CONTRIBUTION N° 21

AN EXCESS SPREAD APPROACH TO NON - PARTICIPATING INSURANCE PRODUCTS

PAR / BY

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Etats unis / United States

UNE APPROCHE PAR LA MARGE EXCÉDENTAIRE DES PRODUITS ASSURANCE NON - PARTICIPATIFS

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RESUME

Au **cours** des deux **dernières décennies, l'activité** de nombreuses compagnies **d'assurances nord ‐** américaines a **été dominée** par des produits **pouvant se** caractériser comme **non - participatifs** et orientés **investissements.** Ces **produits comprennent :** la rente **différée à** prime unique, l'assurance ‐ vie **universelle,** la rente immediate à **prime** unique, l'assurance ‐ vie **entière** à prime unique, la rente à prime flexible, les contrats **d'investissement garantis** et les **contrats** de rente à **terme avec** plan de **retraite. Les actuaires ont cherché** à mesurer la veritable **rentabilité économique** de ces produits en **comparant** la valeur **marchande** des **portefeuilles d'actifs compensateurs** à celle **des** engagements. La **difficulté à laquelle se heurte** cette **méthode, est** la &termination **d'une** valeur marchande **crédible** des engagements.

La méthode par la marge **excédentaire, présentée dans** cet article, vise **à résoudre** ce **problème** en **mesurant** la **rentabilité espérée** en **termes** de difference **entre** le **taux** de **rentabilité des actifs** et la **valeur** de **ce** taux **requise** pour remplir les engagements. Pour de **nombreux** produits **d'assurance, une partie importante** du **coût** des engagements est la **valeur** des options de **taux d'intérêt incluse dans** les polices. **L'approche** par la marge **excédentaire utilise une** technique **de fixation** du **prix des** options, **suffisamment** flexible pour **permettre aux** actuaires **d'étudier les effets** des divers schémas de **comportement des assurés ‐** et de **l'assureur ‐** dans différents environnements de taux **d'intérêt.** En plus de son application à l'analyse continue de la **rentabilité,** la **méthode** par la marge **excédentaire** peut **être utilisée** pour la **conception** du **produit,** la **fixation de son prix** et **l'analyse** du risque.

La rente **différée** à prime unique a **été choisie** comme exemple **d'application** de cette **méthode :** elle **constitue** la **meilleure démonstration de la flexibilité** la **méthode** et elle **montre** l'importance de la **modélisation** du **comportement.** Cette rente **comprend** en effet une option **laissée à l'assuré ‐ l'option** de **rachat ‐** et une option **laissée à** la **compagnie d'assurances ‐** la **capacité de réajustement** des taux **crédités.**

AN EXCESS SPREAD APPROACH TO
NON - PARTICIPATING INSURANCE PRODUCTS

BY MARK W. GRIFFIN, F.S.A.

ABSTRACT

In the past two decades the new business of many North American insurance companies has been dominated by products that could be characterized as non-. participating and investment-oriented. Such products include single premium deferred annuities (SPDAs), universal life, single premium immediate annuities, single premium whole life, flexible premium annuities, guaranteed investment contracts, and pension plan'termination annuity contracts. Actuaries have sought to measure the true economic profitability of these products by comparing the market value of the offsetting asset portfolios against the market value of the liabilities. The difficulty in this approach is in identifying a credible market value of the liabilities.

The excess spread approach introduced in this paper addresses this problem by measuring expected profitability in terms of a spread between the earnings rate on the assets and the rate required to be earned on the assets to satisfy the liabilities. For many insurance products, an important part of the liability cost is the value of interest rate options embedded in the policies. The excess spread approach incorporates an option pricing technique that is flexible enough to allow actuaries to study the effect of policyholder (and insurance company) behavior patterns in different interest rate environments. In addition to ongoing profitability analysis, the excess spread approach can be used in product pricing, design, and risk analysis.

The SPDA is chosen to demonstrate the technique because it provides the best example of the flexibility of the technique and the importance of behavior modelling. The SPDA involves an option in the hands of the policyholder - the surrender option, and an option in the hands of the insurance company - the ability to reset credited rates.

INTRODUCI'ION

The fust section of the paper is a background section. The section begins with a very brief summary of the history of the field of asset/liability matching. Also described in the background section are the SPDA product, which will be used as an example, and a Mortgage Backed Security (MBS) valuation technique, which is adapted in the second section into the excess spread technique. The second section describes the purpose of the excess spread technique and the steps that are involved in the calculations. The third section is an in depth example of how the excess spread technique can be used to design, price, and evaluate different strategies for an SPDA product. The example lengthens the paper considerably but is very important to understanding the capabilities of the technique. Section four demonstrates how the excess spread technique can be used to measure the various risks of the SPDA product.. The fifth section discusses briefly how the excess spread technique can be used on an ongoing basis to measure both the overall profitability and the different components of profitability. Section six is a summary.

I • BACKGROUND

A) Asset / Liability Background

The work done by Macaulay (1) in the late thirties, and Redington (2) in the fifties, introduced the actuarial profession to the field of asset / liability matching. The approach of duration matching they developped is still generally accepted today. Numerous authors have since written on how the calculation of the appropriate duration index will depend on the assumed stochastic process for changes in interest rates (3) (4).

The increase in interest rate volatility in the 1970s increased awareness of the value of interest rate options, both those embedded in callable bonds and MBS, and exchange traded option contracts. As a result, considerable time and effort has been spent in developing option pricing techniques for fixed income securities (5)(6). This research has been greatly aided by the ability to observe, in the fixed income marketplace, prices on option contracts and the effect embedded options have on the price behavior of callable and putable securities.

The increase in policy loan activity that came with rising interest rates awakened actuaries to the value of the fixed rate loan option many policyholders had been granted in traditional life insurance policies. Even though interest rate options clearly existed in many insurance products, the development of insurance liability option pricing techniques has been slow. This is due at least partly to the lack of a secondary market to provide price data for these options.

In 1985, Clancy (7) suggested that the cost of the option package necessary to protect an insurer against the options granted in its policies can be used to quantify the value of these policy options. The adoption of this approach is an important step for actuaries in studying interest-sensitive products. However, due to the nature of some interest-sensitive insurance products, the "necessary option package" often cannot be easily identified from the universe of fixed income options available in the marketplace. (The adjective "interest-sensitive" refers to products with a credited rate reset feature and/or possible disintermediation due to changing interest rates). Also, many of the option pricing techniques developed for particular asset classes are not well suited to valuing such a complicated option package. Fortunately, there is one asset class that has embedded options where the valuation of securities is dependent on a behavior function, the mortgage backed security (MBS). An option pricing mechanism developed for MBS, and described later in the paper, is general enough to be applied to interest-sensitive insurance products, and is incorporated into the excess spread technique.

B) The SPDA

An SPDA is an insurance product that acts like a savings account for the eventual purchase of an annuity. The SPDA purchaser makes a single premium payment which is credited to the policy account value and accrues interest at rates declared by the insurance company. The initial declared rate is usually guaranteed for a period of between one and ten years, after which the rates are reset periodically (usually annually) by the insurance company. Initial guaranteed rates available in the SPDA market move in general with the overall level of rates on assets available to insurance companies in

the fixed income marketplace. The account value accumulates at the declared rates until either the account value is used to purchase some form of like annuity product, the policyholder dies, or the policyholder decides to withdraw the account value. Usually, a scale of fixed surrender charges will be applied to the account value if the money is withdrawn within five to ten years of original deposit. In many cases during the surrender charge period, a portion of the account value can be withdrawn free of, surrender charges. If the proceeds of the SPDA are used to purchase a life annuity, there' is no current taxation of the interest income earned in the SPDA The policyholder may also surrender *one* SPDA and buy another without taxation of any of the proceeds. Depending on the policyholder's current SPDA rate, applicable surrender charges, and SPDA rates available in the marketplace, it may be advantageous f a the policyholder to surrender *one* SPDA and buy *another* one.

C) Mortgage Backed Security (MBS) Valuation

The MBS is a pass through security of principal and interest payments from a pool of residential mortgages. The pool of mortgages will have very similar coupon rates as well as time remaining to maturity. The majority of MBS are comprised of thirty year amortization fixed rate mortgages. Typically, the MBS carries a guarantee of timely payment of principal and interest from an agency of the United States government. Individual mortgage holders ordinarily have the right to prepay their mortgages at any time without penalty, regardless of the level of current interest rates. Therefore, many prepayments result from mortgage holders refinancing into lower rate mortgages. Prepayments as well as scheduled interest and principal amortization amounts are received by the holders of the MBS.

The prepayment right of mortgage holders is identical in nature to a call option on an amortizing bond. However, option valuation models developed for callable corporate bonds and option contracts on bonds are not well suited for use directly on MBS for a couple of reasons: .

i)   Cash flows from MBS are generally recognized to be path dependent. At a given point in time, the expected cash flow from a particular MBS will depend on where interest rates have been during the life of the underlying mortgages.

ii)   The prepayment rights held by the mortgage holders are exercised with only partial efficiency. Often, homeowners will prepay their mortgages for reasons other than refinancing with a chaper mortgage. Also, some homeowners will not prepay when their mortgage carries a much higher than current rate, and it would seem advantageous to refinance *. In general, however, prepayments on MBS with higher rates than current coupon MBS do tend to increase as interest rates go down. **

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* *The underlying mortgages ordinarily contain a "due-on-sale" clause that forces repayment when the property is sold. Any mortgages that default will effectively be prepaid by the guarantor. Also, if the property value has fallen, it may not be possible' for the homeowner to refinance due to lam - to - value limitations.*

** *The term "current coupon MBS" is used to describe MBS that would be securitized from currently originated mortgages.*

These mortgage holder behavior characteristics of the MBS make it necessary to use a Monte Carlo cash flow simulation *technique* to most accurately value the security (8): MBS valuation *can* be organized into four steps:

Step 1: A set of possible future Treasury interest rate paths that do not permit riskless arbitrage are calculated Along each interest rate path, at each point in time, short term (usually 90 day) rates are calculated as well as at least one other longer maturity (say five years). The mean of future rates are implied by the initial Treasury term structure and changes are assumed to be lognormal. Given the assumed volatility and correlation of 90 day and five year rates, normal deviates can be generated that will allow calculation of the required yields at each point in time along each interest rate path. The set of paths calculated will represent a finite sample from the underlying distribution. To ensure that the arbitrage conditions are met (within tolerable limits) by this finite sample, some adjustments will have to be made.

Step 2: Along each interest rate path the expected cash flows from the MBS are calculated. Considerations on projecting cash flows include ; the coupon rate of the MBS relative to current coupon MBS in that interest rate environment, the proportion of the original principal of the pool that remains, seasonality, geographic location, and other factors.

Step 3: The option ˉ adjusted spread (OAS) is calculated to be the spread which, when added to the short ˉ term Treasury rates along each path, will discount the MBS cash flows to its market price.

Step 4: *Once* the OAS has been calculated, the option adjusted duration of the MBS can be determined. This is done by shocking the initial Treasury yield curve both up and down. Typically, the shocks are parallel shocks to the forces of interest, but nonparallel shocks could be used if desired. For each shock, a set of interest rate paths is calculated as in Step 1. For each shock, the expected market price is determined by adding the OAS to the short term Treasury rates, and discounting the cash flows to get the new market values. Duration can then be calculated as negative 100 times the relative market value change for a change in interest rates of one percent.

The purpose of the OAS calculation is to provide a measure of the relative value between MBS with different coupon rates, times to maturity, and market prices. The OAS reflects the expected value of the mortgage holder's call option more accurately than a spread calculated using a static prepayment assumption. The option adjusted duration is calculated to measure the expected price effect of small changes in Treasury rates.

In some respects the SPDA is similar to the MBS. Both instruments *can* be "terminated" at any point by the exchange of fixed percentage of the "account" value, although in both cases this option is not always executed efficiently. In fact, the typical Canadian mortgage is in some ways more similar to the SPDA because it involves a resetting of the rate, usually every five years. (9)

II - THE EXCESS SPREAD TECHNIQUE

IF there were a market value of insurance liabilities, one could calculate an option-adjusted spread on the liabilities by a method very similar to the MBS method. The option - adjusted spread on the liabilities could then be compared to the option-adjusted spread on the supporting assets. Also, a market value of liabilities, when subtracted from the market value of assets, would give the market value of surplus. The market: value of surplus could be calculated periodically to measure the true economic profitability of the insurance product.

However, in the vast majority of cases there is no unambiguous market value of liabilities, therefore another approach must be adopted. The excess spread technique uses a known quantity, the market value of assets, to calculate the required spread on assets (RSA). The RSA is the spread over Treasuries that must be earned on the assets in order to satisfy the liabilities. The RSA is calculated as follows :

a)   Calculate the market value of the asset portfolio as of a certain date. At the point of product pricing and &sign, the market value of assets is simply the premium assumed to be received on the product, less up - front expenses.

b)   Calculate the Treasury forward rates as of the same date.

c)   For interest - sensitive liabilities, develop a set of Treasury interest rate paths (as described in section I.C. step 1).

d)   Calculate liability and expense cash flows. For non-interest-sensitive liabilities this will be simply a vector of cash flows corresponding to different points in rime. For interest - sensitive liabilities this will be a matrix of cash flows, a cash flow at each future period of time along each interest rate path.

e)   Determine the spread which, when added to the corresponding Treasury rates (vector or matrix), will discount the liability and expense cash flow (vector or matrix) to the market value of assets. This spread is the RSA.

In the pricing and design stage of product development, the RSA is an input to the determination of the total spread target above Treasuries for the eventual asset portfolio. Any expenses that are not incorporated in the liability cash flows must also be accounted for. Typically these might include investment expense and profit. In addition, the credit risk of the assets to be eventually purchased must be included in the total spread target.

When periodically analyzing a portfolio of assets and liabilities, the RSA can be calculated using the market value of assets and the projected liabilities at that point in rime. The RSA plus the basis point effect of any other expenses can then be subtracted from the spread above Treasuries being earned on the assets to give the expected profit. Expected profit can change over time as a result of the different relative performance of the assets and liabilities.

III - PRODUCT PRINCING, DESIGN AND STRATEGY

A) SPDA Pricing

The first step in the pricing of the sample SPDA product is to calculate a set of arbitrage-free Treasury interest rate paths. For this application each path will consist of a short - term (three - month) Treasury rate, as well as a one - year coupon Treasury rate

at each time interval (quarterly) wer the 24 year period being studied. The set of interest rate paths is used to calculate the RSA, duration, and mean term of the SPDA product shown in Table 1.

Table 1 shows the step - wise pricing of the SPDA product, each line representing one "step". Ordinarily, some of the steps in Table 1 could be combined. However, more steps are shown here to help the reader gain an understanding of the technique, and develop an intuition for other product features or other applications. For each step, four figures are shown:

1.   **RSA** : The RSA at the time of product pricing can be thought of as the borrowing cost of the liabilities expressed as a spread wer Treasuries. The RSA is expressed in terms of basis points per year (1 basis point = .01%).

2.   Marginal Effect on RSA : This line shows the marginal effect on RSA due to the feature introduced on that line.

3.   Duration : The interest rate duration is calculated as described in Section 1.C. in the fourth step of the MBS valuation procedure. In this case new present values of the liability are calculated while holding the RSA constant.

4.   Mean Term of Liabilities : The mean term of liabilities is the Macaulay duration (present - value - weighted time to maturity) averaged over the set of interest rate paths. The RSA is added to the short term Treasury ram for this calculation Only fa products where there are no interest - sensitive surrenders and the credited rate on the product does not change, will the duration and the mean term be equal. The mean term of liabilities measures the persistency of the liabilities. For example the mean term can be used as the appropriate time period for amortizing aquisition expenses.

The RSA and duration are calculated to provide earnings and duration targets used in choosing an investment strategy and the eventual selection of particular assets. The mean term is calculated to demonstrate what it can be used far, (expense amortization), and to emphasize what it must not be used for in this situation (duration target for the asset portfolio) (see table1).

Line 1 :   In line one the insurance company hypothetically issues a current-coupon five-year Treasury bond at its current market price of $100. Projecting the cash flows from the liability along the interest rate paths is trivial because they are simply the coupons and principal repayment of the five - year Treasury and are independant of the path. In this case, if one adds a spread of 0 basis points to the short term Treasury rates along each path, the cash flows discount to the market pice of the assets - hence the RSA is zero.

         The assets are the single premium received for the policy, which is assumed to be $30,000. The duration and mean term of this simplified liability are the same, equal to that of the five - year current - coupon Treasury bond. This trivial case demonstrates why following the arbitrage conditions in building the interest rate paths is essential. The model indicates that investing the proceeds from the sale of this product in five - year Treasuries would eliminate interest rate risk (duration match) and provide sufficient

earnings power (RSA = 0) to back the product.* (Note that there are no expenses and no profit target at this point). This is the result one would expect intuitively. Sets of paths that do not meet the same criteria can give very different results.

Table 1 - SPDA Product Pricing

Line Number	Description	RSA	Marginal Effect on RSA	Duration	Mean Term
1	Five-yaar Treasury	0 bp		4.0 yr	4.0 yr
2	*Bare Bones™ SPDA	0	0 b p	4.0	8.9
3	*Bare Bones™ SPDA (different base withdrawal assumption)	0	0	4.2	10.6
4	Deduct $1,800 of up-front Commissions and $300 of up-front expenses	73	+73	4.2	10.2
5	Add annual renewal expense of $100	101	+28	4.2	10.0
6	Credit 50 bp below three-year Treasuries	51	-50	4.2	10.0
7	Collect surrender charges upon policy lapse	39	-12	4.2	10.1
8	Include interest-sensitive surrenders	81	+42	3.0	7.4

Line 2 : The second line shows a "bare-bones" SPDA There are no expenses, no profit target, and no surrender charges. On this SPDA product the current five - year Treasury rate is guaranteed for the first five years. After five years the credited rate is reset annually to the then - prevailing one - year Treasury rate. In the model it is assumed that there is an annual "base" surrender rate, *equal* to the credited interest rate. At the end of 24 years any business that remains is assumed to surrender.

The surrender rate assumption and credited rate assumption have been choosen such that the SPDA product being offered at this point is really *nothing* more than a five-year Treasury that when it matures becomes a series of rolling one year Treasuries. It is simple intuition that initially an asset portfolio of five - year Treasuries will give an earnings and duration match for this product Thus one would expect the model to give an RSA which is zero and a duration that is the *same* as the five - year Treasury bond, which it does. There is now a much longer mean term of liabiities because the business is "on the books" for a much

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* *In fact, modelling any of the Treasury bonds that were used in setting the initial Treasury term structure would yield an RSA of zero and the correct interest rate duration.*

longer period due to the reset **procedure.** This **example** clearly demonstrates that f a a product **with** a rate reset like the SPDA, the **duration** (sometimes **known** as the price sensitivity **duration)** is the proper **measure** to use **for asset management purposes, not** the mean **term** of **liabilities.**

Line 3 : **Line** 3 is the **same** "bare-bones" SPDA product, but with a base surrender assumption of 6% per year f a the first **ten** years and 10% per **year thereafter.** In this **example** this change results in no **change** in the RSA, a small change in the **duration, and an increase in the mean term** of liabilities due to **an** overall reduction in surrender rates. Often, such a change in the base surrenders will also **have** a small effect on the **RSA.**

Line 4 : The **fourth** line is the first of a **number** of **pricing** steps. This line **includes** the **effect** of deducting **$1,800** of commission expense and $300 of issue expense **from the** $30,000 **of** single premium. **One** must **now add** a positive **spread** to *short* - term **Treasury rates** in order *to* **discount** the **liability** *cash* flows *to* the **new,** lower **market** value **of assets** ($27,900). The increase in RSA of 73 basis points means **that** one has to earn an **additional** 73 **basis** points **above** Treasuries to recoup the up · **front** expenses. A shortcut method f a estimating this **number** will be very intuitive to **actuaries. One** first expresses the up - front expenses **as an** average percent of the single **premium (7.25%),** and then in effect **amortizes** it over the average *semi* - **annualized mean** term of **the liabilities** (9.97 **years),** to get 73 basis points. The RSA is expressed on a **bond-equivalent a semi-annual basis, so** the mean term should be "semi - **annualized"** *to do* **the estimation,** much the **same** way that **Macaulay duration measures for** non - **callable** instruments must be **modified** to **estimate** the price **change attributable** to a specified **change** in semi · **annual** yields.

Line 5 : Line 5 **shows** the **effect** of **adding an annual** renewal expense of $100 **per policy.** The actual level of this expense will **probably** depend to some extent on **the volume** of this product that is written.

Line 6 : The assumption that **five-year** Treasury **rates** would be a edited initially and one · year **Treasury rates** on **all the** reset dates **was** just a starting point for demonstrating the model. In **fact the** insurer **may** decide *to* credit 50 basis points less than **Treasuries** both **initially** and **an all** reset **dates.** Line 6 shows that the RSA **calculation** produces **exactly** the intuitive effect when this **change** is made, a **cost** reduction of 50 **basis** points.

Line 7 : One feature that will **make the** product **"cheaper",** is **that** in the **early years** surrender charges will be collected **when** policies are surrendered. This SPDA **policy has** a surrender charge **scale** of **6%. 5%, 4%, 3%,** 2%, 1% f a the **first** six years, and the 0% for the **remainder** of **the life** of the policy. The **policyholder** has a **guarantee an return of** principal. Line 7 shows the effect **of** collecting these **surrender** charges.

Line **8** : In line 8 interest - sensitive **surrenders** are introduced. It is assumed that whenever the **credited** rate falls too far below **prevailing** new money rates for **SPDAs,** (assumed to be 50 basis points below one - year **Treasuries)** there will

be interest - sensitive surrenders in addition to the base surrenders. The policyholder is presumed to make their surrender decision based on the ability to recover surrender charges over a three " year period. F a example, in the fourth year, when the surrender charge is 3%, it would take a spread between the policy's credited rate and new money rates of at least 1% before interest - sensitive surrenders would begin to occur. Interest - sensitive surrenders are assumed to be four times the square of the rate gap that exists beyond the surrender charge amortization threshold (expressed as a percent). So if a rate gap of one percent existed, annualized interest - sensitive surrenders of four percent are assumed to occur. A rate gap of 2 percent would cause annualized interest - sensitive surrenders of sixteen percent Interest - sensitive surrenders are capped at an annualized rate of 50%.

In some SPDA products, a portion of the account value can be withdrawn each year free of surrender charges (this feature is not included in this example). This free surrender portion of the business may therefore exhibit different surrender behavior in the first six years than the remainder of the account value on which surrender charges would be levied, and should be modelled accordingly.

Introducing interest - sensitive menders into the cash flows is how disintermediation risk can be incorporated into product pricing and asset management targets. The increase of 42 basis points in the RSA shows that there is definitely a "cost" to disintermediation. Taking account of interest - sensitive surrenders also results in a lower duration. Also, increased surrenders lead to a lower mean term of liabilities.

There are really two components to the marginal cost of 42 basis points. One component is the option cost, the effect of higher surrenders in higher interest rate environments. The second component is the effect of amortizing the up - front expenses over a shorter period of time in higher interest rate environments.

B) SPDA Design and Insurance Company Strategy

The excess spread technique is very useful in addressing product design and insurance company strategy questions. It is possible to test the RSA effect of different combinations of commission, surrender charges, and credited rate levels. It may be possible to calculate a number of essentially "RSA neutral" combinations of product features, and let the marketing arm of the insurance company choose the final product from among them. For example :

	Product Description			
Commission	- Surrender Charge Scale	Initial and renewal spread below Treas.	RSA	Duration
6 2	6%,5%,4 ..	50 basis points	81*bp	3.0*years
7.252	7%,7%,6%,5%,4%...	50 basis points	81	3.2
5.52	7%,7%,6%,5%,4%...	25 basis points	80	3.4

* This is the product priced in Table 1

The calculations above assume that *sales volume does* not change f a the **different product descriptions. One could also** test the effect **on** RSA of different sales volumes resulting from **more** or **less** aggressive credited **rates.** Higher volume *of sales* would decrease initial and periodic expenses to some extent, but these savings would **presumably** be at least partly **offset** by the mare aggressive credited rates *necessary* to generate that volume.

The rate reset strategy is also a very important aspect of insurance company strategy. Rather than resetting the credited rate completely to **one - year** Treasuries **minus** 50 **basis points (new money rate) on every reset date as shown in Table 1, one could adopt** a strategy of changing the rate by a fixed **percentage** of the **difference** between **the** new **money** rate and **the** previous credited rate. It is also **possible, using** the excess spread **approach,** to test more dynamic strategies: one is shown in Table 2.

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Table 2

Different **Credited** Rate Reset Strategies

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Reset Strategy	RSA	Duration	Mean Term
Static Reset Strategies			
Reset completely to the new money rate (pricing assumption shown in Table 1)	81 bp	3.0 yr	7.4 yr
Reset to: two thirds of the new money **rate plus** one third of the previous credited rate	84	3.2	**7.3**
Reset to: one third of the new money rate plus **two** thirds of the previous credited rate	98	3.5	**7.0**
Dynamic Reset Strategy			
Reset to: the new money rate if **rates** go down, but if rates go up, **reset** to one half the previous credited rate	71	**3.1**	**7.0**

The first three lines show that for this sample product and the **surrender** behavior **assumption,** following new money rates less closely gives a longer interest rate duration but a shorter mean **term.** Following new **money** rates less closely **makes** the **policy** mare like a **longer** term fixed rate **financial instrument,** and **produces** a longer interest rate *duration.* However, this **causes** the **business** to run off the books more quickly(shorter mean term) due to **more** interest - sensitive surrenders. In this situation, completely following **new money** rates produces a lower RSA **than** resetting rates one **third** or **two** thirds of the way towards new money rates.

At the bottom of Table 2, a dynamic reset strategy that lowers the RSA is shown. The lower RSA of the dynamic strategy is evidence of the value of the insurance company's option - the option to reset rates differently based on the path of interest rates. With this dynamic strategy the insurer is more selective about changes in the credited rate. In effect, the insurer is taking advantage of inefficient policyholder behavior. Depending an the surrender rate function assumed, in rising rate environments there may be a certain point at which the savings of crediting less to those policyholders who stay, more than offsets the cost incurred by those who surrender.

However, the strategy with the lowest RSA may not necessarily be the best strategy for a couple of reasons.

1) Different asset OASs may be achievable at different sales volumes and at different durations. F a example, higher asset spreads may be achievable for lower sales volume through the ability to put a larger proportion of the asset portfolio into attractive, but scarce, high - yielding private placements. Also, different asset spreads may be available at different durations simply as a market phenomenon of the public and/or private debt market.

2) The insurance company's profit goal may be to maximize total excess spread, and not excess spread per dollar of business. In that case, one should compare strategies using the product of the excess spread and the sales volume. Surplus considerations will probably also affect the level of sales volume that can be considered.

IV - MEASURING RISK

One can measure the risks that affect the SPDA or other products by measuring the exposure of the excess spread to various risk factors. The change in excess spread caused by the change in a risk factor provides a relative measure of risk that can be compared against the expected profit and also against the other risks that are present. The SPDA product and strategy used in Table 1 are the ones used in the following examples of risk analysis.

A) Interest Rate Risk

To measure interest rate risk, one must have an asset to pair with the liability. Assume the insurance company is able to buy acceptable credit quality non - callable bonds that have a duration of 3.0 years and a spread of 150 basis points over Treasuries. The bonds are investment grade and the insurance company's credit analysts feel that 5 basis points is the appropriate deduction from the yield for credit risk. Also assume that the insurance company has a profit target of at least 50 basis points per year f a this type of product. This asset (ABC bond) gives the following expected profitability picture when used to support the SPDA product :

Spread	150 bp
- *Credit* risk	- 5
- Ivestment Expense	- 14
- Required Spread on Assets	- 81
Excess Spread	50 bp

Interest rate risk can be measured by the effect on excess spread of parallel shocks in interest rates. The first step would be to select the shock levels to use, and to calculate the new market value of the asset position at each shock level. For a non - callable bond such as the ABC bond, the market value calculation is relatively easy. The second step is to generate a set of interest rate paths for each interest rate shock, in order to calculate the RSA. This produces the analysis shown in Table 3.

Table 3
Interest Rate Risk

Change in Interest Rate8	-3%	-1%	0%	+1%	+3%
Market Value of Assets	$30,534	$28,751	$27,900	$27,081	$25,516
Spread on Assets	150 bp	150 bp	150 bp	150 bp	150 bp
⁻ Credit Risk	-5	-5	-5	-5	-5
⁻ Investment Expense	-14	-14	-14	-14	-14
- RSA	-97	-84	-81	-83	-146
Excess Spread	34 bp	47 bp	50 bp	48 bp	-15 bp

Duration is a measure of price sensitivity to small changes in interest rates. Table 3 is a good example of how a simple duration matching strategy can give good results within a certain range of interest rate changes, but less acceptable results outside the range. Other risks can be studied in a similar fashion using the excess spread technique (10) (11).

B) Policyholder Surrender Risk

The sensitivity of the RSA to different surrender assumptions should be tested. In pricing the SPDA product, an assumption as to policyholder surrender behavior was made using both a base and interest - sensitive component. Changes in the base component and in the interest-sensitive component should both be tested. The sensitivity of the results to changes in the interest sensitive component could be tested by simply taking a multiple (say .5 and 2.0) of the assumption used in step 8 of Table 1.

However, it is often interesting to test some different types of surrender functions such as those shown in Graph 1.

| ICR	= Insurance company's credited rate
CCR	= Competitor's credited rate (current one - year Treasury minus 50 basis points)
SC	= Prevailing surrender charge
RG	= Rate Gap = CCR - (ICR + (SC / 3))

Assumption	Surrender Rate*	RSA	Duration
A (table 1 line 9)	$400 \times (RG^2)$	81 bp	3.0 years
B	$8 \times RG$	79	3.1
C	.06 when RG>0 +.10 first time RG>0	69	3.6

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* Calculated when RG>0 and always capped at 50%

Graph 1

Surrender Rate (SR)

Rate Gap (RG)
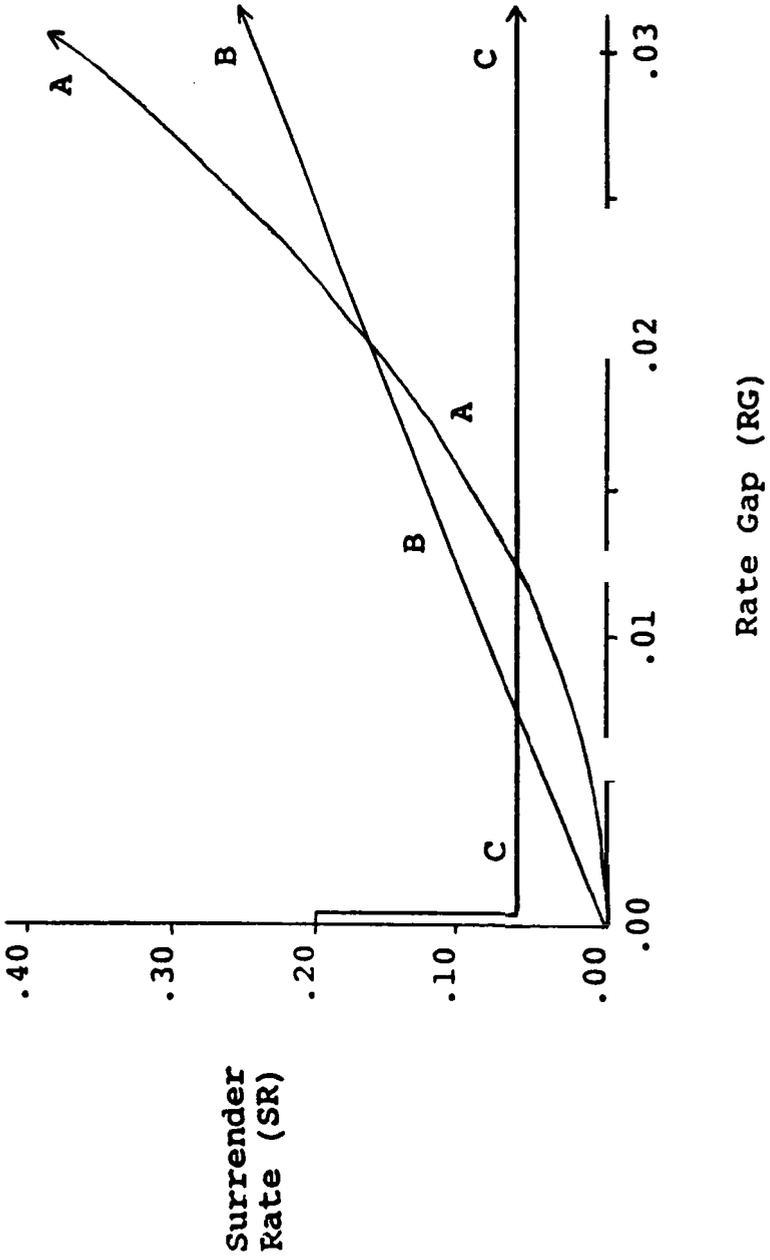
Sometimes the results of resting different surrender behaviors can be surprising. One should remember that interest - sensitive surrenders represent policyholder exercise of a very long dated option. Therefore, quick exercise of the surrender option as soon as it is slightly advantageous to the policyholder (such as assumption C) does not always lead to the most expensive (highest RSA) result. The three assumptions shown all have mean terms that are very close, thereby removing the expense amortization effect from the comparison.

There may be as many different possible surrender rate functions as there are actuaries. One possible approach not shown is to start the analysis with two groups of policyholders, a "hot - money" group, and a "cold - money" group, and apply different surrender functions to each.

C) Expense Risk

Lines 4 and 5 in Table 1 give a good guide as to the cost effect to changing the up - front and ongoing expense assumptions. It may be instructive to calculate the effect on RSA of different than assumed per policy expenses. Per policy expense misestimation may result from different than assumed sales volume for the particular rate crediting strategy, or different than assumed average policy size.

V - MEASURING EXPERIENCE

The excess spread technique can be used to measure the experience on a block of business at any pint in time after issue. The technique can be used nut only to measure the overall experience, but also to determine the respective contributions to the overall performance from a number of sources.

A) Asset Performance

The first comparison to be made is between the spread above Treasury levels at which it was assumed one could invest, and the actual spread at which investments were made. The actual initial spread on assets establishes an excess spread for comparison against actual results.

The path that interest rates have taken since the policy was priced or profitability was last studied will be known. To measure the contribution to profitability of asset performance, one does not need to know anything about actual liability behavior - one merely presumes that the liabilities behaved as assumed, given the interest rate environment that occurred. One begins by measuring the market value at the end of the time period of the amount of assets that would be held if the liability had behaved as assumed. This calculation involves "surrendering" the appropriate percentage of the block of policies based on the assumed surrender rate function and the actual path of rates, and then deducting expected profits, expenses, etc. A new set of interest rate paths based on current interest rate levels will have to be generated in order to project the liabilities and recalculate the RSA. The change in excess spread that results from this calculation can be ascribed to asset performance wer the year. Among the contributing factors to this change are the movement of rates and the relationship between asset and

liability durations. Other possible contributing factors include : any spread change on the assets ; the realization of the credit risk charge (unless the assets defaulted a were downgraded), and the effect of a change in the market's implied volatility *outlook* if the assets involve options.

B) Liability Performance

The liability performance component of the overall profitability result is measured by the change in RSA as a result of using actual liabilities in the calculation rather than assumed liabilities. The major component of this performance in the case of the SPDA will likely be the degree of surrenders experienced. Depending on the level of interest rates and the surrender charges collected, actual surrenders will have a positive or negative effect on the RSA. Unfortunately, it will take a number of observations of surrender experience in different interest rate and surrender charge environments to draw any conclusions about the appropriateness of the originally assumed surrender rate function. For other nonparticipating insurance products, the major component of liability performance will often be mortality.

C) Other Contributors to Performance

There are a number of other possible contributors to the overall performance of the block of business. The marginal contribution of each is determined by measuring the change in RSA caused by incorporating that aspect of experience into the RSA calculation. For example, using actual versus assumed expenses shows the marginal effect on profitability of expense misestimation. *Of course*, expense misestimation *can* derive from a number of sources such as misestimation of volume, average policy size, actual costs, a allocation of expenses among lines of business. Expense (and for that matter surrender) experience may also cause one to rethink the assumptions used for future periods. Changing prospective assumptions during the lifetime of a block of business will also have an immediate effect on the RSA and hence on expected future profitability.

In the case of the SPDA and similar products, an aspect of insurance company behavior that must be monitored is the resetting of credited rates. Resetting to a higher or lower level of rates than previously assumed will interact with the surrender experience and may cause incremental profits or losses. Any change in rate reset strategy should be reflected immediately in the RSA calculation.

VI - SUMMARY

This paper proposes a pricing and valuation technique that measures the total cost of an insurance product, including any interest - sensitive features. The technique can be used for product design, product pricing, strategy assessment and risk analysis. The excess spread technique can also be used throughout the life of the product to provide an ongoing report card on the economic health of the product. The same methods discussed in this paper f a the SPDA can also be used for pricing, design, strategy, risk and experience analysis f a other ncm - participating products (10) (12).

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With the emergence of interest-sensitive products, the study of policyholder and insurance company **behavior** in different interest rate environments will **became** an important part of actuarial science. The excess spread approach provides not just a **pricing** and valuation methodology, but also a **tool** to measure the **economic** effect of **different behavior** patterns.

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