CONTRIBUTION N° 45

ARBITRAGE CONDITIONS FOR OPTION PRICING ON THE SOFFEX

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CONDITIONS D'ARBITRAGE LORS DE L'EVALUATION D'UNE OPTION DU SOFFEX

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ABSTRACT

Arbitrage opportunities (associated with the lower limit on calls, convexity conditions and put - call parities) are analyzed for the three months from August 29 to November 30 1988 on the Soffex or Swiss Options and Financial Futures Exchange. A first series covers 12 underlying securities and a second 13 ; in each case there is a total of 9,0192 possible arbitrage positions. Basic data includes prices as quoted every 15 minutes, two riskfree rates (Eurofranc rate and domestic money market rate) and dividends actually distributed during the period under consideration Transaction costs are partly taken into account, but bid / ask price spreads are not considered.

While rejecting the hypothesis that prices used for options and underlying securities are synchronized, the study identified arbitrage opportunities with positive outcomes after deduction of transaction costs based on convexity conditions and put - call parities, operations of these kinds being more complex than those based on limit conditions. The study thus brought to light some inefficiencies in the Soffex market. Nevertheless, these results should be accepted only with caution since actual arbitrage operations would have to be based on bid and ask prices and not, as here, on trade prices the potential gains would thus be lower, assuming they existed at all.

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July 1989

1. INTRODUCTION

On May 19, 1988, the Swiss Options and Financial Futures Exchanges' (SOFFEX) started its activities. Option contracts are presently traded on 13 underlying stocks and on the Swiss Market Index (SMI)[(1)]. As it is the case for other similar markets, arbitrageurs are expected to play a fundamental role. Because of their activities, one can hypothesize that option prices must meet the boundary conditions derived by Stoll (1969), Merton (1973b) and Klemkosky and Resnick (1979). If these conditions were not satisfied, hedging strategies would generate profits before transaction costs expressing non-synchronization in prices and potential market inefficiencies.

As shown by Phillips and Smith (1980), arbitageurs' transaction costs are not negligible. Hence, hedging strategies by efficient traders cannot disregard these costs. Clearly, market inefficiencies would result when persistent above normal hedging profits after transaction costs are detected.

This paper examines the boundary conditions on call prices and the call-put parity relationships for the second quarter of activity on the SOFFEX market, the first three and a half months being considered as a starting period during which the market operators learned how to trade more efficiently on this new market. Because a complete set of data on bid and ask prices was not available, tests were conducted with intradaily observations of market prices. Section 2 presents the arbitrage relationships tested in the paper. The market structure and the data used are described in Section 3. Testing methodology and strategies are explained in Section 4. Section 5 contains the empical results of both ex-post and ex-ante tests with or without transaction costs. Finally, Section 6 offers some concluding remarks. This study investigates the impact, if any, of several variables such as option maturity, dividends and prices of underlying stocks on the frequency and size of option prices violations.

2. ARBITRAGE RELATIONSHIPS

2.1 Lower bounds for American call options

Merton (1973a) proved several necessary conditions f a rational pricing of options (calls and puts) in perfect markets. These conditions .do not rely on a specific pricing model and give boundaries for option values. The lower boundary conditions were expanded and generalized by Galai (1978) for American calls with no compensating adjustments if dividends are paid to the stockholders during the life of the option, which is the situation for the SOFFEX options.

The first lower bound for an American call option is that its value cannot be less than the larger of zero or the stock price minus the striking price, that is the value of the call if exercised immediately:

$$C(S, T, X) \geq \max(0, S-X),$$

where C (S, T, X) is the market value of the American call option with exercise price X and time to maturity T and S is the current market price of the underlying stock. Galai

(1978) refers to the above condition as the "weak dominance condition".

The second lower bound condition, the "strong European call dominance' condition", assumes that the call will be held to maturity. If it is known with certainty that the stock will pay n dividends Dj at calendar time t_j over the life of the option, then :

$$(2) \quad C(S,T,X) \geq \max\left[0, S - Xe^{-rT} - \sum_{j=1}^{n} D_j e^{-rt_j}\right],$$

where r is the default-free interest rate.

A third condition, the "strong early exercising dominance condition", examines whether it is optimal to exercise the unprotected American call just before the stock goes ex dividend. With n dividends D_j, this lower bound is :

$$(3) \quad C(S,T,X) \geq \max\left\{0, \max_j\left[0, S - Xe^{-rt_j} - \sum_{i<j} D_i e^{-rt_i}\right]\right\}.$$

2.2 Calls convexity condition

Call prices are convex in the striking price for identical maturity options on the same underlying stocks. For a defined by,

$$X_2 = \alpha X_1 + (1-\alpha) X_3$$

with $0 \leq \alpha \leq 1$ and $X_1 < X_2 < X_3$ being striking prices, we should have :

$$(4) \quad C(S,T,X_2) \leq a\,C(S,T,X_1) + (1-a)\,C(S,T,X_3).$$

Note that these call prices depend on the same stock price as simultaneity of the option prices must be observed here.

2.3 Put-Call parity

Assuming that it is optimal to hold to maturity both the put and the call options to make up a hedge, Klemkosky and Resnick (1979) derived the following *boundary* conditions :

$$(5) \quad C(S,T,X) \leq P(S,T,X) + S - Xe^{-rT} - \sum_{j=1}^{n} D_j e^{-rt_j} \quad \text{(long hedge condition)},$$

$$(6) \quad P(S,T,X) \leq C(S,T,X) - S + Xe^{-rT} + \sum_{j=1}^{n} D_j e^{-rt_j} \quad \text{(short hedge condition)}.$$

A sufficient condition for no premature exercise of the call is that the present value of the dividends foregone is less than the present value of the return that could be earned from investing the exercise price at the risk-free rate :

$$(7) \qquad \sum_{j=1}^{n} D_j e^{-rt_j} < X[1-e^{-rT}],$$

condition equivalent to that of Roll (1977) under perfect market assumptions [(2)]. In contrast, rational premature exercise of the put is always possible. If at inception,

$$(8) \qquad C(S,T,X) < X[1-e^{-rT}] - \sum_{j=1}^{n} D_j e^{-rt_j},$$

it would be to the put holder's advantage to exercise the put immediately against the hedger. It is also possible that this inequality fails to hold at inception but holds later with an interim call price. Hence, it is impossible to establish a perfect short hedge since one is not able to assume that the put would never be prematurily exercised.

3. MARKET STRUCTURE AND DATA

3.1 Market structure

Options traded on the SOFFEX over the studied period are dividend payout unprotected American options on stocks [(3)]. If dividends are paid to owners of the underlying stocks during the lives of options, no adjustments are made on the striking price. Each option contract represents rights to 5 shares of the underlying stock. At any time, options an the same underlying stock (options class) are available with 4 expiration months, the longest maturity an option can have being 6 months [(4)]. In its expiration month, an option normally expires on the Saturday immediately following the third Friday of the month.

For each option class and each maturity, there is a minimum of 3 striking prices. When a new maturity is proposed, 3 different striking prices are introduced based on the last closing price of the underlying stock on the Zurich Exchange. For an option series with a particular expiration date, when the closing price of the underlying stock on the Zurich Exchange is smaller (or greater) than the smallest (or greatest) striking price during two consecutive business days, a new smaller (or greater) striking price is introduced, except when the maturity of this option series is less than 3 weeks.

The SOFFEX trading is organized around two basic groups : brokers and market makers. Brokers act both as agents and as principals. They receive and execute customers' orders. They are also authorized to trade for their own account. Market makers are responsible for providing liquidity in options of underlying stocks assigned to them. It is their duty to maintain reasonable bid and ask prices throughout the hours of trading. They have no ability to accept orders from the public. Furthermore, in meeting their market responsabilities or during their hedging activity, market makers are faced with transaction costs, including the expenses of the bid-ask spread. Unfortunately, these latter costs are ignored in this study. But one can expect that they are smaller than those incurred by other market operators.

When examining the efficiency of the SOFFEX, one must consider the above normal profits earned by market makers, who are the traders incurring the lowest trading costs. Supposing that market makers cannot obtain any above normal gains after transaction costs with hedges based on violations of boundary conditions or put-call parity relationships, hence it is a fortiori impossible for other traders with higher trading costs to realize abnormal gains on the same market.

The direct transaction costs for a market maker who can trade directly on the Swiss Stock Exchanges are SF 0.50 per option contract, plus SF 8 each time an option is exercised, in addition to taxes equivalent to 0,09 ‰ of the underlying stock price at which the trade occurs (5).

3.2 Data

This study covers 68 trading days from August 29, 1988 to November 30, 1988. The data consists of three sets of 90192 intradaily observations made up respectively of call, put and underlying stock market prices that were in effect on the SOFFEX at the end of each 15 minutes intervals of time starting at 12:00 a.m. until 2:00 p.m. (6). In this manner, option prices and stock prices are matched together. Dividends used are actual dividends instead of estimates. The Eurofranc rate and the Swiss domestic monetary rate matching with the option maturities are both used for the risk-free rate (7).

4. TESTS AND STRATEGIES

4.1 Ex-post and Ex-ante tests

Ex-post tests of boundary conditions, convexity conditions or call-put parity relationships state that the price of an option is not less than the computed rational boundary. It is assumed that the observed violations and the corresponding hedging strategies are based on the same prices. For this reason, these ex-post tests are not valid for testing market efficiency since the observed prices at t are not necessarily the prices for the next transactions. Persistent violations can only indicate that prices of options and underlying stocks are not well synchronized or are not equilibrium prices.

When studying market efficiency, a relevant question is whether an arbitrageur obtains abnormal arbitrage profits by trading at time t+1 given the information at time t. For ex-ante tests, any violation registered at time t is taken as a signal for a transaction that is assumed to be executed at time t+1. There is no guarantee that a profit identified at t will imply an abnormal return at t+1 since the market prices generally change between t and t+1. Trading based on information at time t will result in a risky position at time t+1. (8) Thus, hedging becomes a risky activity and it is difficult to distinguish between the normal profit which represents a normal reward for this risk, and the abnormal gain which can be associated with market inefficiency.

4.2 Boundary and convexity conditions

For an immediate exercise, the tested hypothesis is :

$$(9) \qquad \varepsilon_1 \equiv S - X - C - M \leq 0,$$

where M is the transaction cost. To profit from a mispriced call, hence inducing $\varepsilon_1 > 0$ (type 1 violation), the strategy consists of buying the underpriced call, exercising it immediately and selling the stock thus acquired.

The tested hypothesis for the strong European condition is :

$$(10) \qquad \varepsilon_2 \equiv S - Xe^{-rt} - De^{-rt} - C - M \leq 0,$$

provided that the stock will be subject to no more than one dividend payment D at time t. If $\varepsilon_2 > 0$ (type 2 violations), the profit strategy consists of buying the call, shorting the stock and lending an amount equal to the sum of present values of the exercise price and the expected dividend, and holding this portfolio until expiration.

The tested hypothesis related to the strong dominance condition may be written as :

$$(11) \qquad \varepsilon_3 \equiv S - Xe^{-rt} - C - M \leq 0,$$

where $\varepsilon_3 > 0$ (type 3 violation). The profit strategy is analogous to the preceding one except that if premature exercise is optimal, the hedge is terminated at t instead of T.

The call convexity test is :

$$(12) \qquad \varepsilon_4 \equiv C(X_2)\, \alpha (X_1) - (1 - \alpha)\, C(X_3) - M \leq 0.$$

when type 4 violations occur ($\varepsilon_4 > 0$), the trading strategy for obtaining profits is to sell one contract of the call with the striking price X_2 and buy a contracts with the lowest exercise price and 1- a contracts with the highest exercise price.

4 3 Call-put parity relationships

When call prices are too high relative to put prices, an arbitrageur can set up a profitable long hedge by selling a call, assuming a long position in the underlying stock, buying a put and financing in part the position by borrowing at the risk-free rate the amount $Xe^{-rT} + De^{-rt}$, where D is the dividend payment at t. To prevent such a profitable strategy, the following condition must hold :

$$(13) \qquad \varepsilon_5 \equiv C - P - S + Xe^{-rt} + De^{-rt} - M \leq 0.$$

In case many type 5 violations of this condition are observed, hedging profits are guaranteed if condition (7) holds at inception (9).

The conversion of a call into a put is used if the put price is too high relative to the call price, providing that the put will never be prematurely exercised. This short hedge consists of purchasing the call, shorting the underlying stock, lending at the risk-free rate an amount, $Xe^{-rT} + De^{-rt}$, and selling the put. This strategy is unprofitable if :

(14) $\varepsilon_6 \equiv P - C + S - Xe^{-rt} - De^{-rt} - M \leq 0.$

In contrast to a call, possible rational premature exercise of a put may not be determinable at inception. For this reason, it is impossible to establish a perfect short hedge and test options market efficiency using (14) except when condition (8) fails to hold for all interim sets of values. Consequently, if (8) fails to hold at inception, the short hedge profit calculated at that time is only conditional.

5. RESULTS

5.1 Lower boundary conditions

Ex-post test

For each of the 90192 registered market call prices, ex-post hedges were constructed using conditions (9) and (10). There are no violations (type 1) of the immediate exercise condition. However, 2293 violations (type 2) of the strong European dominance condition are detected before trading costs. When dividends are paid to owners of underlying stocks during the remaining lives of call options (48840 observations), no ex-post violations (type 3) of the strong early exercising dominance condition occur. After trading costs, no profitable type 2 violations are detected. These results are summarized in table 1.

TABLE 1: Number and Average Amounts of Type 2 Violations Classified by Type of Test, Costs and Dividends

Type of Test	Number of Violations			SF Average Amount of Violations		
Ex-Post	Without Dividends	With Dividends	Total	Without Dividends	With Dividends	Total
Before costs	1070	1223	2293	6.9	1.6	4.1
After costs	0	0	0	0	0	0
Ex-Ante						
Before costs						
Profits	870	1006	1876	7.2	1.6	4.2
Losses	60	62	122	(17.0)	(10.6)	(13.8)
Total	930	1068	1998	5.63	0.88	3.1
After costs	0	0	0	0	0	0

The absence of type 1.2 and 3 violations after costs and the presence of only a smaller number of type 2 violations before costs show that option prices are generally within their rational boundaries. Consequently, the hypothesis that call prices are synchronized with their underlying stock prices cannot be rejected although these, prices are not necessarily fully synchronized.

The preeminence of type 2 violations was previously describes for Canadian transaction data by Halpem and Turnbull (1985) who reported that 58 % of their observed violations occured in the type 2 category. By expressing type 2 in terms of type 1 violations, they showed that there will tend to be more type 2 than type 1 violations, especially for long maturity options. This explanation cannot fully account for results presented in table II, since before trading costs, only 20,4 % of the calculated ex-post type 2 violations occur for options with maturity of over 60 days.

Table II - Number (followed by Percentage) of Ex-Post Type 2 Before Costs Violations Classified by Dividends, Number of Days to Maturity, and In-the-Money Category

SF Amount of Violation / Number of days to maturity	Without dividends							With dividends						
	0 - 5		6 - 10		over 10		Total	0 - 5		6 - 10		over 10		Total
	DI	I	DI	I	DI	I		DI	I	DI	I	DI	I	
0-7	92 (8.6)	259 (24.2)	0	0	0	0	351 (32.8)	163 (13.4)	607 (49.6)	0	0	0	0	770 (63.0)
8-30	6 (0.6)	85 (7.9)	0	12 (1.1)	0	0	103 (9.6)	40 (3.9)	216 (17.7)	0	0	0	0	264 (21.6)
31-60	0	111 (10.4)	0	62 (5.8)	0	19 (1.8)	192 (18.0)	0	73 (6.0)	9 (0.7)	42 (3.4)	16 (1.4)	5 (0.4)	145 (11.9)
over 60	21 (2.0)	55 (5.1)	39 (3.6)	63 (5.9)	19 (1.8)	227 (21.2)	424 (39.6)	20 (1.6)	18 (1.5)	3 (0.2)	3 (0.2)	0	0	44 (3.5)
Total	119 (11.2)	510 (47.6)	39 (3.6)	137 (12.8)	19 (1.8)	246 (23.0)	1070 (100)	231 (18.9)	914 (74.8)	12 (0.9)	45 (3.6)	16 (1.4)	5 (0.4)	1223 (100)

Notes : DI denotes Deep-In-the-Money. I denotes In-the-Money.

In table II are examined the amounts and frequencies of type 2 before costs violations with respect to maturity, dividends and in-the-money category (10). As there is no definition of deepin-the-money, it was decided to consider a call as deep-in-the-money if the stock price was more than 20 % above the exercise price. The largest frequency of before costs violations is obtained when calls are in-the-money (83,4 % when no dividends are paid and 78.8 % when dividends are paid). In addition, hedging profits per contract are in majority less than or equal to SF 5 (58.8 % when no dividends are paid and 93.7 % when dividends are paid). Observed type 2 violations are more important for long maturity hedges with no dividends remaining than for short maturity hedges with dividends. Theses results differ from those of Galai (1978) who found that the number of violations increased for deep-in-the-money and short maturity options. However, given the absence of profitable violations after costs, it is of little interest to extrapolate guidelines for hedging based on prices, time to maturity and dividends.

Ex-ante test

Without transaction costs and based on signals given by ex-post gains at time t, 1998 hedging strategies were constructed using t+1 prices. Profits are obtained for 1876 hedges before costs with an average profit of SF 4.2 per contract although losses occur in other cases (see table I). However, there are no profitable hedges after transaction costs. Thus, it is clear that, for the studied period, a market maker using ex-post profits before instead of after transaction costs as signals for the construction of a hedge would have undoubtly incurred large losses. The absence of positive returns associated with ex-ante risky hedging strategies are in favour of the market efficiency hypothesis.

Table III - Number (followed by Percentage) of Ex-Ante Type 2 Before Costs Violations Classified by Dividends, Number of Days to Maturity, and In-the-Money Category

SF Amount of Violation / Number of days to maturity	Without dividends							With dividends						
	0 - 5		6 - 10		over 10		Total	0 - 5		6 - 10		over 10		Total
	DI	I	DI	I	DI	I		DI	I	DI	I	DI	I	
0-7	81 (9.3)	212 (24.4)	0	0	0	0	293 (33.7)	143 (14.2)	514 (51.0)	0	0	0	0	657 (65.2)
8-30	4 (0.5)	63 (7.2)	0	10 (1.1)	0	0	77 (8.8)	42 (4.2)	173 (17.2)	0	0	0	0	215 (21.4)
31-60	0	80 (9.2)	0	53 (6.1)	0	17 (2.0)	150 (17.3)	0	44 (4.4)	8 (0.8)	36 (3.6)	14 (1.4)	4 (0.4)	106 (10.6)
over 60	17 (2.0)	35 (4.0)	35 (4.0)	54 (6.2)	16 (1.8)	193 (22.2)	350 (40.2)	14 (1.4)	8 (0.8)	3 (0.3)	3 (0.3)	0	0	28 (2.8)
Total	102 (11.8)	390 (44.8)	35 (4.0)	117 (13.4)	16 (1.8)	210 (24.2)	870 (100)	199 (19.8)	739 (73.4)	11 (1.1)	39 (3.9)	14 (1.4)	4 (0.4)	1006 (100)

Notes : DI denotes Deep-in-the-Money. I denotes In-the-Money.

The impact of dividends, time to maturity and in-the-money category of options on the average magnitude and frequency of ex-ante type **2** violations is reported in table III. Comparison with table II shows no marked differences between **ex-ante** and ex-post hedges. But such a comparison may mean very **little** due to the fact that after trading costs, all profits disappear.

5.2 Call convexity **tests**

Among the **43104** triplets of calls written on the same underlying stock and with identical maturity, **2661** type 4 violations were found before transaction costs, this number falling to **1880** when these costs are maintained at their minimum level (table IV, case 1). In any case, when trading costs are included, the number of type 4 violations **seems** to be more **important** and **profitable** than other categories of violations detected for other boundary conditions. The ex-ante tests confirm these results since it is observed that before costs **1803** hedges generate profits, number which turns out to be **1278** in case **1**, 708 in case 2 and **563** in case **3,** the **latter** being the least favourable situation considered here for hedgers.

TABLE IV : Number (Followed by Percentage) and Average Magnitude of Convexity Violations Classified by Type of Test

Type of test / SF Amount of Violation	0 - 5	6 - 15	16 - 30	31 - 50	51 - 75	over 75	Total	SF Average Magnitude of Violation
Ex-Post								
Before costs	1270 (47.7)	588 (22.1)	398 (15.0)	200 (7.5)	116 (4.4)	89 (3.3)	2661 (100)	16.9
Case 1	681 (36.2)	463 (24.6)	357 (19.0)	188 (10.0)	106 (5.7)	85 (4.5)	1880 (100)	25.0
Case 2	196 (19.6)	305 (30.5)	228 (22.8)	113 (11.3)	100 (10.0)	58 (5.8)	1000 (100)	29.3
Case 3	203 (25.9)	258 (32.9)	173 (22.0)	80 (10.2)	47 (6.0)	24 (3.0)	785 (100)	22.2
Ex-Ante								
Before costs	749 (41.6)	414 (23.0)	302 (16.7)	163 (9.0)	101 (5.6)	74 (4.1)	1803 (100)	19.6
Case 1	369 (28.8)	312 (24.4)	281 (22.0)	153 (12.0)	94 (7.3)	70 (5.5)	1278 (100)	30.1
Case 2	95 (13.4)	208 (29.4)	173 (24.4)	99 (14.0)	85 (12.0)	48 (6.8)	708 (100)	32.2
Case 3	111 (19.7)	181 (32.1)	145 (25.8)	67 (11.9)	41 (7.3)	18 (3.2)	563	23.5

Notes :

Case 1 : Transaction costs are assumed to be SF2 per hedge with the proportion of the lowest exercice price α equal to 0.5.

Case 2 : Transaction costs are assumed to be SF10 per hedge with the proportion of the lowest exercice price α equal to 0.1.

Case 3 : Transaction costs are the mean of costs incurred when calls are bought or sold with α equal to 0.5 and the purchased calls are exercised.

In type 4 violations, around 50 % of calls are in-the-money but less than 10 % are out-of-the-money, and more than 60 % have a life to maturity between 8 and 30 days. The only marked difference between hedges with or without dividends is the average magnitude of profits, which is higher when dividends are to be paid during the remaining lives of calls. Before costs, the higher percentage of violations is associated with arbitrage profits less than or equal to SF 5 per contract. After trading costs and in cases 2 and 3, profits are concentrated between SF 6 and SF 30 per contract.

Contrary to our previous results, these type 4 violations seem to be a source of profitable ex-ante arbitrages. During the period covered by this study, an arbitrageur who had relied on ex-post violations after costs in order to build up hedges, would have obtained profits in 77 % of the hedging strategies for case 1, 81 % for case 2 and 82 % for case 3 (11). With respect to the convexity condition, the SOFFEX seemed to present some inefficiencies during this period. This suggests that, because the arbitrage is more complex to implement, the market makers may have been less concerned by the convexity condition than by the lower boundary conditions, in contrast to the findings of Galai (1979) and Bhattacharya (1983) who reported only a few type 4 violations for the CBOE call-options.

5 3 Call-put parity

Ex-post tesl

Following Klemkosky and Resnick's procedure (1979), out of the 45422 potential profitable long hedges identified, 6372 were eliminated because condition (7) failed to hold at inception (12). The average gain before trading costs for each of the 39050 remaining hedges is SF 72.2 per contract. As shown in table V, the majority of these arbitrage profits before trading costs (23561 hedges representing 60.3 % of the sample) are, however, less than or equal to SF 40 per contract. The price range does not seem to have a deterministic effect on type 5 violations, despite the finding that 68.4 % of the violations occur for underlying stock prices less than or equal to SF 3500 with a surge reaching a maximum of 9603 hedges for prices between SF 1500 and SF 2000. For the sample studied, dividends do not explain the observed violations since 93.2 % of the profitable long hedges do not include any dividends. Moreover, the percentage of violations is higher (32.3 %) for options with time to maturity between 30 and 60 days. When trading costs are introduced, 20634 hedges still have profits after costs, the average gain per contract being SF 85.5. In addition, profits follow the same pattern as before trading costs with respect to dividends, option's remaining life and price range of the underlying stock prices.

tabla V - Number of Lx- Post Profitable Long Hedges (Before or After Costs)
Classified by Amount of Violation, Stock Price Range, Dividends and Time lo Expiration

Factors \ SF Amount of Violation	0-40		41-100		101-500		501-1000		over 1000		TOTAL	
	B	A	B	A	B	A	B	A	B	A	B	A
Stock Price Range (SF)												
250-500	4181	1705	96	87	11	7	0	0	0	0	4288	1799
1000-1500	2	3	2	1	0	0	0	0	0	0	4	4
1500-2000	7434	4223	1808	824	361	252	0	0	0	0	9603	5299
2000-2500	725	443	291	195	168	116	0	0	0	0	1184	754
2500-3000	3004	1435	1257	830	872	630	13	10	0	0	5146	2905
3000-3500	3953	1990	1822	694	711	515	0	0	0	0	6486	3199
over 3500	4262	1338	2448	1501	4948	3424	619	364	62	47	12339	6674
Total	23561	11137	7724	4132	7071	4944	632	374	62	47	39050	20634
Value of Dividend (SF)												
0	23169	10652	7062	3668	5755	3868	348	200	62	47	36391	r u n
13	0	0	0	0	0	0	0	0	0	0	0	0
25	245	354	452	314	184	149	0	0	0	0	881	817
72,5	147	131	210	150	1132	927	289	174	0	0	1778	1382
75	0	0	0	0	0	0	0	0	0	0	0	0
108	0	0	0	0	0	0	0	0	0	0	0	0
120	0	0	0	0	0	0	0	0	u	0	0	0
Total	23561	11137	7724	4132	7071	4944	632	374	62	47	39050	20634
Time to Expiration (days)												
0-7	1923	140	75	19	19	0	0	0	0	0	2017	159
8-14	1949	467	304	182	229	109	[illegible]	0	0	0	2482	758
15-21	2098	854	583	335	584	397	67	32	18	14	3350	1632
22-30	2136	1302	809	636	1155	884	129	55	16	16	4245	2893
31-60	8396	4174	2654	1168	1555	824	4	0	0	0	12609	6166
61-90	5227	2701	1785	747	1698	1248	55	31	0	0	8765	4727
91-120	766	619	606	447	669	451	0	0	0	0	2041	1517
120-150	624	495	519	416	1105	870	87	82	28	17	2363	1880
151-190	442	385	389	182	58	161	289	174	0	0	1178	902
Total	23561	11137	7724	4132	7071	4944	632	374	62	47	39050	20634

The total number of short conversions is 44020. Because condition (8) holds at inception, 9549 short hedges were eliminated. Since our data covered only three months, it was impossible, as described by Klemkosky and Resnick (1980). to verify if this condition holds for the beginning of each interim week and each hedge until the expiration date of the options. In considering that the put will not be prematurily exercised to cause a destruction of the hedge, profits obtained from conversion strategies tend to be overevaluated for the 34471 remaining short hedges for which condition (8) fails to hold at inception. The average gain per hedge before costs is SF 106.8.

Table VI - Number of Ex-Post Profitable Short Hedges (Before or After Costs)
Classified by Amount of Violation, Stock Price Range, Dividends and Time to Expiration

Factors \ SF Amount of Violation	0-40		41-100		101-500		501-1000		over 1000		TOTAL	
	B	A	B	A	B	A	B	A	B	A	B	A
Stock Price Range (SF)												
250-500	2725	1161	0	0	0	0	0	0	0	0	2725	1161
1000-1500	1	1	1	0	11	11	0	0	0	0	13	12
1500-2000	3504	2269	1402	784	1279	1125	39	30	0	0	6223	4208
2000-2500	629	285	191	142	202	158	0	0	0	0	1022	585
2500-3000	2925	928	782	567	777	591	42	24	0	0	4526	2110
3000-3500	3013	1377	1445	843	789	480	20	8	0	0	5267	2708
over 3500	4764	1740	3126	1563	5593	,3273	900	738	312	262	14695	7576
Total	17561	7761	6947	3899	8651	5638	1000	800	312	262	34471	18360
Value of Dividend (SF)												
0	17258	7486	6699	3673	7565	4894	857	693	308	262	32687	17008
13	1	0	0	0	0	0	0	0	0	0	1	0
25	156	120	72	54	355	334	7	7	0	0	590	515
72.5	142	155	176	172	731	410	136	100	4	0	1189	837
75	0	0	0	0	0	0	0	0	0	0	0	0
108	4	0	0	0	0	0	0	0	0	0	4	0
120	0	0	0	0	0	0	0	0	0	0	0	0
Total	17561	7761	6947	3899	8651	5638	1000	800	312	262	34471	18360
Time to Expirations (days)												
0-7	2651	127	170	33	26	12	0	0	0	0	2847	172
8-14	1204	434	527	97	199	112	8	0	0	0	1938	643
15-21	925	594	588	304	631	369	101	100	27	18	2272	1385
22-30	845	676	606	500	1261	815	230	198	141	130	3083	2319
31-60	4883	1785	1838	535	1408	822	102	63	19	17	8250	3222
61-90	4309	1494	1142	589	1263	818	238	230	79	57	7031	3188
91-120	1482	1350	780	639	1517	1008	1	0	0	0	3780	2997
120-150	839	870	713	624	1338	979	166	117	10	6	3066	2596
151-180	423	431	583	578	1008	703	154	92	36	34	2204	1838
Total	17561	7761	6947	3899	8651	5638	1000	800	312	262	34471	18360

Notes : B denotes before costs A denotes after costs.

Slightly more than 50 % of the hedging profits are less than or equal to SF 40 per hedge, as can be seen in table **VI.** Moreover, 57.4 % of the **profits** occur when underlying stock prices are less than or equal to **SF** 3500 with a surge reaching 6223 **profitable** hedges in the **SF** 1500-SF **2000** stock **price** range. In addition, the percentage of short hedging profits is 94.8 % when no dividends are paid and 44.3 % when the option's expiration **time** ranges between 30 and 90 days, **results** being comparable to those obtained for long hedges. After trading costs, **18360** short hedging **returns** remain **positive with** an average profit of **SF** 138.8 per contract. Moreover, violations follow the same pattern as before trading costs withrespect to dividends, lives of the options and price range of the underlying stocks.

Table VII - **Distribution of** Ex-Post Profitable **Long** Hedger **According** to Ex-Ante **Long Hedges**

SF Amount of Ex-Ante Profits or Losses \ SF Amount of Ex-Post Violation	0-40		41-100		101-500		501-1000		over 1000		TOTAL	
	B	A	B	A	B	A	B	A	B	A	B	A
under 120	68*	49	29	24	37	37	0	3	0	0	134	113
(120-41)	143	257	99	61	31	37	4	0	0	0	476	355
(40-0)	1070	2025	276	133	68	19	2	0	0	0	1416	2197
0-40	15330	6531	1508	650	113	47		0	0	0	16951	7429
41-100	1500	835	4323	2145	540	398		1	0	0	6364	3379
101-500	107	57	535	411	5321	1655	73	101	0	0	6036	4224
501-1000	0	0	-	0	79	106	458	215	11	9	548	330
over 1000	0	0	0	0	0	0	12	11	42	31	54	42
Total	20418	9754	6769	3624	6189	4319	550	aai	53	40	33979	18068

Notes : B denotes before oorts. A denotes after oortr .

- 68 is tho numbr of ox-ante losses greater than SF 120 per oontraot for whloh the corresponding profitable ex-post profits are less than or equal to SF 40 per oontraot. All other values in this table are oomputed in the same way.

Ex-ante tests

Using ex-post profitable hedges before trading costs as signals for trading, 33979 **ex-ante** long hedges were **constructed** from h e 39050 **profitable** ex-post hedges. Among them, 4026 losses (11.8 % of the sample) and 29953 gains (88.2 % of the sample) are **detected. Table** VII demonstrates that almost all losses occur **when** small **ex-post** gains **(87.1 %** are less than or equal to **SF** 40 per **contract)** end up as small losses (84.8 % are less than **or** equal to **SF** 40 per contract).

When ex-ante arbitrages are based on positive ex-post hedging returns after trading costs, the 20634 ex-post long hedges provide 18068 ex-ante **long** hedges from **which** 15403, or 85.3 %, are profitable and 2665, or 14.7 %, are unprofitable **(see** table VIII). **As** is the case before costs, unprofitable hedges are concentrated in small losses (82.4 % are less than or equal to **SF** 40 per contract) and generally originate from **small** ex-post gains (87.8 **%** of these profits are less than or equal to SF 40 per contract).

Gains before and after trading costs continue to follow the same pattern. For example, the percentage of these profitable hedges before and after costs is, respectively, 68.5 % and 67.7 % for stock prices less than or equal to SF 3500, 93.2 % and 89.4 % in the SF 0 dividend class as well as 54.8 % and 52.9 % when the option's expiration time is comprised between 30 and 90 days (see table VIII).

Amont the 29797 ex-ante short hedges examined, when ex-post profitable hedges before trading costs serve as signals for trading, 25765 (86.5 % of the sample) produce profits before costs and 4032 (13.5 % of the sample) are unprofitable. After trading costs, hedges are profitable in 13119 cases (83.1 % of the sample) and unprofitable in the remaining 2669 cases (16.9 % of the sample). With or without trading costs included, around 80 % of the ex-ante unprofitable hedges originate from ex-post gains less than or equal to SF 40 per contract.

Table VIII - Number of Ex-Ante Long Hedges (before or after costs)
Classified by Amount of Profits or Losses, Stock Price Range, Dividends and Time to Expiration

Factors \ SF Amount of Profits or Losses	under 120		(120-41)		(40-0)		0-40		41-100		101-500		501-1000		over 1000		Total	
	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A
Stock Price Range (SF)																		
250-500	0	0	0	0	602	387	3041	1105	84	73	10	6	0	0	0	0	3787	1572
1000-1500	0	0	0	0	1	0	1	2	1	0	0	0	0	0	0	0	3	2
1500-2000	13	7	26	34	708	604	5726	3087	1559	709	313	220	0	0	0	0	8845	4661
2000-2500	8	4	5	4	103	81	532	298	288	159	139	96	0	0	0	0	1020	642
2500-3000	10	11	45	50	547	358	2108	964	1052	676	748	536	10	8	0	0	4520	2550
3000-3500	-	7	75	71	584	431	2027	1208	1924	943	618	441	0	0	0	0	5636	2801
over 3500	100	84	325	196	871	341	2716	731	1906	1199	4208	2925	538	322	54	42	10718	5840
Total	134	113	476	355	3416	2197	16951	7420	6364	3379	6836	4224	548	330	54	42	33979	18060
Value of Dividend (SF)																		
0	105	90	424	292	3324	2106	16690	7112	5839	3080	4929	3309	298	174	54	42	31663	16148
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	4	10	29	43	186	284	391	267	160	129	0	0	0	0	770	713
72.5	29	33	48	53	63	48	75	32	134	82	947	786	250	156	0	0	1546	1210
75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
108	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
120	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total	134	113	476	355	3416	2197	16951	7420	6364	3379	6836	4224	548	330	54	42	33979	18060
Time to Expiration (days)																		
0-7	0	0	0	1	112	28	1552	180	64	17	16	0	0	0	0	0	1744	141
8-14	1	2	2	6	156	78	1558	341	261	147	192	91	1	0	0	0	2168	665
15-21	7	4	9	3	151	119	1673	636	510	292	303	339	60	29	16	12	2931	1434
22-30	8	10	16	11	190	142	1659	1004	698	544	1005	763	108	46	14	14	3698	2534
31-60	17	10	199	68	1147	755	6000	2897	2251	975	1331	702	4	0	0	0	10949	5407
61-90	30	29	126	180	1302	715	3329	1588	1888	581	1440	1075	50	28	0	0	[illegible]	[illegible]
91-120	11	11	26	12	118	154	550	392	493	363	580	392	0	0	0	0	1778	1324
120-150	41	33	57	44	123	136	887	306	404	317	924	782	75	71	24	16	2085	1685
151-180	19	14	41	30	117	73	244	236	300	143	45	130	250	156	0	0	1016	784
Total	134	113	476	355	3416	2197	16951	7420	6364	3379	6836	4224	548	330	54	42	33979	18060

Notes . B denotes before costs   A denotes after costs.

Table IX – Distribution of Ex-Post Profitable Short Hedges According to Ex-Ante Short Hedges

SF Amount of Ex-Ante Profits or Losses \ SF Amount of Ex-Post Violation	0-40 B	0-40 A	41-100 B	41-100 A	101-500 B	101-500 A	501-1000 B	501-1000 A	over 1000 B	over 1000 A	TOTAL B	TOTAL A
under 120	43*	47	21	27	32	39	3	3	2	3	101	119
(120-41)	293	325	98	83	47	57	1	2	1	1	440	468
(41-0)	3077	1728	320	261	90	92	3	1	1	0	3491	2082
0-41	10189	3645	1421	808	194	146	2	5	0	1	11806	4605
41-100	1410	799	3234	1544	933	670	3	4	1	0	5581	3017
100-500	211	148	939	655	5991	3715	144	106	3	3	7288	4627
500-1000	2	3	5	6	151	109	645	518	27	18	830	654
over 1000	0	1	2	0	8	8	34	22	216	185	260	216
Total	15225	6696	6040	3384	7446	4826	835	661	251	211	29797	15788

Notes : B denotes before costs. A denotes after costs.

* 43 is tho number of ax-ante losses greater than SF 120 per contract for which the corresponding profitable ex-post profits are less than or equal to SF 40 per contract. All other values in this table are computed in the samr way.

Almost all profitable hedges before or after costs are without dividends. Tie to maturity tends to be a little longer for short hedges than for long ones, and less concentrated in stock prices with values less than or equal to SF 3500. (see table X).

Table X - Number of Ex-Ante Short Hedges (Before or After Costs)
Classified by Amount of Profits or Losses, Stock Price Range, Dividends and Time to Expiration

Factors \ SF Amount of Profits or Losses	under 120 B	under 120 A	(120-41) B	(120-41) A	(40-0) B	(40-0) A	0-40 B	0-40 A	41-100 B	41-100 A	101-500 B	101-500 A	501-1000 B	501-1000 A	over 1000 B	over 1000 A	Total B	Total A
Stock Price Range (SF)																		
250-500	0	0	0	0	598	196	1766	918	0	0	0	0	0	0	0	0	2364	1080
1000-1500	0	0	1	1	1	0	0	1	1	0	11	11	0	0	0	0	14	13
1500-2000	0	1	26	20	715	413	2807	1569	1186	658	1088	943	35	26	0	0	5858	3629
2000-2500	5	7	17	a	100	92	446	159	156	101	157	124	0	0	0	0	881	486
2500-3000	10	12	42	36	519	340	2019	459	632	461	644	473	36	22	0	0	3902	1808
3000-3500	0	0	69	IT	586	424	2081	760	1182	480	442	392	17	5	0	0	4547	2238
over 3500	86	99	285	331	972	617	3257	847	2424	1122	4726	2684	744	601	260	216	12734	6817
Total	101	119	440	468	3491	2082	11806	4605	5581	3017	7288	4627	830	654	260	216	29797	15788
Value of Dividends (SF)																		
0	64	76	378	416	3416	1999	11671	4460	5417	2870	6378	4036	708	568	256	216	28288	14628
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	7	5	27	32	77	56	55	41	307	289	6	6	0	0	479	429
72,5	37	43	60	47	48	51	58	81	109	106	603	312	116	85	4	0	1085	731
75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
120	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total	101	119	440	468	3491	2082	11806	4605	5581	3017	7288	4627	830	654	260	216	29797	15788
Time to Expiration (days)																		
0-7	0	0	3	0	110	28	2182	89	144	22	22	8	0	0	0	0	2461	147
8-14	6	0	7	13	181	65	889	208	459	84	163	79	7	0	0	0	1662	546
15-21	4	4	7	13	150	111	655	410	499	251	323	290	95	95	25	16	1948	1180
22-30	11	4	4	38	205	129	551	468	907	407	1079	641	198	163	121	113	3671	1998
31-60	13	14	73	154	1268	543	3087	927	1452	378	1167	680	88	51	13	11	7141	2758
61-90	20	32	220	108	1288	617	2428	639	857	480	1058	670	107	102	48	48	6068	2728
91-120	7	11	30	26	116	246	1176	958	653	528	1325	855	1	0	0	0	3307	2624
120-150	54	42	55	64	189	204	607	556	367	483	1108	802	136	98	7	3	2647	2247
151-180	6	12	41	62	139	119	287	278	464	439	841	362	135	60	29	28	1092	1579
Total	101	119	440	468	[illegible]	[illegible]	11806	4605	5581	3017	7288	4627	830	654	260	216	29797	15788

Notes : B denotes before costs. A denotes after costs.

Based on the present study, no clear relationship can be established between dividends, number of weeks to expiration, stock price range and the amount and frequency of type 5 and 6 violations, which is consistent with the previous works of Klemkosky and Resnick **(1979, 1980).** In comparison with these authors' results, the greater number of violations reported in this study cannot be explained only on the basis of a greater number of observations used. Furthermore, the higher amounts of arbitrage profits recorded here may be partly due to higher underlying stock values (see table V). As an example, for arbitrage gains greater than SF 1000, the corresponding underlying stock price is generally greater than SF 10000 per share, with 5 shares per option's contract unit. Nevertheless, there is no indication that this latter explanation is self-s cient.

The number and amounts of type 5 and 6 violations after transaction costs tend to indicate that, over the studied period, the SOFFEX presented some inefficiencies regarding the call-put parity relationships. Ex-post violations would not express non-synchronization of options and underlying stock prices, but rather the difficulties encountered by market makers in order to make up hedges. Moreover, the fact that a market maker using ex-post violations as signals for building up long and short hedges, could have obtained arbiuage profits in **88.2 %** and **83.1 %** of the cases respectively may imply market inefficiency.

6. CONCLUSION

During the period covered by this study, no evidence was obtained from the ex-post tests of the lower boundary conditions for the rejection of the hypothesis stating that option prices are synchronized with those of the underlying stocks, even if it could not be asserted that synchronization was perfect. However, the amount and number of recorded violations of the convexity condition and of the call-put parity relationships tend to demonstrate that ex-ante after costs profitable hedges could have been realized based on ex-post profitable hedges, thus leading to the suggestion that the SOFFEX market contained some inefficiencies. These inefficiencies could be related to the market maker's reduced interest to construct more complex hedges not based on lower boundary conditions.

Unfortunately, the signifiance of this study is limited because of the short sample period, the use of market prices instead of bid-and-ask prices and the inability to report all prices quoted during a trading day. Further research should be conducted in order to examine the influence of the above factors on the results obtained in this paper.

Footnotes

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** Both assistants at the University of Geneva.

1 - This article examines options trading in 12 underlying securities, contracts being traded on the BBC security only since November 17, 1988. An additional stock (Alu Suisse) was recently introduced on May 19, 1989. Furthermore, options on the Swiss Market Index (SMI) are traded since Decmeber 7, 1988. The SMI is a value weighted market index constructed with a total of 24 securities including the 13 underlying stocks of the SOFFEX.

2 - As noted by Klemkosky and Resnick (1980), the Merton and Roll (MR) sufficient condition for no premature exercise of a call is identical to inequality (7) if the discount factor for dividends applicable between the j^{th} exdividend date and the j^{th} dividend payment date in the MR condition is introduced under perfect capital market assumptions.

3 - Common Stocks and Certificates of Participation.

4 - The present month, the two following months and the next month in the calendar choosed among a January/April/July/October expiration cycle. For example, in March, trading occurs f a options which expire at the end of March, April, May and July.

5 - Transaction costs (except those expressed by the bid-ask spread) are estimated for each arbitrage according to the nature of the transaction and the price of the underlying securities, contrary to Klemkosky and Resnick (1979, 1980) and Bhattacharya (1983) who used average costs.

6 - We only record the last market price quoted in the last 15 minutes for each option and underlying stock. In certain cases, because of the unavailability of underlying stock prices, the arithmetical mean of the last bid-and-ask underlying security price was used.

7 - Because results obtained with both rates are similar, only those calculated with the Eurofranc rate, in use by the market makers on the SOFFEX, are reported in this paper. Moreover, it is assumed that a year is made up of 360 days. Calculations computed considering 365 instead of 360 days did not yield significantly different resuts.

8 - It is assumed that a decison to make a transaction at time t is executed 15 minutes later with the market prices prevailing at that time (see footnote 6).

9 - When condition (7) fails to hold at inception, there is a probability different from zero that the call may be exercised just before the ex-dividend date following the Merton (1973a) and Roll's (1977) contention or, as shown by Klemkosky and Resnick (1980), for some interim put prices, risk free rate and term to maturity, if ever :

$$p < De^{-rt} - X [i - e^{-rt}]$$

10 - 41667 calls are out-of-the-money but none of these calls are part of type 2 violations.

11 - Using ex-post profitable violations as signals to build up hedges, the 2368 before costs possible hedges are split respectively into 1803 profitable and 565 unprofitable ex-ante hedges. The numbers of profitable and unprofitable ex-ante hedges turn out to be respectively 1278 and 382 in case 1, 708 and 166 in case 2, and 563 and 126 in case 3.

12 - In these cases, earned profits can be computed assuming, for example, that long hedges are maintained until ex-dividend dates. By doing that, 44 % of the 6372 long hedges eliminated are in fact profitable, with an average profit before costs amounting to SF 45.2.

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