CONTRIBUTION Nº 27

THE DEVELOPMENT OF A MARKET YIELD CURVE - THE SOUTH AFRICAN SOLUTION

PAR / BY

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Afrique du Sud / South Africa

ELABORATION D'UNE COURBE DES TAUX DU MARCHE - LA SOLUTION SUD AFRICAINE

ELABORATION D'UNE COURBE DES TAUX DU MARCHÉ - LA SOLUTION SUD - AFRICAINE

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RESUME

En Afrique du Sud, le second marché obligataire a commencé à se développer au début des années 80 et a connu une expansion rapide, en volume et comme en maturité. Une caractéristique de ce marché est la concentration de l'activité sur certaines obligations. Une première tentative de definition d'un indice de mesure des performances de ce marché a échoué, à cause de difficultés de conception et de production.

En 1987, il a été recommandé A l'Actuarial Society od South Africa de créer deux nouveaux indices : l'indice des performances des obligations des actuaires JSE (JSE-Actuaries Bond Performance Index), comme standard de référence pour la mesure des performances des portefeuilles, et la courbe des taux des actuaires JSE (JSE-Actuaries Yield Curve), pour donner une description et assurer un enregistrement historique des niveaux des taux d'intérêt. L'indice de performance et la courbe des taux ont été mis en place et sont entrés en application en octobre 1988.

La mtthode de construction de la courbe des taux devait tenir compte des caractéristiques des marchés locaux, en particulier l'absence de négociabilité de nombreux éléments. On a étudié la possibilité d'utiliser un certain nombre de courbes de formes différentes, qui ont été testées par rapport à cinq critères. La courbe existante la plus prometteuse Ctait celle élaborée en Grande - Bretagne par les chercheurs de Grieveson Grant. Avec l'aide de consultants locaux, une autre méthode de construction de courbes des taux a été élaborée, comportante l'utilisation d'une analyse factorielle et de splines cubiques. La méthode s'est avérée supérieure aux autres méthodes testées et pourrait avou des applications dans d'autres domaines de construction de courbes.

L'article décrit en detail la procedure d'ajustement de la courbe, en en donnant une formulation mathématique complète.

La forme d'ajustement de la courbe a été utilisée avec succès pour deriver une courbe taux - durée, qui constitue la base de la mise en oeuvre d'une politique d'immunisation d'une partie des actifs et engagements d'un bureau d'assurances - vie, visant A réduire le risque du taux d'intérêt.

THE DEVELOPMENT OF A MARKET YIELD CURVE

THE SOUTH AFRICAN SOLUTION

BY HEATHER D. MCLEOD.B.BUS. SC. (HONS), FIA.

SYNOPSIS OF THE PAPER :

The secondary market in bonds in South Africa began developing in the early 1980's and has grown rapidly, both in volume and sophistication. A feature of the market is the concentration of trade in certain bonds. An early attempt at a market index for performance measurement was unsuccessful due to the design and production difficulties.

In 1987 it was recommended to the Actuarial Society of South Africa that two new indices be created : the JSE-Actuaries Bond Performance Index as a standard against which to measure portfolio performance, and the JSE-Actuaries Yield Curve to provide a description and historical record of the level of interest rates. The Performance Index and Yield Curve were implemented and came into use in October 1988.

The method of drawing the Yield Curve had to take account of the characteristics of the local markets, in particular the lack of tradeability of many issues. A number of different forms of curve were investigated and tested against the five criteria chosen. The most promising existing yield curve was that developed by Grieveson Grant researchers in Britain. With the aid of local consultants an alternative method of drawing yield curves was developed which incorporates the use of cluster analysis and cubic splines. The method was found to be superior to others tested and could have wider application in curve drawing.

The paper describes the curve fitting process in detail, providing a full mathematical formulation.

The form of curve fitting has been used successfully in deriving a duration yield curve which formed the basis for the practical implementation of a policy of immunising a portion of the assets and liabilities of a life office, thereby reducing interest rate risk

1 - THE SOUTH AFRICAN BOND MARKET

An active secondary market in South African Government bonds and semi-government utility bonds (especially electricity and transport) began developing in the early 1980's. Some 2000 bonds are currently listed on the Johannesburg Stock Exchange (JSE) and are traded on a separate Gilt Floor. The total market capitalisation at the end of 1988 was some R 70 billion. (R1 billion equals approximately $360 million).

Although bond trade on the JSE frequently exceeds R1 billion a day, it is estimated by market participants that this is perhaps only 40% of the total trade in bonds in South Africa. Since 1983 a substantial "over-the-counter" market has developed with major borrowers, commercial and merchant banks, discount houses, life offices and major pension funds participating in a screen based market. As there is no central clearing house for trade there is no source of data for this "over-the-counter" market at present. Under legislation promulgated in 1989 the markets will be supervised under a self-regulatory framework and it is expected that the newly formed Bond Market Association will establish central data collection and clearing facilities;

The accompanying graph shows the growth in trade in the bond market relative to that in the equity market in South Africa. The bond data shown is for the JSE only and thus underestimates total bond trade.

A feature of the market is the extent to which trade occurs in only a few bonds. Despite the large number of bonds some 75 % of trade occurred in three long-dated bonds for much of 1986 and 1987. In more recent years trade has been dominated by a single long bond. The medium-dated bonds have a poor level of marketability and shortdated bonds are tightly held by banks to fulfil statutory requirements.

2 - BOND MARKET INDICES IN SOUTH AFRICA

The Actuarial Society of South Africa and the Johannesburg Stock Exchange have jointly been responsible for the introduction of the definitive indices in use in the South African equity and bond markets. The actuaries take responsibility for the design and applicability of the indices and the JSE for the production of the indices on a daily basis and their dissemination to the media.

2 - 1 - The Jse - Actuaries Fixed Interest Index

The Fixed Interest Index was launched in January 1983 with data from 1980 onwards, for the purpose of providing a benchmark for performance measurement. Modelled on the UK bond indices, separate categories for gilts and semi-gilts were provided for and price and interest components were shown separately. No weightings were used in the construction. Although adequate for performance measurement, the Fixed Interest Index did not gain wide acceptance amongst fund managers due to the difficulty in calculating performance and in large measure to the production difficulties experienced in the early months. Traders did not use the index as it was quoted on price in a market that traded in yield The increasing sophistication in the market made it necessary to reconsider the applicability of the Index

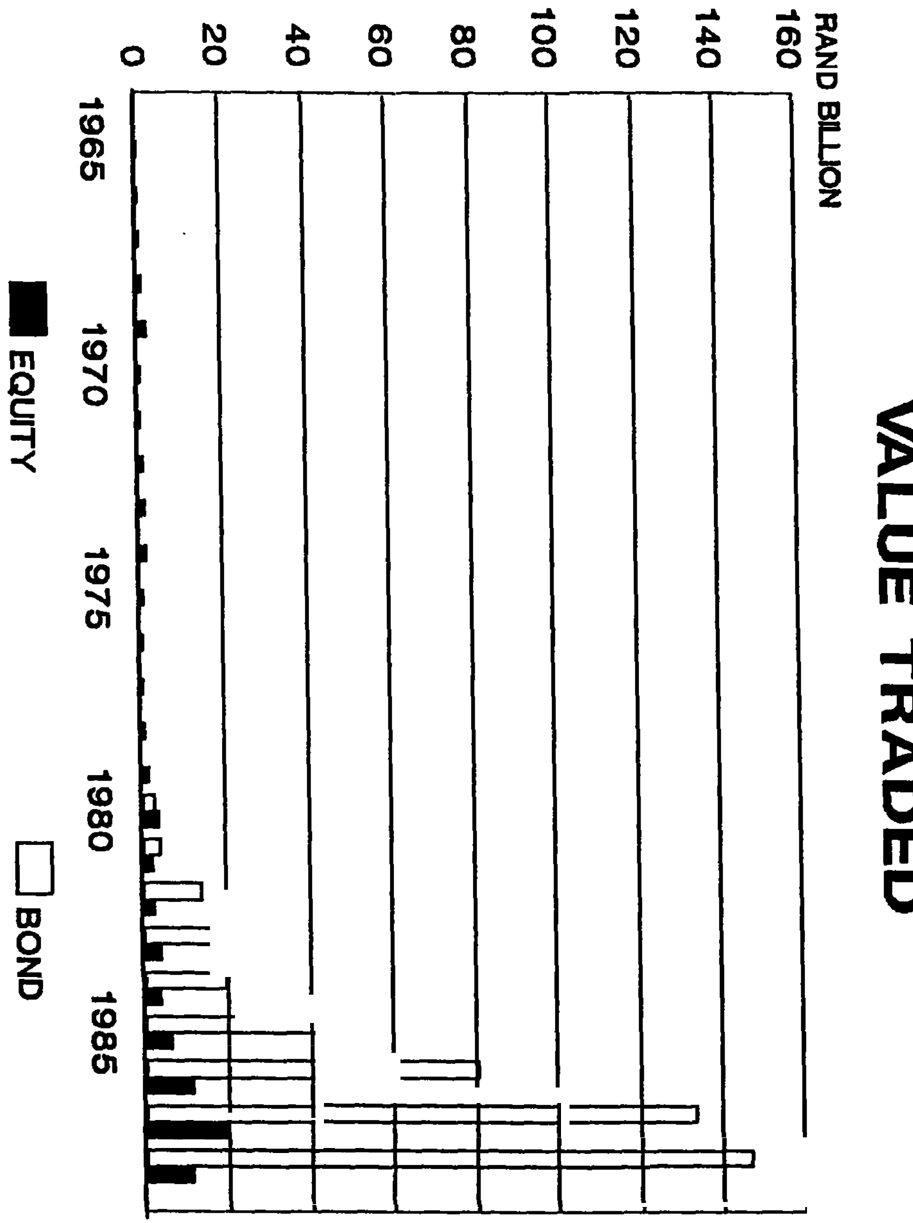
VALUE TRADED
RAND BILLION
0
20
40
60
80
100
120
140
160
1965
1970
1975
1980
1985
EQUITY
BOND

DIAGRAM 1

Following research by the Actuarial Society and consultations with market participants it was recognised that, unlike in the equity market, a single index could not satisfy the needs of investors, traders and researchers. The Fixed Interest Index was discontinued at the end of 1985 and replaced with the JSE-Actuaries Bond Performance Index and the JSE-Actuaries Yield Curve. Historical data from January 1986 were provided on the new basis when the Performance Index and Yield Curve were launched in October 1988.

2 - 2 - The revised Jse - Actuaries Bond Performance Indices

The first major objective of an index is to provide a standard against which performance can be measured. This form of index will allow the building of notional portfolios so that further aspects of performance can be investigated will enable an approximate valuation of a portfolio to be made and will facilitate the determination of the terms of entry into and exit from an internal fund that is deemed to have been invested in bonds.

The Bond Performance Index was determined as a market-capitalisation weighted index with separate and more easily understood price and income components. Given the lack of marketability in the local market and hence the difficulty in obtaining data, as well as the lack of a differential between Government and semi-government bonds, the number of sub-categories was reduced. An All Bond Index was introduced that enabled the completely passive management of bonds to be used as a performance benchmark Further details, the mathematical formulation and historical data are available from the Actuarial Society of South Africa.

3 - THE NEED FOR A JSE - ACTUARIES YIELD CURVE

The need was expressed by researchers, economists and actuaries for a description of interest rate levels in the bond market as well as a consistent record of interest rate levels over time. The use of actual bonds for gathering data was frustrated by changes in marketability and at times the inability to obtain reliable closing yields. A daily market yield curve would provide the description of the interest rate structure needed by a variety of participants. A credible market yield curve can be used as an aid in setting the terms of new issues, as an aid in determining an appropriate yield for trading unmarketable bonds or other interest-sensitive products and as a standard valuation mechanism for bond portfolios. In addition it can assist actuaries in the determination of interest rates for products and it facilitates research work by bond analysts and economists. There is also the possibility of using points from the yield curve as the basis f a financial instruments, for example financial futures.

4 - THE SEARCH FOR A YIELD CURVE

The lack of reliable data on closing yields from which to determine a yield curve was a source of some frustration to the actuaries. The JSE agreed to integrate the capture and storage of data with the needs of the Performance Index and the Yield Curve to resolve this difficulty. The availability of historical data for evaluating alternative yield curves was resolved by using data collected privately by a major life office and a leading stockbroker. Data for all trading days in 1986 was used as well as simulated data for a wide variety of market conditions.

4 · 1 · Alternatives investigated

The traditional approach to curve fitting is to find a suitable formula or group of formulae that describe the general pattern of the data The parameters for the curve are usually derived from a process that minimises the weighted sums of squares.

In most curve fitting exercises, data are readily available for each interval along the x-axis. In the South African market groups of stocks tend to trade together and data are available for only certain sections of the yield curve. There are sections where no bonds are trading at all. This clustering of the data means that the yield curve must respond to the heavy clusters while remaining relatively unresponsive in the areas where there are no data points.

A problem in fitting yield curves is that the broad shape can change wer a short period of time. The curve may need to move from an inverse position to a more normal yield curve shape wer a matter of weeks. Many of the curve fitting formulae behave poorly under such extremes.

A substantial amount of work was done by a statistical consultant in evaluating various forms of curve fitting. The approach sometimes taken in graduating mortality tables in which several curves are fitted and the result is blended to form a single curve, was felt would be unsuitable for this application. The possibility of using more than one independent variable was also rejected The inherent limitations of using one formula over a wide variety of market conditions led to the rejection of a number of curve types.

The most promising existing yield curve was found to be that developed by researchers at Grieveson Grant in Britain. Modifications were made to the basic Grieveson Grant model which gave a better solution under South African conditions. However the data problems in the local market remained a constraint and an alternative method of curve fitting, using cluster analysis and cubic splines, was developed *during* the research.

4 · 2 - The criteria for evaluation

Five criteria were used in evaluating the forms of curve fitting :

a. **Goodness-of-fit: the** curve should reflect the underlying trend of the data points without any significant deviations. Tested mathematically as the weighted average of squares of deviations of actual data points from the calculated yield curve.

b. Smoothness : the curve should not display any rapid changes in curvature. Tested mathematically as the *sum* of the absolute third differences of the calculated yield curve points at monthly intervals.

c. Stability : the curve should not shift significantly when constituents change. Tested by removing constituents at random and measuring the resultant change to the calculated points on the yield curve.

d. Future applicability : the curve should be flexible enough to adapt rapidly to different shapes or market conditions without a change in formula or approach.

e. **Ease of** understanding : the curve should be easily explainable at the conceptual level and relatively easily implementable by market practitioners.

Points a, b and c were measured mathematically while d and e were of a subjective nature. Points a and b were seen as critical and typically resulted in a trade-off; the smoother the curve the worse the goodness - of - fit.

4 - 3 - Results of the investigation

Two candidates survived the initial selection phase, namely the modified Grieveson Grant curve and the cluster analysis - cubic spline approach The former had the advantage of a simpler shape and thus greater smoothness. A major concern was its volatility from day to day with the short end forming a spoon shape under certain conditions. From a layman's point of view it was sometimes difficult to justify the shape of the curve in the short and medium ends as it clearly did not follow the pattern of the data. The curve was derived for bond market conditions in Britain and did not seem to be fully transferable to South African conditions.

The method of curve fitting chosen for the JSE-Actuaries Yield Curve was found to be superior to all others tested in that a substantial improvement in goodness-of-fit was obtained for a relatively small loss of smoothness. The future applicability and ease of understanding of the method were decisive factors in its choice.

5 - THE JSE-ACTUARIES YIELD CURVE

5 - 1 - A non - technical description

A yield curve is a means of describing the relationship between the term to maturity of the bonds in the market and their yields. A distinct pattern emerges which changes over time according to economic conditions. The yield curve provides a summary of the interest rate pattern prevailing in the market at a particular point in time.

Conceptually the method of drawing the JSE-Actuaries Yield Curve is simple to understand. The market tends to trade in groups of bonds and these groups or clusters form the starting point for curve drawing. The statistical technique of cluster analysis is used to determine which cluster each bond belongs to and the centre points of those clusters. [See Diagram 2 below.] These centre points are then pined to obtain the Yield Curve. The "joining-the-dots" is achieved by means of natural cubic splines. [See Diagram 3.]

The format in which the results are presented or stored can be varied to suit a particular need. Once the yield curve has been produced a number of points can be read off (e.g. 3 year, 5, 10 and 20 years). For storage and later re-creation the most simple form of output is to sample the curve at one year intervals from 0 to 30 years.

5 - 2 - Constituents: the Jse - Actuaries Selected Bond List

The JSE-Actuaries Selected Bond List is a list of the most marketable bonds traded on the floor of the Johannesburg Stock Exchange. The list is drawn up quarterly in advance by a subcommittee of the Actuarial Society together with JSE representation. The bonds on the List are used in the calculation of the Performance Index and in drawing the Yield Curve.

JSE – ACTUARIES YIELD CURVE

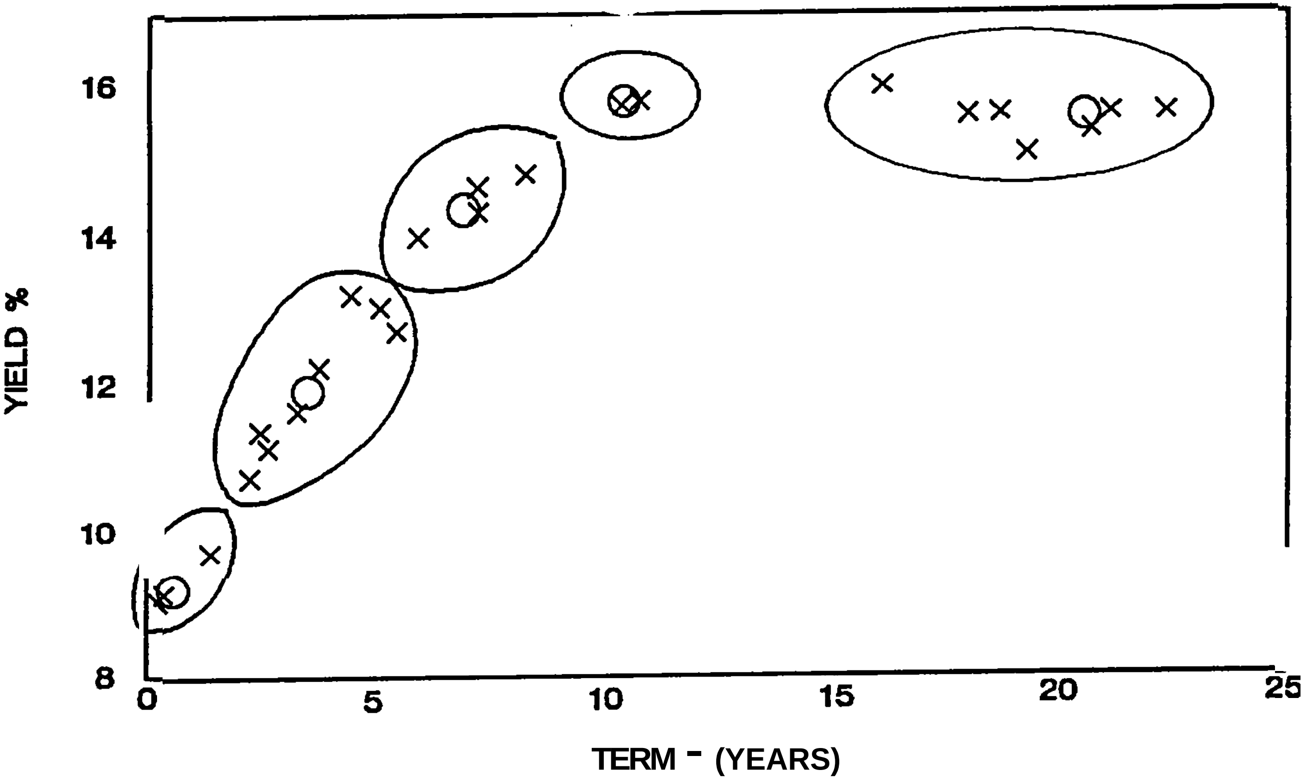

DIAGRAM 2

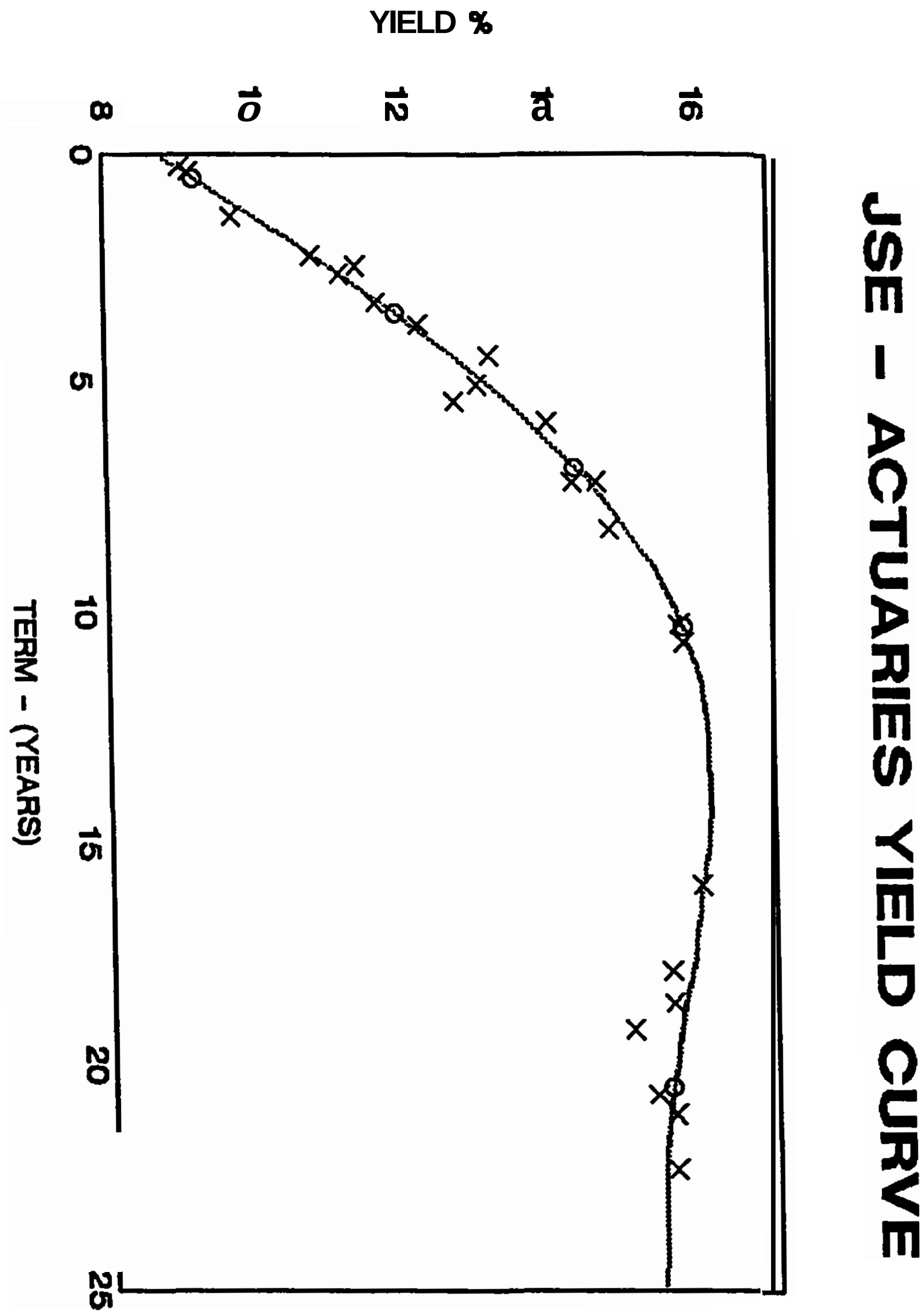
JSE – ACTUARIES YIELD CURVE
YIELD %
8
10
12
16
TERM – (YEARS)
0
5
10
15
20
25

DIAGRAM 3

In order to obtain reliable information on yields the bonds must by definition be traded frequently in the market. Marketability is considered to be the most important criterion for selection. Along with recent trade volumes, consideration is given to knowledge generally available in the market as to tap issues, consolidations a any other activity likely to substantially affect the marketability of particular bonds.

Instruments eligible for inclusion are Government debt, semi-government utilities and any other debt guaranteed by Government. Interest must be at a fixed rate, payable at six monthly intervals and the redemption date and amount must be fixed at inception. As the derivation of the Yield Curve requires that data is spread over the full maturity spectrum, the criterion of marketability must be tempered by the need to provide this spread of data.

The Selected Bond List was found in 1988 to cover some 65 % of the market capitalisation of the bond market and in excess of 90 % of the trade.

Weightings are assigned by the Subcommittee to each bond to reflect the amount available for trade in the market and hence the bond's relative importance in the market. The nominal amount in issue held in public hands, in Rand millions, is used to determine the weighting. Tap issues, consolidations or other activities leading to material changes in the amount in issue are taken into account. It is 'he intention to reflect the amount available for trade over the next quarter, not purely the amount available at the beginning of the period.

The derivation of the Yield Curve requires that a money market security with a term of 90 days be included together with the bonds in the Selected Bond List. The market in Treasury Bills in South Africa is particularly thin and reliable quotations can not always be obtained on this instrument. Instead, the 90-day Bankers Acceptance rate is used and the rate is determined from money market participants at the time that the bond yields are collected. A weighting is assigned to this security that reflects its importance as an anchor f a the short end of the Yield Curve.

5 - 3 - Mathematical formulation

By convention the yields quoted in the bond market are interest rates payable half-yearly and this practice is carried through to economic time series and most sources of interest rate information. It is important to note that the yields published from the Yield Curve are also nominal half-yearly rates. Effective annual rates can be obtained using the standard compound interest conversion.

Note that the symbol "*" is used to indicate multiplication.

The Bankers Acceptance is treated as an additional bond in the descriptions that follow. The quoted discount rate must be adjusted to a nominal half-yearly interest rate as follows:

$$\text{interest rate} = 2 * \left[\frac{1}{\left(1 - \text{discount rate} * \frac{91}{365}\right)^{2}} - 1 \right]$$

Cluster analysis

Five seed points **are** provided to enable **the** cluster analysis to begin. **The seed** points are **terms** to **maturity** expressed in years. **They are** independent of **the data** or **the date of** analysis. At **present** there are five seed points **and** it is envisaged that **the** seed **points** will remain **unchanged unless substantial** changes in **market** conditions occur over time.

n	= number of clusters
sp[j]	**= seed point for cluster j (j runs from 1 to n)**
At present :	**sp[1]** = 0.0
	sp[2] = 3.5
	sp[3] = 7.0
	sp[4] = 12.0
	sp[5] = 21.0
m	= number of bonds used in drawing the curve
Curve Date	= **date** for which curve is being determined
Maturity Dale	= **date** of maturity of the bond
D[i]	= (Maturity date in days - Curve date in days) + 365.25 **i.e.** term to **maturity** in years **(i runs** from 1 to m)
Y[i]	= closing yield of **bond on Curve** Date **Note that** yields are naninal half-yearly rates. (i **runs** from 1 tom)
W[i]	= weight attached to bond (i runs from 1 **tom)**

The cluster analysis proceeds in four stages **as follows** :

I - Objective : Determine the two bonds closest to each seed point using the criterion term to maturity.

Calculate for each bond at each seed **point** :

$$g[i,j] = \frac{W[i]}{(D[i] - sp[j])^2}$$

(i runs from 1 to m)
(j runs from 1 to n)

Take care that division by zero does **not** *occur.*

For each cluster **[j]** choose **the** two bonds giving the highest value f a g[i,j]

Rename those bonds **as** [k,j] (**k** runs from 1 to 2)
(j **runs from** 1 ton)

Important Note : It is possible that a bond may give the highest or second highest value for more than one cluster. **There** is no preferential allocation process and that bond must be allowed to contribute to more than **one** cluster. It will simplify **manipulation** to make a copy of **the data** of that **bond** and treat the copy **as** an additional bond added to the list **i.e.** increase the number of bonds 'm'.

II - Objective : compute the centre point of each cluster of two bonds

For each cluster [j] calculate the weighted average term to maturity AD[j] and weighted average yield AY[j].

For this step set all p[j] = 2

$$TW[j] = \sum_{k=1}^{p[j]} W[k,j]$$ (j runs from 1 to n)

$$AD[j] = \frac{\sum_{k=1}^{p[j]} (D[k,j] * W[k,j])}{TW[j]}$$

$$AY[j] = \frac{\sum_{k=1}^{p[j]} (Y[k,j] * W[k,j])}{TW[j]}$$

III - Objective : allocate remaining bonds to clusters.

For all bonds not already belonging to a cluster, determine the closest cluster in terms of distance over the dimensions term to maturity and yield.

Rename the unallocated bonds [z] (z runs from 1 to m-2n)

Calculate for all clusters [j]

$h[z,j] = (D[z] - AD[j])^2 + (Factor * (Y[z] - AY[j]^2)$ (j runs from 1 to n)

where the Factor is set at 0.50.

For each bond [z], determine for which cluster [j] the value of h[z,j] is a minimum Allocate the bond to cluster [j].

Rename the bonds as [k,j] (k runs from 3 as there are already two bonds in each cluster, but can never exceed m-2n)

Note that there is not necessarily an equal number of bonds in each cluster. In Step 3 all bands remaining are uniquely allocated to clusters unlike in Step 1 where a band may belong to more than one cluster.

IV - Objective : determine the centre point of each cluster.
For each cluster [j] calculate the weighted average term to maturity AD[j] and weighted average yield AY[j].

p[j] = total number of bonds in cluster [j] (j runs from 1 to n)

Note that the sum of all p[j]'s must equal m, the number of bonds used to fit the curve.

Calculate TW[j], AD[j] and AY[j] as above. (j runs from 1 to n)

The AD[j] values form the x co-ordinates and the AY[j] values the y co-ordinates for the fitting of the cubic spline.

The cluster analysis is performed using five seed points which results in five co-ordinate pairs. To extend the curve at the long end to a defined end point it is necessary to add a sixth co-ordinate pair before fitting the cubic spline. The last co-ordinate pair has an x co - ordinate of 30.00 and y is set equal to AY[n].

The JSE requires the Yield Curve to be determined for all points between 0 and 30 years f a certain internal applications. As none of the constituents has a term greater than 25 years, the Yield Curve is only published f a terms up to 25 years.

Fitting the cubic spline

A natural cubic spline fits a mathematically smooth curve to a set of (x,y) data points. There is a separate cubic polynomial curve linking each pair of points in turn. Each cubic polynomial is joined to its successor at a data point and fits smoothly into it. For six data points there ate five cubic polynomials which, when pined together at the data points, make up the cubic spline.

Note : the symbols used have a different meaning to those in the cluster analysis.

Note : the symbol "*" is used to indicate multiplication so as to avoid confusion with x.

n	= number of data points minus one	
x[j]	= x coordinate of the j'th data point	(j runs from 0 to n)
y[j]	= y coordinate of the j'th data point	(j runs from 0 to n)
h[j]	= x[j+1] - x[j]	(j runs from 0 to n-1)
k[j]	= 2 * (h[j] + h[j-1])	(j runs from 1 to n-1)
d[j]	= y[j+1] - y[j]	(j runs from 0 to n-1)
b[j]	$= 6 * \left(\frac{d[j]}{h[j]} - \frac{d[j-1]}{h[j-1]} \right)$	(j runs from 1 to n-1)
c[j]	= intermediate calculations	(j runs from 0 to n)
S[j](x)	= cubic polynomial j evaluated at x	(j runs from 0 to n-1)

Set up and solve the following **tri-diagonal matrix** system far the **c[j] intermediate** values:

$$\begin{bmatrix} k[1] & h[1] & 0 & 0 & \cdots & 0 & 0 \\ h[1] & k[2] & h[2] & 0 & & & \vdots \\ 0 & h[2] & k[3] & & & & \vdots \\ \vdots & 0 & h[3] & & & & 0 \\ \vdots & \vdots & & & h[n-3] & k[n-2] & h[n-2] \\ 0 & 0 & 0 & \cdots & 0 & h[n-2] & k[n-1] \end{bmatrix} * \begin{bmatrix} c[1] \\ c[2] \\ \vdots \\ \vdots \\ c[n-2] \\ c[n-1] \end{bmatrix} = \begin{bmatrix} b[1] \\ b[2] \\ \vdots \\ \vdots \\ b[n-2] \\ b[n-1] \end{bmatrix}$$

Solve the system of equations by standard matrix solving methods i.e. invert the matrix on the left and then multiply by the matrix of **b[j]'s** to **obtain** the matrix of **c[j]'s.** In the absence of matrix handling facilities in the **computer** system or language being used, the technique of Gaussian elimination should be used

The formula f a each cubic polynomial is given by

$$S[j](x) = \left[\frac{c[j]}{6 * h[j]} * (x[j+1] - x)^3\right] + \left[\frac{c[j+1]}{6 * h[j]} * (x - x[j])^3\right]$$

$$+ \left[\left(\frac{y[j+1]}{h[j]} - c[j+1] * \frac{h[j]}{6}\right) * (x - x[j])\right]$$

$$+ \left[\left(\frac{y[j]}{h[j]} - c[j] * \frac{h[j]}{6}\right) * (x[j+1] - x)\right]$$

where j runs from 0 to n - 1 ;

c[0] = c[n] = 0 ;

and x is limited between **x[j]** and **x[j+1].**

F a x between 0 and **x[0]** use cubic **polynomial S[0](x).**

Specific points **on** the **curve** are obtained **from** the **S[j](x) formula where** x is the term to **maturity** requested. A **curve** can be **plotted** by evaluating the polynomials **S[j]** *at* regular one year intervals and joining each set of adjacent evaluated **points** by means of straight line segments.

It is advisable to carry out all **calculations** to the full number of decimal places available in the particular environment In particular there should be no **rounding** of the results *of* the cluster analysis before the cubic spline is fitted The only rounding necessary is that to the **S[j](x)** values which are rounded **to** *two* decimal places.

5 - 4 - Calculation and dissemination

Yields are determined in the market for the roughly 30 bonds on the JSE-Actuaries Selected Bond List at the close of trade each day. These bonds are used to determine the JSE-Actuaries Yield Curve for the day. The Curve is then used by the JSE to determine yields on the balance of the 2000 bonds in the market, using a table of differentials to allow for differences in risk raring and marketability.

The Yield Curve is sampled at yearly intervals and this series is published in the financial press and on Reuters. In the general press only the most important yields are quoted (3, 5, 10 and 20 years). The 20 year or Long Bond Yield is quoted daily on press and television. An annual publication updates the historical series and provides details of the mathematical formulation and rules for selection of bonds. Further information is available from the Actuarial Society of South Africa.

6 - FURTHER APPLICATIONS: THE USE OF A DURATION YIELD CURVE IN THE MANAGEMENT OF INTEREST RATE RISK IN A LIFE OFFICE ENVIRONMENT

The sale of term-certain annuities at guaranteed interest rates represents a form of gearing for the with-profits policyholders of the life office. While this non-profit business remained a relatively small part of the total, the assets were pooled with those from the with-profit business. As a result of restrictive tax legislation, life offices in South Africa developed "back-to-back" products consisting of an annuity and a linked endowment. As the volume of this business increased dramatically the necessity to split the assets and reduce the risk of loss on the annuity business became imperative.

A number of techniques were explored for the reduction of risk and although they are well established in theory, their practical application was constrained by the nature of the local bond market. The technique of immunisation, familiar to actuaries, was found to offer some particular problems in the practical implementation that necessitated a relook at the valuation of the assets and liabilities.

Immunisation requires, amongst other criteria, that the present value of the assets is at least equal to the present value of the liabilities. We took issue with the calculation of present values, insisting that the assets and liabilities be measured on a consistent basis. This lead to an examination of various methods of determining the appropriate interest rate at which to discount the assets and the liabilities.

The market value of assets is readily determinable from closing yields each day in the market. The liabilities do not have a market value and there is difficulty in determining an appropriate discount rate on a frequent basis The rate at which new annuities are sold is not necessarily applicable as it is influenced to a large extent by competitive forces. Market value was thus not seen as being usable for the consistent valuation of assets and liabilities.

Market theorists will recognise that the duration of a bond is a more appropriate measure than term to maturity. The method of curve fitting described in the paper was successfully used for fitting a duration yield curve to the bond data. The duration of the instrument rather than term to maturity, together with the yield, is input into the curve fitting process. Modifications are required to the seed points and the terminal point of the curve.

The liabilities can be valued using the concept of duration and determining an appropriate discount rate from the daily duration yield curve. The same mechanism can readily be used to value the assets. If interest-bearing instruments other than conventional bonds are used their valuation is facilitated by the use of the duration yield curve for obtaining discount rates. This "duration curve value" ensures that the assets and liabilities are valued on a consistent basis.

7 - EXPERIENCE TO DATE

The JSE-Actuaries Yield Curve has been run on daily data from January 1986 to the present, a period during which the Yield Curve has flattened and become inverted. An area of concern has been the unusually large weighting of one bond in the market and the subsequent "pulling effect" on the curve. This situation arose from the consolidation of a utility issue and the subsequent creation of derivatives on the physical bond This unhealthy concentration of activity is likely to decrease as Government bonds are also consolidated and derivatives are created on notional bonds.

It is acknowledged that other forms of yield curve may give a superia result in specific instances where a researcher has control over the curve fitting process and can massage the curve parameters before using the result. However where the curve is determined on a regular basis without intervention we believe the inherent flexibility in the curve fitting process is a distinct advantage wer other methods available.

8 - ACKNOWLEDGEMENTS

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