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LIFE INSURANCE, INFLATION AND INVESTMENT

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ASSURANCE - VIE, INFLATION ET INVESTISSEMENT

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RESUME

Cet article traite du problème de l'assurance - vie et de l'inflation. Le modèle mathématique présenté peut être utilisé pour élaborer des produits dans lesquels les bénéfices et les primes peuvent être ajustées par le même indice. L'application du modèle dépend des actifs disponibles pour l'investissement des réserves. Le modèle est général et applicable à la plupart des formes d'assurances - vie et de pensions.

Le principal résultat est qu'il est possible de construire des produits intégralement adaptés à l'inflation, dont le tarif des primes dépend uniquement du taux d'intérêt réel et non du taux d'intérêt inflationnaire ou nominal. Le taux d'intérêt réel joue donc un rôle très comparable au taux d'intérêt nominal dans les produits traditionnels. La politique d'investissement de la compagnie doit donc être modifiée en sorte de réaliser des gains non plus sur la base d'un taux de rentabilité nominal, mais sur celle d'un taux de rentabilité réel.

La nécessité d'une telle indexation est fondée sur la nature à long terme des contrats d'assurance - vie et sur le fait qu'ils sont normalement souscrits pour remplacer un revenu, lors du décès, du départ en retraite, etc. L'auteur a participé à l'élaboration de tels produits, il y a quelques années en Islande.

On utilise d'abord l'équation aux différences de Thiele, pour déterminer les réserves d'assurance à terme et à capital différé. Sur cette base, on est capable de calculer les primes : on observe alors que ces primes dépendent uniquement du taux d'intérêt réel k_t, avec :

$$1 + k_t = (1 + i_t)/(1 + j_t)$$

où i_t est le taux d'intérêt nominal et j_t l'inflation. Nous trouvons ensuite facilement des formules de prime où k_t peut être constant ou nul. Nous observons que les mêmes résultats peuvent être donnés pour la plupart des formes d'assurance - vie.

Dans les deux derniers chapitres, nous considérons deux applications du modèle, et nous commentons certains instruments d'investissement disponibles en Norvège. Les deux applications sont des produits avec indexation garantie et des produits dont l'indexation n'est pas garantie, mais est un objectif de la compagnie.

LIFE INSURANCE, INFLATION AND INVESTMENT

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1 - INTRODUCTION

We will address in this paper the problem of life insurance and inflation. A mathematical model will be presented that may be used to develop products where both the benefits and premiums may be adjusted by the same index. The application of the model is dependent on the assets available for investing the reserves. The model is general and may be applied to most forms of life insurance and pensions.

The main result given is that it is possible to construct fully inflation adjusted products where the premium tariff only depends on the real interest rate and not the inflation or nominal interest rate. The real interest rate has therefore much the same role as the nominal interest rate in traditional products. The investment policy of the company must therefore be changed from earning a nominal rate of return to a real rate of return.

2 - THE NEED OF INFLATION - INDEXING

A life insurance contract guarantees a payment at a future unknown date at death, disability, retirement or if the insured is alive. The contract is long term with a duration from a few years up to several decades. The policy is typically written to replace the income that is lost when the sum is due. To achieve this, it is necessary to uphold the real value of the *sum* insured, and therefore the *sum* should be adjusted after an inflation index.

There will of course also be written policies to cover fixed obligations with no need of inflation indexing.

3 - HISTORICAL SOLUTIONS

For individual life and pension insurance in Norway we have used two different approaches to inflation indexed products.

For individual life insurance the premium is adjusted annually after an inflation index. The premium then is increasing nicely, but the problem is that the sum is increasing less than the inflation. As we are getting near the end of the premium paying period, the premium is increased in line with the inflation, and the *sum* hardly increased at all.

For individual pension the *sum* or pensions are indexed after a social security index. In this case we get the opposite problem that the sum increases, smoothly, but that the premium increases more and more. Near retirement age the increase of the premium is prohibitive, and the indexing is stopped seven years before retirement age.

For some products sold the last two years these problems are somewhat reduced. The surplus is credited the policy each year. For indexed life insurance the surplus is used to additional *increase* of the sum and for indexed pension used to reduce the premium increases. Depending on the relation between the interest rate assumed in the premium tariff, the interest earned and the inflation, this may a may not give the same yearly indexing of the sum and premiums.

4 - THE HISTORY OF THE MODEL

The reason for my engagement in developing a model for life insurance and inflation is that I have been involved in *setting* up a new life insurance company in Iceland. In a country with historical high inflation, life insurance has been almost non-existing. Our challenge was to develop a product that was viable also in a high inflation environment. The model presented in this paper is based on the work done to develop such an insurance product

5 - DEFINITIONS

We first make some definitions

P - fust year annual premium

P_t - annual premium in year t

S - fust year sum insured

S_t - sum insured in year t

V_t - the retrospective reserve

q_x - the probability that an insured of age x should die within one year

p_x - the probability that an insured of age x is alive after one year

${}_tp_x$ - the probability that an insured of age x is alive at age $x+t$

i_t - interest rate in year t $\quad t = 1,2,3\ldots$

j_t - inflation in year t $\quad t = 1,2,3\ldots$

$$f(t) = \prod_{h=1}^{t} (1 + j_h) \qquad t = 1,2,3\ldots$$

$f(0) = 1$

k_t - real interest rate in year t where

$$(1 + i_t) = (1 + k_t)(1 + j_t) \qquad t = 1,2,3,\ldots$$

$$g(t) = t\prod_{h=1}^{t} (1 + k_h) \qquad t = 1,2,3,\ldots.$$

$g(0) = 1$

Another possible definition of the real interest rate is

$k^1_t = i_t - j_t$

We observe that

$(1 + i_t) = (1 + j_t) + k^1_t$

$(1 + i_t) = (1 + j_t) + k_t(1 + j_t)$

or

$k_t = k^1_t / (1 + j_t)$

It is however necessary to use the defined real interest rate k_t to develop our model as we will see in chapter 7.

6 - THE MODEL

Our model is quite simple that :

$P_t = P\,f(t)$ $\qquad t = 1,2,3,...$

$S_t = S\,f(t)$ $\qquad t = 1,2,3,...$

This is the same as saying that both the premium and sum should always have the same real value.

We wish to find the formula for the first year premium and show that under certain condition it is independent of the inflation j_t.

7 - PREMIUM RESERVE FOR TERM AND ENDOWMENT INSURANCE

We assume a discrete model where the premium Pt is paid yearly in advance and the sum is paid at the end of the year it is due. The same results could also be given in a continuous model. We are also only looking at the net premium reserve. The gross premium reserve could have been calculated in the similar manner.

The reserve is calculated by this recursive formula from Jordan (1975)

(1) $(V_t + Pf(t))(1 + i_{t+1}) = q_{x+t} Sf(t+1) + p_{x+t} V_{t+1}$

$V_0 = 0$

which we solve for V_{t+1}

(2) $V_{t+1} = ((V_t + Pf(t))(1 + i_{t+1}) - q_{x+t} Sf(t+1)) / p_{x+t}$

$V_0 = 0$

Formula (2) may then be written as

$$V_{t+1} = (V_t(1+i_{t+1}) + f(t+1)\,(P\frac{1+i_{t+1}}{1+j_{t+1}} - q_{x+t}S))/p_{x+t}$$

or

$$V_{t+1} = (V_t(1+i_{t+1}) + f(t+1)\,(P(1+k_{t+1}) - q_{x+t}S))/p_{x+t}$$

$$V_0 = 0$$

For simplicity we introduce

$$P^A_{x+t} = q_{x+t}\, S$$

$$P^B_{x+t} = P(1+k_{t+1}) - P^A_{x+t}$$

We then get this formula

$$(3)\quad V_{t+1} = (V_t(1+i_{t+1}) + f(t+1)\, P^B_{x+t})/p_{x+t}$$

$$V_0 = 0$$

which gives

$$(4)\quad V_{t+1} = \sum_{s=0}^{t} \left(\prod_{h=s+2}^{t+1} (1+i_h)\right) f(s+1)\, P^B_{x+s}/{}_{t+1-s}p_{x+s}$$

which again may be written as

$$(5)\quad V_{t+1} = f(t+1)\, g(t+1) \sum_{s=0}^{t} P^B_{x+s}/(g(s+1)\,{}_{t+1-s}p_{x+s})$$

8 - PREMIUM FORMULAS FOR TERM AND ENDOWMENT INSURANCE

We then find the premium formulas for term and endowment insurance using *(5)* and that

$V_n = 0$ for term insurance

$V_n = f(n)\, S$ for endowment insurance

where n is the duration of the contract. For term insurance we solve this equation for P

$$\sum_{s=0}^{n-1} P^B_{x+s}/(g(s+1)\,{}_{n-s}p_{x+s}) = 0$$

$$\sum_{s=0}^{n-1} (P(1+k_{s+1}) - P^A_{x+s})/(g(s+1)\,{}_{n-s}p_{x+s}) = 0$$

$$(6)\quad P = \left(\sum_{s=0}^{n-1} P^A_{x+s}/g(s+1)\,{}_{n-s}p_{x+s}\right)/\left(\sum_{s=0}^{n-1} 1/(g(s)\,{}_{n-s}p_{x+s})\right)$$

We find the premium for endowment insurance by solving this equation

$$f(n)\, g(n) \sum_{s=0}^{n-1} P^B_{x+s}/g(s+1)\,{}_{n-s}p_{x+s}) = f(n)\, S$$

$$(7)\quad P = \left(\sum_{s=0}^{n-1} P^A_{x+s}/g(s+1)\,{}_{n-s}p_{x+s} + S/g(n)\right)/\left(\sum_{s=0}^{n-1} 1/(g(s)\,{}_{n-s}p_{x+s})\right)$$

As we observe, the premium (6) and (7) are only dependent on the real interest rate k_t and not the inflation j_t, or interest rate i_t.

9 - THE REAL INTEREST RATE IS CONSTANT

When the real rate is constant i.e.

$k_t = k$ for $t = 1,2,3,\ldots$

then

$g(t) = (1+k)^t$ for $t = 1,2,3,\ldots$

Then (6) and (7) which are the premium formulas for term and endowment insurance get

$$(8)\quad P = \left(\sum_{s=0}^{n-1} P^A_{x+s}/(1+k)^{s+1}\,{}_{n-s}p_{x+s}\right)/\left(\sum_{s=0}^{n-1} 1/((1+k)^s\,{}_{n-s}p_{x+s})\right)$$

$$(9)\quad P = \left(\sum_{s=0}^{n-1} P^A_{x+s}/(1+k)^{s+1}\,{}_{n-s}p_{x+s} + S/(1+k)^n\right)/\left(\sum_{s=0}^{n-1} 1/((1+k)^s\,{}_{n-s}p_{x+s})\right)$$

10 - THE REAL INTEREST RATE IS ZERO

When the real interest rate is zero for all t i.e.

$k_t = 0$ for $t = 1,2,3,\ldots$

then

$i_t = j_t$ for $t = 1,2,3,\ldots$

then

$g(t) = 1$ for $t = 1,2,3,\ldots$

Then (6) and (7) which are the premium formulas for term and endowment insurance get

$$(10)\quad P = \left(\sum_{s=0}^{n-1} P^A_{x+s}/{}_{n-s}p_{x+s}\right)/\left(\sum_{s=0}^{n-1} 1/{}_{n-s}p_{x+s}\right)$$

$$(11)\quad P = \left(\sum_{s=0}^{n-1} P^A_{x+s}/{}_{n-s}p_{x+s} + S\right)/\left(\sum_{s=0}^{n-1} 1/{}_{n-s}p_{x+s}\right)$$

We have now, when the interest and the inflation is the same, found the premium formula for term and endowment insurance where both the premium and sum insured is annually increased by the inflation as set up in the model. We do also observe that the first years premiums are independent of both the interest i_t and inflation j_t, for all t.

11 - OTHER FORMS OF INSURANCE

The results above were based on (1) which may be written

$$(V_t+Pf(t))\,(1+i_{t+1}) = P^A_{x+t}\, f(t+1) + p_{x+t}\, V_{t+1}$$

$$P^A_{x+t} = q_{x+t}\, S$$

$$V_0 = 0$$

when calculating the premium formulas (6) - (9) we have only used P^A_{x+t} and never the information that $P^A_{x+t} = q_{x+t}\, S$, We may thererore use the same formula and draw the the same conclusions if P^A_{x+t} is another function of x + t.

If for example

$$P^A_{x+t} = P_{x+t}\, S$$

and $V_n = 0$, formula (1) is actually the difference formula for the reserve of an immediate yearly deferred annuity with a duration of n years. We may use formula (6), (8) or (10) to calculate the premium. We do also observe that the first year premium P is independent of i_t and j_t when $i_t = j_t$ and only dependent on k_t else.

We easily see that the technique may be used to give similar results for most kinds of life insurance products including deferred annuities, widower pensions, children's pensions.

12 - INTERPRETATION OF THE RESULTS

The result in formulas (10) and (11) that the first year annual premium is independent of the interest rate i_t and inflation j_t when these are equal is quite obvious. All the elements premium, sum insured and the premium reserve are adjusted by the same index. We are only revaluing all the elements and nothing material changes. In a sense we are just changing the reference system. The first year annual premium is the same as if we assumed no inflation and zero interest rate. It is not easy to give a straightforward interpretation of the general premium formulas (6) - (9).

13 - THE INVESTMENT ASPECTS

This paper may be regarded as a general paper on life insurance that has nothing special to do with investments.

We have however shown how important the expected return of our assets are when developing an inflation adjusted product. We have formally shown that if you are able to get a real rate of return on your assets, you may also develop a product where both the premium and sum insured may be indexed by the same inflation index.

14 - APPLICATIONS OF THE MODEL

There may be several applications of this model. We will look at two types of products f a which we may use these results.

First we assume that it is possible to invest the reserves in inflation adjusted assets possibly with a real rate of return. We may then develop products where we guarantee that the premium and sum insured both will be indexed after the inflation. When calculating the premiums (6) and (7), we use the real interest rate k_t we expect in the future. When deciding which value of k_t to use, it is important be careful about assumptions for the future in the same manner as when assessing the interest rate in premium tables for traditional products. It is also important to only guarantee the inflation adjustments as long as there are possibilities to invest the premium reserve in inflation adjusted assets.

The second application is if it is not possible to invest the reserve in inflation adjusted assets. We may then not guarantee indexing for the future. However, using (6) and (7) and assuming a realistic real interest rate for the future, for instance 4%, we may guarantee the nominal value insured S. If the premium is adjusted by the inflation, the interest earned is credited a policy account yearly and the surplus is used to increase the sum insured, the sum will also be increased by the inflation if the real rate of interest is as assumed in the premium formulas (6) or (7).

15 - INDEXED BONDS

We have this summer seen issued indexed bonds in Norway. The real interest rate is guaranteed for 5 years. This investment instrument is not used before by life insurance companies in Norway. If the reserves is invested in such bonds, we may develop a product as described in the first part of chapter 14 with guaranteed indexing for 5 years. To do this we are however depending on reinvesting the interest earned and investing new premiums paid the first 5 years in indexed bonds too.

BIBLIOGRAPHY

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