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ANALYSIS OF SOLVENCY GURANTEED PROVIDED BY THE INSURANCE INDUSTRY

PAR / BY

Phelim P. BOYLE, Angelien. KEMNA

Canada / Pays - Bas / Netherlands

ANALYSE DES GARANTIES DE SOLVABILITE FOURNIES PAR LES COMPAGNIES D'ASSURANCE

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P. P. BOYLE, WATERLOO, CANADA

RESUME

Cet article traite des garanties de solvabilité des compagnies d'assurances. On examine des situations où des indemnisations d'assurés de compagnies non solvables, sont payés à partir des actifs de compagnies solvables. Le modèle utilisé, qui peut être adapté pour inclure les risques des investissements et des engagements, est dans l'esprit d'un récent article de Cummins (1988). Un modèle de détermination des prix des options est utilisé pour valoriser les indemnisations des assurés et les assurés eux-mêmes, sur les actifs de l'entreprise. Dans le cas d'une compagnie unique, les fonds propres correspondent à une option d'achat, alors que les indemnisations des assurés correspondent à une créance à risque. On élabore un modèle explicite, dans l'hypothèse particulière de partage du risque entre compagnies. Dans le cadre de cette hypothèse, les indemnisations des sinistres peuvent Ctre analysés comme des options complexes dont les rendements dependent de plusieurs variables d'état. On utilise des exemples numériques pour explorer les propriétés de ces indemnisations et on discute des incitations créées par une telle coopération. On trouve que les assureurs ont plus de chances d'opérer selon ce mode de coopération, s'ils sont plus ou moins comparables en termes d'exposition au risque. A noter Cgalement que les dispositifs de réassurance et la supervision gouvernementale imposent des limites et des assurent contrôles qui sont des éléments favorables à la mise en place de tels accords de partage mutuel des risques. Le modèle peut également être utilisé pour explorer les relations entre les obligations de réserve et de solvabilité des différentes lignes d'activité de compagnies d'assurance proposant plusieurs lignes de produits.

Phelim P. Boyle*

University of Waterloo
Waterloo, Ontario
Canada N2L 3G1
519 885 1211 (ext. 6513)
PBOYLE @ WATDCS

Angelien Kemna

Erasmus University Rotterdam
P.O. Box 1738
3000 DR Rotterdam
The Netherlands
31 10 4081287
SPRONK @ HROEUR5

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ABSTRACT

This paper discusses insurance company solvency guarantees. We examine situations where the unsettled claims of the policyholders of insolvent firms are paid from the assets of the solvent firms. The model used can be adapted to include both investment risk and liability risk and is in the spirit of a recent (1988) paper by Cummins. An option pricing model is used to value the claims of the insurance company's shareholders and the insurance company's policyholders on the assets of the firm. In the case of a single firm the equity resembles a call option while the policyholders' claims correspond to risky debt. We develop an explicit model for one particular assumption concerning the intercompany risk sharing. The claims that arise under our proposed arrangement can be analyzed as complex options whose pay - offs depend on several state variables. We use numerical examples to explore the properties of these claims and we discuss the incentives created by such an arrangement. We find that insurers are mare likely to operate in this cooperative fashion if they are more or less comparable in terms of riskiness. We also note that reinsurance arrangements as well as government supervision provide limits and controls that are conducive to the existence of such mutual risk sharing arrangements. Our model can also be used to explore the relationships among the surplus and solvency requirements of different lines of business of a multi - l i i insurance company.

1 - INTRODUCTION

The solvency of financial intermediaries is a topic of current interest. The financial difficulties of the Savings and Loan industry in the United States have been widely publicised. It has been estimated that the cost of the rescue operations for these institutions will run into many billions of dollars and add significantly to that country's national debt. In several countries the supervisory authorities have become concerned about the financial health of the banking firms under theu jurisdiction. Threats to solvency have .arisen from third world loans and from the proliferation of so - called off balance sheet items. In fact new capital adequacy rules for banks have been proposed (f a details see Hull [1989]) by a few of the world's leading central banks. Furthermore, because of the movement towards global financial markets, large financial firms now operate in many countries and domestic supervisory authorities may have difficulty in regulating such entities. Within a number of countries there has been a blumng of traditional roles for deposit taking institutions and this has also posed problems for regulation.

Since the purpose of insurance contracts is to transfer certain risks from individual economic agents p a n insurance company, the solvency of insurance companies is of considerable interest.

intrusion by the regulators may provide them with more information about the firms in the industry. The firms may not welcome this since the existence of asymmetric information between the regulator and the industry may be beneficial to the industry. A second justification for the industry acting in this way relates to the objectives of bureaucratic involvement Even if the regulators are efficient, it may be the case that social objectives diverge from industry objectives. For example, there could be differences as to the optimal number of firms in the industry or the degree of competition. The industry may be able to exercise more control wer its member firms if it agrees to bail out the policyholders of insolvent firms. With this type of informal arrangement in place, the insurance industry has incentives to police its own membership and has some control wer entry. Of course it should be stressed that in most jurisdictions the insurance industry is already subject to a considerable amount of regulation and that these arguments relate to the incremental involvement of the regulators in the affairs of the industry.

This paper develops a model to explore some of the characteristics of the solvency guarantees provided by the insurance industry. We recognize that the actual dynamics of the process are much more subtle and much more complex than our model suggests. As such this model can be regarded as a preliminary first step.

We assume a set or system of insurers who cooperate in such a way that the policyholders of insolvent insurers will be reimbursed by the system as a whole. Of course, if the system of insurers as a whole becomes insolvent, then not all claims can be met. The model assumes that the unpaid claims of the insolvent insurers will be reimbursed by the solvent insurers and that the contribution of the solvent firms is in proportion to their solvency. This arrangement puts a very high priority on honouring the claims of the policyholders in the system. Under this sharing rule the burden of rescue falls heaviest on the strongest firms. This ties in with the fact that when the policyholders of financially troubled insurers are reimbursed this can take the form of a merger with one of the stronger insurers in the system

We assume in our model that policyholder protection will always be forthcoming if the insurance system as a whole is solvent. In practice this is by no means the case. Our risk sharing arrangement is much more rigid and mechanical than those found in the real world. In spite of this, the model can be used to identify circumstances which would put a lot of stress on the arrangement and perhaps lead to its collapse.

To some extent our proposed model resembles a guaranty fund. Many of these funds operate on a post - assessment basis (Cummins [1988] and Duncan [1984]). Under a guaranty fund the arrangement is more formal and explicit. However, the solvent companies still end up paying the unsatisfied claims of the insolvent insurers. We assume that each member of the system agrees to act in a cooperative fashion. When we work out the costs and benefits of this arrangement we can identify the incentives for individual insurance firms to act in a non - cooperative fashion. One implication of our model is that the policyholders of the riskier firms are protected and that this protection is provided by the equityholders and/or participating policyholders of the stronger firms.

However, if a government body makes available such insurance to financial institutions, such as insurance companies, this can seriously distort incentives as the recent Savings and Loan situation in the United States attests. The existence of government insurance encouraged excessive risk taking by the stockholders and managers of many of these firms.

A fairly recent development has been the emergence of insurance guaranty funds. These funds provide reimbursement to the policyholders of insolvent insurance companies. Duncan [1984] describes the insurance guaranty funds in the United States. Cummins [1988] examines the fee structure of such funds using an option pricing approach In this model the stockholders of the insurance company have a call option on the assets of the company, whereas the policyholders' claim is equal to the value of the liabilities less a put option. Cummins shows that the current flat fee assessment is more likely to encourage high - risk strategies and he recommends a risk - based premium structure along the lines of similar recommendations by researchers who have studied the deposit insurance system in the United States.

In some jurisdictions the insurance industry has, from time to time, stepped in to protect the policyholders of a financially troubled insurer. Sometimes one of the stronger companies in the system will merge with the ailing company. In other situations the insurance industry will compensate some or all of the policyholders who have suffered financial loss because their insurer is insolvent or is in financial trouble. Typically the policyholders who are given the highest priority in these circumstances are those with the lowest financial resources. Such rescue operations are not guaranteed in advance and the insurance industry does not precomit itself to protect the policyholders of any insurer in financial difficulties. In addition the willingness of the insurance industry to mount rescue operations of this nature varies over time and also varies across jurisdictions. Since the negotiations which precede such mergers take place behind closed doors empirical data on both successful and failed rescue operations is very difficult to obtain However if rescue operations of this nature have taken place in the past, there is reason to believe that, unless circumstances change, they will occur again in the future. Hence these implicit guarantees, provided on a discretionary basis by the industry, have some value to policyholders. The purpose of our paper is to develop a simple model which captures some aspects of such industry guarantees.

The question as to why the insurance industry sometimes acts in this fashion is an interesting one. A common reason, given by insurance executives, is that it protects the good name of the insurance industry. The economic theory of regulation has been the focus of recent research[3] and provides alternative perspectives. One possible motivation for the insurance industry to act in this way is to prevent additional government intervention. The firms may reason that if they do not protect the policyholders of the insolvent firms the government will step in with a scheme such as a mandatory guaranty fund. Such an

[3]For a discussion of some of the recent work see Spulber [1989] and Baron [1984]. These new approaches apply the tools of micro - economic theory and game theory to derive new insights into the characteristics and consequences of regulation. These studies tend to be of a general nature and do not take into account the detailed institutional structure of the insurance industry.

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Hence, there will be a strong incentive for the less risky firms in the group to monitor the risk taking behaviour of the other firms in the system. Clearly, government supervision puts upper limits on the degree of risk taking as well. In addition reinsurance companies help to reduce the risk of individual insurance firms and to monitor the risk taking behaviour of managers.

In the present paper we use an option pricing approach to model the situation. Both Doherty and Garven [1986] and Cummins [1988] have used this methodology. We assume a single period model for convenience. The option pricing approach provides market based estimates for the value of the stockholders' claims and the value of the policyholders' claims when both assets and liabilities are assumed to be risky (and correlated to each other). The basic intuition can be obtained by assuming that there is only one stochastic variable[4] that determines the financial results [cf. Cummins]. Hence, we consider only one[5] stochastic variable in the case of each insurance firm. The outline of the paper is as follows. Section 2 develops the analytic structure of the model and lays out in detail a particular example of a sharing rule. The value of the claims of the individual firms' stockholders and policyholders can be expressed in terms of complex options. The payoff under these options depends on all the companies in the system and the structure of these options may be of some interest in their own right. In section 3 we indicate how these options can be valued numerically and we illustrate the impact of our proposed risk sharing arrangements using numerical examples. These examples are used to discuss the properties of the proposed model and examine the incentives that are created

2 - CLAIMS VALUATION UNDER A SPECIFIC RISK SHARING RULE

In this section we assume that we have n insurance companies that agree to pay the claims of policyholders of insolvent firms. We assume a one period model. The group of policyholders are better off with such an arrangement than when there is no mutual agreement(see Doherty and Garven [1986]). When there is such an arrangement the stockholders' position is worse unless the policyholders pay for the additional protection. [We would expect that the cost of this additional benefit security is reflected in additional premiums paid by policyholders]. Our analysis sheds some light on this issue but we do not pursue it in detail here. Cummins (1988) discusses the additional solvency premium in the case of a guaranty fund[6].

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[4] See Cummins (1988) here. This variable can be defined as the ratio of the assets to the liabilities.

[5] We could extend the analysis to allow for explicit stochastic variation in the liabilities at the cost of greater complexity.

[6] When equityholders want the same rate of return with or without this agreement, it is clear that the policyholders have to provide an extra initial premium. However, the exact mechanism of premium payments is complicated. If the policyholders pay an additional flat percentage of t k premium this makes the policyholders of the less risky firm worse off. A risk - adjusted system implies that the policyholders of the riskier firms provide the extra capital that is needed.

At the start of the period we assume that the insurance companies have initial assets equal to A_i^0, $i = 1,\ldots, n$.[7] For each company the assets are equal to the initial equity and premiums paid by the policyholders of the firm in question Our model permits us to investigate the impact of the intercompany risk *sharing* arrangement on the claimholders' positions. To calculate the market value of the equityholders' claim E_i^0, $i = 1,\ldots, n$, and the market value of Ute policyholders' claim, p_i^0, $i = 1,\ldots, n$, we analyze the possible outcomes at the end of the period. The values for the claims at the end of the period E_i^1 and p_i^1, form the terminal conditions necessary to value the claims as at the start of the period.

At the end of the period the policyholders of the insolvent firms are compensated as much as possible[8]. We explicitly consider the possibility that the industry as a whole defaults. We define insurance company i as insolvent when the assets (A_i^1) are less than the liabilities (L_i)[9] at the end of the period. The solvent firms will pay the policyholders of the insolvent firm the necessary amount, (L_i - A?). For the remainder of this section we suppress the tine index, 1, for the end of period values. If the assets of all the firms in the system are less than the total liabilities. the system defaults. The policyholders of the insolvent firms receive the balance, after the policyholders of the solvent firms have been paid

We can summarize this arrangement more concisely if we divide the n insurance companies into a group of solvent and a group of insolvent firms. We define the following, end of period, values:

A_S (A_I) = the value of the assets of the solvent (insolvent) firms

L_S (L_I) = the value of the liabilitles of the solvent (insolvent) firms

E_S (E_I) = the stockholders' claims of the solvent (insolvent) firms

p_S (p_I) = the policyholders' claims of the solvent (insolvent) firms

and

$$A = A_S + A_I$$
$$L = L_S + L_I$$

For these groups we can determine four distinct situations at the end of the period:

I All firms are solvent

$$A_S \geq L_S \quad A_I = L_I = 0$$
$$E_S = A_S - L_S, \; p_S = L_S \quad E_I = p_I = 0$$

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[7] *The assets of each firm do not necessarily have the same characteristics, although the correlation between the assets of any two firms will normally be high.*

[8] *Our model does not discriminate between large and small policyholders of the insurer. In practice, large policyholders may have the resowces and the incentives to become better informed. Furthermore, rescue operations may give a higher priority to the unsettled claims of smaller (poorer) policyholders.*

[9] *Notice that L_i is equal to the expected end of period liability. Ow model treats this as deterministic.*

II Some firms are insolvent and the system is solvent

$A \geq L, A_S \geq L_S A_I < L_I$

$E_S = A - L, p_S = L_S, E_I = 0, p_I = L_I$

III Some firms are insolvent and the system is insolvent

$A < L, A_S \geq L_S A_I < L_I$

$E_S = 0, p_S = L_S E_I = 0, p_I = A - L_S$

IV All firms are insolvent

$A_S = L_S = 0, A_I < L_I$

$E_S = p_S = 0, E_I = 0, p_I = A_I - L_I$

In the first situation, no firm is insolvent, which means that policyholders receive their promised benefits in full and stockholders the balance. There is no wealth transfer from one group to the other. For both groups the stockholders' and policyholders' claims add up to the total assets. In the second situation, the stockholders of the solvent firm, have to pay the policyholders of the insolvent firms. The stockholders position equals $A_S - L_S - (L_I - A_I) = A - L$; only the solvent firms have non - zero equity. Note that here the stockholders' and policyholders' position do not necessarily add up to the assets for each group since wealth transfers take place from one group to the other.

In the third situation, the entire system defaults and the policyholders of the insolvent firms receive what is left after the required payment to the policyholders of the solvent firms.In the fourth case, when there is no solvent firm, the policyholders of each company own the entire assets of the company and there is no wealth transfer. We can summarize these cases as follows:

(2.1) $E_S = \mathrm{Max}(A - L, 0)$

(2.2) $p_S = L_S$

(2.3) $E_I = 0$

(2.4) $p_I = L_I - \mathrm{Max}(L - A, 0)$

The stockholders of the solvent firms have a claim whose payoff is analogous to a call option on the total assets and the policyholders of the insolvent firms have a claim that resembles risky debt, where the put option is written on the total assets[10]. If there were no risk sharing arrangement, put - call parity would apply to each the individual firm. With our proposed risk sharing arrangement put - call parity only applies to the total system :[11]

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[10] *These options are one -period European options.*

[11] *Of course there may be special configurations for which the put - call parity relationship applies as well to each individual firm. We discuss this further in Section 3.*

$$(2.5)\quad A = E_S + E_I + p_S + p_I$$
$$= \text{Max}(A - L, 0) + 0 + L_S + L_I - \text{Max}(L - A, 0)$$
$$= C^1 + L - P^1$$

where C^1 and P^1 are the terminal values of a call and a put option on the end of period assets, A with an exercise price of L.

Thus far, we have only determined the claimholders' position for the group of solvent and insolvent firms. To determine the claims for an individual firm we need to specify how much the stockholders of the solvent firms are going to pay in case one of the firms in the system is insolvent and how much the policyholders of an insolvent firm are going to receive. Many sharing rules are possible. We have analyzed a particular arrangement where the contribution of a particular firm[12] to the unsettled claims of the system as a whole is proportional to the solvency of the firm in question.

We first illustrate the operation of our proposed risk sharing arrangement by means of simple numerical examples. Then we provide algebraic expressions for the end - of - period values of the stockholders' and policyholders' claims. We have already observed that no intercompany risk sharing takes place if every firm in the system is solvent or if every firm in the system is insolvent. Therefore let us assume that, at the end of the period, there is at least one insolvent firm and that there is also at least one solvent firm. The values given in Table 2.1 correspond to this situation. Here we have three firms in the system each with end - of - period liabilities of 100.

Table 2.1 Numerical example for system of three firms ; System as a whole solvent

Value at end of period (before any transfer)	Firm 1	Firm 2	Firm 3
Assets	180	120	60
Liabilities	100	100	100
Surplus (Deficit)	80	20	(40)

In this example the policyholders of Firm 3 have a shortfall of 40. Under our purposed risk sharing arrangement this shortfall is made up by contributions from the two solvent firms where each firm pays a share in proportion to its solvency. Thus the contribution from Firm 1 is

$$\frac{80}{80+20} \times 40 = 32$$

The contribution from Firm 2 is

$$\frac{20}{80+20} \times 40 = 8$$

[12] *This particular firm is, of course, assumed to be solvent.*

After the intercompany risk sharing has taken place all the policyholders' claims are satisfied. The shareholders of Firm 1 receive 48 (80 - 32), the shareholders of Firm 2 receive 12 (20 - 8) while the shareholders of Firm 3 receive zero. The result of the intercompany risk sharing is that the shortfall in the payments to Firm 3's policyholders is made up by the surplus of the two solvent firms. For these numerical values the system as a whole is solvent and so all the policyholders' claims are met.

Table 2.2 provides another possible set of outcomes. In this case the system as a whole is insolvent since the total of the available assets, 295, falls short of the total liabilities. In this case the surplus of the solvent firm is divided up to pay (part of) the unsettled claims of the other two firms using the same pro - rata rule as before.

Table 2.2 Numerical example for system of three firms ; System as a whole insolvent

Value at end of period (before any transfer)	Finn 1	Firm 2	Finn 3
Assets	105	98	92
Liabilities	100	100	100
Surplus (Deficit)	5	(2)	(8)

Thus the policyholders of Firm 2 are entitled to all the assets of Firm 2 plus an additional amount, computed as :

$$\frac{2}{2+8} \times 5 = 1$$

In the same way the policyholders of Firm 3 are entitled to the assets of Firm 3 plus an additional amount, computed as :

$$\frac{8}{2+8} \times 5 = 4$$

Hence when the dust settles, all of the shareholders receive nothing. The policyholders of the solvent firm are paid in full. The policyholders of Firm 2 receive 99 while those of Firm 3 receive 96.

It turns out that this type of risk sharing arrangement can be summarised in a compact manner. In the case of firm i, the end - of - period value of its equity holders under our proposed risk sharing scheme is :

$$(2.6) \quad E_i = \frac{\text{Max}\,[(A_i - L_i),\,0]}{\sum\limits_{k=1}^{n} \text{Max}[(A_k - L_k),\,0]} \quad \text{Max}[(A - L),\,0]$$

Note that if either firm i, or the system as a whole is insolvent, the end - of - period equity value is zero. If all of the firms in the system are solvent then $E_i = A_i - L_i$. If both the system as a whole and firm i are solvent and there is at least one insolvent firm then we see that $E_i < (A_i - L_i)$. Under these circumstances the factor

$$\frac{\text{Max}\,[(A - L), 0]}{\sum_{k=1}^{n} \text{Max}[(A_k - L_k), 0]}$$

is less than one. Note that the numerator corresponds to the boundary condition for an option on a portfolio while the denominator corresponds to a portfolio of options.

There is an analogous expression for the end - of - period value of the policyholders' claims. For firm i, the policyholders' claim corresponds to the (expected) value of the liabilities minus a complex put option. This put option incorporates the proposed risk sharing arrangement. We have

$$(2.7) \quad p_i = \quad L_i - \frac{\text{Max}\,[(L_i - A_i), 0]}{\sum_{k=1}^{n} \text{Max}\,[(L_k - A_k), 0]} \; \text{Max}[(L - A), 0]$$

Note that if the system is solvent or if firm i is solvent the value of pi, is equal to L_i. If every firm in the system is insolvent the policyholders of firm i just receive A_i. We can check that equations (2.6) and (2.7) give the correct values for the numerical examples we discussed in this section.

Equations (2.6) and (2.7) provide the end - of - period values of the contingent claims which represent the interests of the stockholders and policyholders. To value these complex options we can use the risk neutral approach of Cox and Ross [1976]. The present value of these options is obtained by discounting their terminal expected values at the risk free rate where the expectation is taken over the equivalent martingale measure [cf. Harrison and Kreps [1979]]. Hence the current value of the equity is given by

$$(2.8) \; E_i^0 = e^{-r} E\,[E_i]$$

where E is the expectation operator under the equivalent martingale measure, r is the continuously compounded interest rate and the time to maturity is one unit. An analogous expression exists for the put option which is included in the value of the policyholder liabilities.

3 - NUMERICAL EXAMPLES

In the last section, we derived boundary conditions for the complex contingent claims, which represent the interests of the equityholders and policyholders. These claims correspond to options whose pay - offs depend on several underlying state variables. To value these options we use an algorithm developed by Boyle Evnine and Gibbs [1989]. The Boyle Evnine Gibbs method involves an extension of the Cox Ross and Rubinstein binomial method. It is assumed in our model that the underlying asset returns have a multivariate lognormal distribution. This is a natural extension of the assumptions made by Doherty and Garven [1986] and Cummins [1988] for the case of a single firm. The parameter estimates used in our numerical simulations correspond to those employed by Cummins [1988].

Consider a generic firm denoted as firm i. Initially assume that it operates on its own without any risk sharing with other insurers. We assume the following values at the beginning of the period :

$A_i^0 = 120$

$L_i = 100$

$r = 0.005$ (assumed interest rate[13])

$\sigma_i = .10$ (volatility of firm i[14])

$T = 1$ year (time to option maturity[15])

Under the option pricing model the value of the stockholders' claim is equal to a European call on the assets of the firm. The policyholders' claim corresponds to the discounted value of the (expected) claims minus a European put option. Using the put-call parity relation for European options we have :

(3.1) $A_i^0 = C_i^0 + [L_i e^{-rT} - P_i^0]$

where C_i^0 is the value of the European call option at the start of the period

P_i^0 is the value of the European put option at the start of the period

The numerical values of these options can be obtained using the Black Scholes model. For the above example the call value equals 20.63 and the put value equals 0.13. Note that the put option value corresponds to that obtained by Cummins [1988], (Table 1). This implies that the stockholders' claim is worth 20.63 and the policyholders' is equal to 99.50 - 0.13 = 99.37. If the policyholders' liabilities were fully guaranteed the value of their claim would be 99.50. Because there is a possibility of default the value of their claim is 99.37.

We now assume that there are three companies in the system. The parameters of each company correspond to those just described for company i. We also assume that the correlations between the assets of all the companies are equal to 0.5. The values of the stockholders' equity and policyholders' liabilities for the three companies when there is no risk sharing arrangement are given in the fust panel of Table 3.1.

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[13] Our parameter r corresponds to r in the Cummins paper.*

[14] The relevant variance is given by $\sigma_i^2 = \sigma_A^2 + \sigma_L^2 - 2\ \rho_{AL}\, \sigma_A\, \sigma_L$, where σ_A is the volatility of the assets, σ_L, is the volatility of the liabilities and ρ_{AL} is the correlation between the assets and the liabilities. However, we simply assume one stochastic variable.

[15] We assume a one period model. Our analysis could in principle be extended to several periods.

These values are identical with those just derived for a single company since there is no intercompany risk sharing. The second panel of Table 3.1 presents the results[16] when there is intercompany risk sharing under the proposed arrangement.

Table 3.1 Value of Stock - and Policyholder claims f a three companies

WITHOUT INTERCOMPANY RISK SHARING

Company	Assets	Stockholders claim	Policyholders claim	Total
1	120.00	20.63	99.37	120.00
2	120.00	20.63	99.37	120.00
3	120.00	20.63	99.37	120.00

WITH INTERCOMPANY RISK SHARING

Company	Assets	Stockholders claim	Policyholders claim	Total
1	120.00	20.53	99.47	120.00
2	120.00	20.53	99.47	120.00
3	120.00	20.53	99.47	120.00

Note that in view of the symmetry the three companies have identical value for the claims. In each case the value of the policyholders' claims has increased from 99.37 in the individual case to 99.47 with intercompany risk sharing. The figures in the bottom panel of Table 3.1 incorporate the probability that the system as a whole will default whereas those in the top panel incorporate individual firm default. In this respect our model differs from that of Cummins, who assumes that the guaranty fund provides default free insurance. Under our arrangement the policyholders of each firm have been granted an additional benefit worth 0.10 units. Because of the symmetry this is reflected in an identical drop in the value of each firm's equity by 0.10 units. The symmetry ensures that for each firm the fall in value of the stockholders' call option is equal to the change in value of the short put position held by the policyholders. Thus put - call parity is valid for each of the three firms even with the intercompany risk sharing.

If we depart from symmetry the size of the change in each firm's equity need not correspond to the size of the change in the value of that firm's policyholders claims. This is illustrated in Table 3.2. We have assumed here that the volatility of firm 3 is 0.20 while all the other parameters remain the same as those assumed for Table 3.1. We present the results without and with intercompany risk sharing.

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[16] *These numbers were obtained using the Boyle Evnine Gibbs[1989] method for valuing options depending on several underlying state variables.*

Table 3.2 Value of Stock - and Policyholder claims for three companies when company 3 has a higher volatility (0.20)

WITHOUT INTERCOMPANY RISK SHARING

Company	Assets	Stockholders claim	Policyholders claim	Total
1	120.00	20.63	99.37	120.00
2	120.00	20.63	99.37	120.00
3	120.00	20.54	97.46	120.00

WITH INTERCOMPANY RISK SHARING

Company	Assets	Stockholders claim	Policyholders claim	Total
1	120.00	19.80	99.44	119.24
2	120.00	19.80	99.44	119.24
3	120.00	22.49	99.02	121.51

In the first panel of Table 3.2, when there is no intercompany risk sharing, the policyholder claims of company 3 are lower than those of the other two because of the higher volatility of company 3. For the same reason the stockholders of company 3 have a more valuable claim than the stockholders of the other two companies. When we assume that the companies pool risks according to our model we see that the policyholders all benefit. The new arrangements are especially beneficial to the policyholders of company 3, since the value of their claim increases from 97.46 to 99.02. The source of this benefit is the stockholders of the less risky firms. Note that the impact on the stockholders of company 3 is very slight. The value of their claim only decreases to 22.49 from 22.54.

The figures in the bottom right hand comer of Table 3.2 denote the sum of the values of the stockholders' equity and the policyholders' liabilities in the case of each firm when there is intercompany risk - sharing. This is obviously not an equilibrium configuration. For example the assets of firm 1 amount to 120 whereas the market value of its equity and policyholders' liabilities is just 119.24. This discrepancy is generated by the risk sharing arrangement we have introduced. We can restore equilibrium by increasing the assets of company 3. The results are given in Table 3.3.

Table 3.3 Value of Stock - and Policyholders claims for three companies with unequal volatilities[17]

WITHOUT INTERCOMPANY RISK SHARING

Company	Assets	Volatility	Stockholders	Policyholders	Total
1	120.00	0.10	20.63	99.37	120.00
2	120.00	0.10	20.63	99.37	120.00
3	149.00	0.20	49.69	99.31	149.00

WITH INTERCOMPANY RISK SHARING

Company	Assets	Volatility	Stockholders	Policyholders	Total
1	120.00	0.10	20.54	99.46	120.00
2	120.00	0.10	20.54	99.46	120.00
3	149.00	0.20	49.54	99.46	149.00

The figures in Tables 3.2 and 3.3 provide insight into the viability of our proposed arrangement. If the arrangement involves too large a deviation from the initial situation with no risk sharing it is unlikely to be viable. For example under the conditions of Table 3.2 the policyholders of Firm 3 are being heavily subsidized by the shareholders of the other firms. There is a strong incentive for both these firms to dissuade Firm 3 from engaging in high risk strategies. If Firm 3 persists in such maverick behaviour there will be pressure to expel it from the system. This threat may act as a deterrent. Another response is to have Firm 3 increase its capital base until equilibrium is restored in the system The intuition here is that insurers with the riskier assets(and also with riskier liabilities) need a higher level of surplus to participate in an equitable basis in the mutual risk sharing.

The situation illustrated in Table 3.2 would strain the type of risk sharing we have proposed. One response is that, if such risk sharing is in place, there are institutional restrictions as well as incentives to prevent the occurrence of such an asymmetric profile. We also note that the existence of the reinsurance market not only serves to reduce the risk of individual firms but also tends to make the risk exposure more homogeneous. In addition the reinsurance market provides a mechanism for the more solvent firms to monitor and reduce the riskiness of the less solvent companies. Thus, we would expect an arrangement such as we propose to prevail as long as these mechanisms for risk monitoring and control are effective and as long as there is not too much asymmetry in the system. When these conditions are not fulfilled the pressure will cause the proposed risk sharing system to break down and at this point the regulator may step in with different arrangements.

4 - SUMMARY

This paper has examined one possible arrangement for spreading the risks of insurance company insolvency. We developed a simple model to investigate the implications of a particular type of risk *sharing* arrangement[18], whereby the solvent firms in the system cover the losses of the policyholders of the insolvent firms. We used option pricing theory to value the complex contingent claims that arise under this type of system. Our model enables us to identify the stress points of such a structure and we were able to suggest some of the incentives that would emerge under such an arrangement. In particular there will be pressure from the other firms in the system to encourage an individual firm to refrain from excessive risk taking.

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[17] *Table 3 3 illustrates the revised values of the stakeholders' claims when the capital of Firm 3 is increased to 149 from 120. Note that it would be irrational for Firm 3's stockholders to add 29 units of capital since the value of their common stock only* increases by **27.20.**

[18] ***O w model can be modified to handle different types of risk sharing arrangements. For example, the assessment might be based on the total Table assets (or the liabilities) of the solvent insurance firms.***

The existence of government supervision which puts limits on the riskiness of individual insurers provides conditions favourable f a the existence and viability of our proposed risk sharing structure.

The model proposed here could be applied to other situations. One application might be to the analysis of the different lines of business of a multi - line insurance company. There is considerable current interest in determining the capital and surplus requirements of the different lines of business. We can address such issues within the framework of our model. For a prescribed level of insurer solvency we can use the model to determine the capital requirements of the different lines of business in a consistent manner. The intuition is that the riskier lines of business will require more surplus and our model provides a quantitative framework f a establishing the amount of the surplus.

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