans l'estimation de la volatilité, en sorte d'entraîner de mauvaises performances (déficits pertinents de presque 10%) de la stratégie d'amélioration du rendement du portefeuille.

En outre, alors qu'une majoration de sécurité d'un niveau tout à fait acceptable de 5% sur le prix d'option de vente pure, suffirait pour assurer une couverture contre un comportement défavorable de 10% de la volatilité, pour échapper aux coûts transactionnels, l'existence d'un march6 oil l'indice est négocié, ou au moins la possession d'un certain fond d'actions (ouvert) constitué en sorte de reproduire l'indice, est absolument nécessaire - si l'on veut être certain d'améliorer le rendement du placement

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RESUME.

This paper concerns stock index portfolio insurance, precisely portfolio insurance on Comit, a broad based italian market index.

The lack of a market for stock indexes as well as for options on these assets, denies the opportunity to obtain in Italy direct portfolio insurance ; to reach this goal you ought to resort to a "synthetic insurance" along the lines of the now classic approach proposed in the path breaking paper of Brennan and Schwartz (1976).

The theory, based on fundamental results of the theory of option pricing, suggests precisely a strategy of continuous revision of a mixture of the no risky asset and the reference index, that is able to duplicate at maturity the insured portfolio, whatever the stochastic path of the reference portfolio.

The theory is based on such not realistic assumptions as : the opportunity to trade continuously without transaction costs and taxes on the reference portfolio, and the non stochastic behaviour of the volatility of the index and of the term structure of the interest rates.

Leaving aside the interest rate risk (which deserves a special treatment based on the so called immunization theories), practice could depart from theory because of : 1) the need to trade only a small number of stocks, following discrete adjustment strategies to avoid heavy transaction costs, and : 2) adverse errors in the estimation of the volatility of the reference index.

Intuitively a good candidate real world strategy would be to select a proper portfolio, built with a number of easy tradable stocks, small but at the same time able to behave as a proxy of the reference index, and to revise proportions between this proxy and the no risky asset only when the difference between the current composition and the one theoretically optimal exceeds some fixed bound (owing to the availability of weekly data on Milan stock exchange, at most weekly revisions have been considered here).

Particularizing this intuition and with reference to several periods of different lenght and characterized by different stock market behavior (bull, bear, steady, up and down...) within a 169 weeks period starling from 1/1/86 and ending at 20/3/89, numerical computations have been performed with a proxy of 48 italian blue cheaps (which turns out to be a very good proxy of the Comit index) and with three values of the bound (3%, 2% and 0% a unconditional weekly revision), assuming transaction costs proportional to the volume of transactions induced by recalibration. Two different a priori estimated values of volatility have been used : 20% and 30%, the last one very close to the real ones prevailing in any of the tested periods.

The ending value (gross and net of the global transaction costs) of the different investment strategies f a both values of volatility have been computed fa each tested period.

Finally gross and net of transaction costs indexes of efficiency have been derived computing the (net or gross) difference between those ending values and the maturity value of the insured Portfolio (max between Comit and the guarantee), or of the insured proxy (max between proxy and the guarantee).

Comments to the data obtained in this "efficiency test" are given in some detail in the final section of the paper ; here it is enough to signal that for any choice of the bound the impact of transaction costs may be relevant especially if combined with adverse errors in volatility estimation, so as to produce bad performances (relevant deficits at almost 10%) of the portfolio insurance strategy.

Moreover it seems that while to hedge against a 10% adverse behavior of volatility, a safety loading on the pure put premium could be charged at a quite acceptable level of 5%, to escape from transaction costs a market where the index is traded, or at least some (open) equity fund tailored to duplicate the index (if one is satisfied to insure the fund) is absolutely needed.

"This paper derives from a joint research effort of the authors, and reflects their common point of view on the subject. Anyway going in some detail F. Pressacco wrote chapters 1, 3, and 5, while P. Stucchi collected and refined data, improved computer programs, obtained numerical results and wrote sections 2 and 4 of the paper".

1 - This paper concerns stock index portfolio *insurance*, precisely portfolio insurance on Comit, a broad based Italian index, which is a weighted average of the about 320 stocks currently quoted at the Milan stock exchange.

As a premise keep account that partially due to the lack of a market for stock indexes as well as for options on these assets, stock index portfolio insurance is not developed in Italy.

At our knowledge the only opportunity related to our subject matter was offered some years ago by I.N.A. (Istituto Nazionale Assicurazioni), which created a life policy with an insured capital linked to the value of an investment fund (Interbancaria Azionario) and with a guaranteed minimum (Longo 1985).

Yet despite the name the above fund should be considered more a balanced fund than a stock fund, so that it captures only partially the evolution of a stock market index.

This being the situation a financial institution intended to offer to savers and intermediaries portfolio insurance on an Italian stock index is forced to keep charge of the whole risk involved that the capital value obtained by the investment of resources turns out to be at maturity lower than the amount to be paid to the customer.

Of come the theory of portfolio insurance, based on the ideas developed in the path breaking paper of Brennan and Schwartz (1976) grants that there is a way to hedge completely the above risk.

The hedge is provided by an investment policy that continuously recalibrates resources between the sure asset and the reference portfolio in such a way to duplicate exactly (whatever the stochastic path of the reference portfolio) the maturity value of the liability, i.e. the greater of the stock index value and the guarantee.

But should a real world financial institution safely rely on this theory ? Indeed the theory is based on the (someway unrealistic) assumptions that :

a) it is possible to trade on the reference portfolio ;

b) it is possible to revise continuously and without transaction costs and taxes the chosen proportion of no risky asset and reference portfolio under perfect matching between the real and estimated volatility parameter of the (diffusion) process governing the evolution of the reference portfolio ;

c) the reference portfolio does not pay dividends ;

d) the sure rate of return is non stochastic along the time horizon until to the maturity of the portfolio.

Leaving aside problems arising from the last two points and thus assuming non stochastic paths of the term structure of interest rates and negligible effects of dividends, we try in this paper to define an operational strategy conveniently approximating assumptions a) and b) above and then to test its efficiency in reaching closely enough the desired stock index insurance target.

The above strategy is based on the following simple ideas :

1) define a "proxy" portfolio built with a relatively small number (less than 50) of italian "blue chips", so as to make possible both to obtain returns close enough to the Comit index, and to trade whenever needed (see later) between the proxy portfolio and the sure asset ;

2) choose the timing of such recalibration according to a simple control strategy, when the difference between the current amount of risky (proxy) portfolio and the desired one according to theoretical suggestions exceeds some fixed bound (given as a fixed percentage of the initial value of the reference index).

We faund that at least according to tests based on a three years (precisely 169 weeks) data base the proposed strategy reveals high efficiency.

The structure of the paper is as follows : para 2 is devoted to a very short recall of the theory of portfolio insurance, para 3 describes the operational strategy chosen to reach our Comit stock index portfolio insurance target. Some technical questions concerning italian financial markets linked to tests of efficiency of the above strategy are discussed in para 4. Finally chapter 5 resumes with a short comment the results of the tests.

2 - Let us assume that the Comit index follows a diffusion process described by the following stochastic differential equation :

$$dI_t = mI_t\,dt + \sigma I_t\,dw_t \qquad 1$$

with w_t a standard Wiener process and m, σ, constant parameters of drift and volatility of the process. Without loss of generality assume $I_0 = 1$, at the beginning of the contract.

The index is defined as K,T insured if there is some grant to obtain at maturity T the payment of max (IT, K). In the following we treat the case $K = I_0 = 1$. Being :

$$\max(I_T,1) = I_T + \max(1-I_T,0) \qquad 2$$

the insured index turns out to be the sum of the (non insured) reference index and an european put option on the reference with maturity T and exercise price 1.

Hence the theoretical price at t = 0 of the insured index is given by :

I, +P(T,1,1, σ , δ)=1+P(T,1,1, σ , δ) 3

with P (T- t, I_t, ,K, a, 6) denoting the theoretical price of an european put option as a function of the time to maturity T - t, the current price of the underlying asset I_t, the exercise price K, the volatility σ of the diffusion process, and the constant istantaneous sure rate of interest 6 prevailing until maturity.

Note that even put options on the index are not traded, the theory provides the way to build an insured index following a convenient strategy of investment of resources.

Precisely at any time t, you ought to invest in the reference index an amount :

$H_c(t)\ I_t$ 4

where:

$H_c(t)=N(a_t)=$ Prob $N(0,1) \leq a$, 5

with:

$$a_t = \frac{\log\left(\frac{I_t}{K}\right)+\left(\delta+\frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} \qquad 6$$

while the remaining wealth is invested in the no risky asset

To understand the rule described by 4, recall that as previously said an insured index is the sum of the reference index and the related put option.

In turn a put option is duplicated by a mixture of the no risky asset and of the reference index. The amount devoted to the index is obtained applying the so called hedging ratio $H_p(t)$ of the put option to the current value of the reference index itself.

Hence summing up, an insured index could be seen as a mixture of a global proportion of $1+H_p$ applied to the reference index and of the no risky asset

After that to derive the rule 4 exploit the following key relation between the hedging ratios of the corresponding european put and call options :

$H_c = 1+H_p$ 7

Finally to check that the relevant call hedging ratio is given according to 5,6 (see e.g. Cox, Rubinstein 1985 pag. 205).

At the end of this chapter note that the theory requires among other technical conditions that you are able to trade the reference index (or an equivalent portfolio) revising continuously the amount invested there.

3 - As previously said neither of the two conditions is currently attainable without a heavy burden of transaction costs ; but we hope to show that it is possible to go very close to the desired targets, adopting a proper transaction costs saving strategy as follows.

First of all select a portfolio proxy of the Comit index built by a convenient mixture of a small number of italian blue chips and let it play the role of the reference index, then choose the timing of portfolio recalibration according to a convenient control strategy.

Going into some details concerning the first point the proxy was derived as a mixture of the prominent 48 ordinary stocks quoted at the Milan stock exchange at the date of 1 - 1 - 1986, with relative weights initially proportional to their capitalization (same updating was done later just to keep account of new facts as the end of a firm previously belonging to the proxy or the emergence of some new relevant stock ; moreover it goes without saying that the relative weights of original stocks change following properly their quotations change). Table 1 reports the composition of the proxy at the beginning of the period (1 - 1 - 86) and one year later.

What do we mean saying that the proxy plays the role of the reference index ? Merely that the hedging ratios are both computed and applied to the value of the proxy rather than to the value of the reference index, or formally that the value of the proxy J_t rather than I_t, is inserted in 4 and 6 to determine the amount that should be invested in the risky (proxy) portfolio.

Moreover this is linked with the timing problem : a redistribution among the proxy and the no risky asset of the total accumulated value at time t is done if and only if the difference between $J_t N(a_t(J_t))$, and the global value A_t of the risky assets derived by the investment of $J_\tau N(a_\tau(J_\tau))$ at the time $\tau < t$ of the last revision, exceeds a fixed bound b (f a some reasons that will become apparent later computations were done for b = 0, 02, 0, 03 and 0. Obviously b = 0 implies a not path dependent weekly revision).

Note that as :

$$A_t = J_\tau N(a_\tau(J_\tau))\, J_t/J_\tau = N(a_\tau(J_\tau))\, J_t \qquad 8$$

the rule is formalized as follows : revise if and only if for some $t > \tau$ (time of last revision) :

$$\left| N(a_t(J_t)) - N(a_\tau(J_\tau)) \right| J_t \geq b \qquad 9$$

Coherently with the idea to leave out interest rate risk, the istantaneous rate of return of the money invested in the no risky asset is assumed to be exactly 6.

4 - We tried to test the empirical efficiency of the adaptive investment strategy previously introduced on a data base of weekly quotations of Milan stock exchange for the period 1/1/86-20/3/89 (the test is going on). We quickly resume some technical points about.

1) The ability of the proxy portfolio to duplicate the returns offered by the Comit index is very high (Check table 2 that reports the sequence of the natural logarithm of the weekly price relatives respectively for the Comit index and the proxy portfolio).

Indeed the equation of the regression line of the proxy on Comit (see, fig.1) is :

$$y = 0{,}000526 + 1{,}034276\, x \qquad (\rho^2 = 0{,}961) \qquad 10$$

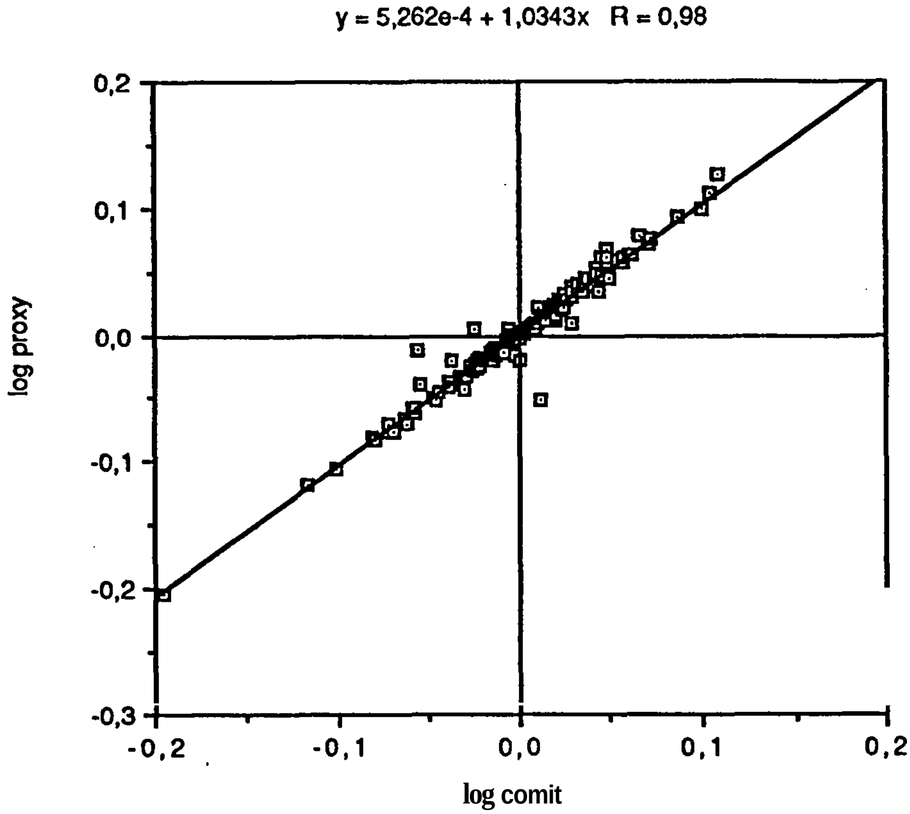

fig. 1

2) Both values of the annualized mean square deviation of return over the whole period are about 30%, slightly greater for the proxy. Unfortunately this index is very volatile (see table 3 for a comparison of the values along shorter periods). For computation of option prices and hedging ratios values of volatility of 30% and 20% respectively have been used. Apparently the use of a volatility meaningfully smaller than the real one should impair efficiency.

3) On the basis of the behavior of domestic financial markets, characterized by a high stability of the short term monetary rate of interest, the instantaneous risk free rate of interest used both to compute options prices, hedging ratios and the effective return of resources invested in the sure asset was assumed to be δ = 0,09568 corresponding to an annual rate of 10,041%.

5 - As a test of efficiency we have computed the final accumulated wealth obtained from the implied *investment* policy f a several periods of differentlenght and stock market behavior (e.g. bear, bull, *steady*, up and down) within the global period 1/1/86-20/3/89(see table 4).

After that, deviations between these values and the values of the insured Portfolios at its respective maturities (with each initial value normalized to one plus the relative value of the put option) have been computed.

These values could be seen as the percentage (gross of transaction costs) gain that a financial institution selling stock index portfolio insurance to private or institutional savers would have realized following the described strategic behavior.

Moreover sane evaluations of the transaction costs &riving from portfolio revisions, on the basis of admittedly quite simple assumptions about a transaction costs function, have been computed too. An homogeneous linear cost function precisely given by :

$$0.01 \left| N(a_t(J_t)) - N(a_\tau(J_\tau)) \right| J_t \qquad 11$$

has been used. Both the gross and net index of efficiency are reported in table 5.

Of course an alternative index of efficiency is given by the difference between the accumulated final value derived from the investment strategy and the respective end of period value of the proxy (clearly here the risk and conversely the reward coming from differences between the proxy and the reference index are left to the customers). For the sake of comparisons, some figures are given concerning this index too (table 6).

Some comments are in order :

1) each of the three strategies that have been tested (weekly control at 3%, 2% or 0% respectively) offer essentially the same final accumulated values (with negligible differences)f a both levels of volatility used for computations and for any tested period, except for the combination of small volatility (20%) and bull market (first period tested), where the zero control reveals meaningfully better returns (see tables 4.1, 4.2).

2) for any period the overall returns are uniformly greater for the greater value of volatility. Intuitively this &rives from the about 5% differences between the respective put premiums(see tables 4.1, 4.2 bottom lines).

Once more an.exception is found for the small volatility-bull market case, where the zero control has a meaningfully better performance in correspondence to the smaller volatility(see tables 4. 1, 4.2 again, first column).

3) for the higher value of volatility (0,3) each strategy is able to insure (gross of transactions costs) the proxy index, i.e. to give a final value no lesser than max (J_T, 1). with negligible deficits only over the first tested period for every strategy, and for the 3% control over the second and fourth period Anyway the deficit never reaches 1% (see table 6.1).

On the contrary a deficit of about 5% uniformly &rives from the use of the smaller volatility, except for the first period, where a smaller deficit comes from 2% and 3% control, and a positive performance from 0% control (see table 6.2).

4) as the proxy sistematically beats the Comit, with an especially relevant difference (13%) between returns over the whole period, the tested strategies a fortiori reveal (gross of transaction costs) ability to insure the Comit index during a bull market (see 5.1 and 5.2 first column).

Concerning the other periods, things are exactly the same as discussed sub the third remark above, as both proxy and Comit are uniformly lesser than the unitary guarantee. Indeed tables 5 and 6 differ only in the first column.

5) intuitively the overall value of transaction costs should be a monotone decreasing function of the control bound b. This is exactly what can be seen in all columns of tables 4.1, 4.2, but it is apparent that our transaction costs function 11 does not produce relevant differences between the tested control strategies. Quite likely the addition of a constant cost factor for any revision is needed to generate a superiority of the control strategies with higher value of the bound. Indeed the examples reported in tables 7.1, 7.2 show that a 2% control implies only 28 weekly revisions along a time period of 168 weeks or, respectively, 59 out of 146 weeks.

6) Finally a glance to tables 5.4 or 6.4 reveals that a combination of transaction costs and adverse errors in volatility estimation may result in bad performances (relevant deficits) of the portfolio insurance strategy.

Moreover it seems that while to hedge against a 10% adverse behavior of volatility, a safety loading on the pure put premium could be charged at a quite acceptable level of 5%, to escape from transaction costs a market where the index is traded, or at least some (open) equity fund tailored to duplicate the index (if one is satisfied to insure the fund) is absolutely needed.

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