CONTRIBUTION N° 39

RELIABILITY
OF
DYNAMIC HEDGING

PAR / BY
J. BERTHON, G. GALLAIS - HAMONNO
France

ROBUSTESSE
DES TECHNIQUES
DE DYNAMIC HEDGING
RELIABILITY OF DYNAMIC HEDGING

JEAN BERTHON AND GEORGES GALLAIS - HAMONNO

ABSTRACT

The basic technique used to cover portfolio risk is dynamic hedging. In the first part of this paper, the authors set out the theoretical basis of the technique, stressing the fundamental importance of choices regarding volatility and interest rate levels in the design of a hedging package.

The second part is devoted to the statistical appraisal of the impact of mistaken forecasts concerning these two parameters. 278 one-year hedging plans and 182 three-year programs (obtained by moving forward systematically one month at a time) based on the CAC 240 index are simulated for the 27 years from 1962 to 1988.

For each duration (one year or three years), four different simulations are made:

- one reference simulation based on observed volatility and interest rates
- two simulations analyzing the respective impact of volatility and interest rates, on the basis of historical levels of the parameter concerned
- one simulation analyzing the combined effects of errors concerning interest rates and volatility.

Overall, results show that dynamic hedging is remarkably reliable, both over one year and over three years. It also appears that the relative importance of volatility and interest rates is inverted over time; volatility is the crucial factor in one-year hedges, but not over three years. Finally, errors regarding volatility and interest rates apparently balance each other out.
"L’assurance de portefeuille" est fondée sur l’utilisation de la Technique du "Dynamic Hedging (DH)".

Dans une première partie nous rappelons les bases théoriques du D.H. afin de montrer l’importance du choix des niveaux de volatilité et de taux d’intérêt dans la mise en place d’un programme d’assurance.

La seconde partie a pour objet de mesurer statistiquement les effets des erreurs de prévisions sur ces deux paramètres.

A cet égard, 278 programmes d’assurance à 1 an et 182 programmes à 3 ans (obtenus par décalage systématique d’un mois) portant sur le CAC 240 sont simulés sur la période globale de 27 ans couvrant 1/2 - 1988.

Pour chacune de ces périodes de programme (1 an ou 3 ans), 4 séries de simulations sont effectuées :

- une simulation de référence utilisant la valeurs ex post de la volatilité et du taux d’intérêt,
- deux simulations analysant les effets respectifs de la volatilité et du taux d’intérêt, en utilisant comme valeur du paramètre étudié sa valeur historique,
- une simulation analysant les effets conjugués des erreurs sur ces deux paramètres.

Les résultats obtenus font apparaître globalement une robustesse remarquable du D.H., que ce soit sur des périodes d’un an ou de trois ans.

Il semblerait que l’importance relative de la volatilité et du taux d’intérêt s’inverse en fonction de la longueur du programme d’assurance : sur un an l’impact de la volatilité est prédominant alors qu’il ne le paraît plus sur trois ans.

D’autre part est apparu un phénomène de compensation entre les erreurs de volatilité et celles de taux d’intérêt.
INTRODUCTION

The financial futures and options markets are completely changing the management of stocks or bonds portfolios and they lead to the creation of new management techniques. Presently, the more popular is represented by "Index Funds", either "tilted" or not.

According to our belief, a more promising technique - even if it is presently less popular - is "Dynamic Hedging", also called "Portfolio Insurance".

Definition

To be more precise, Dynamic Hedging (D.H.) is a systematic management technique, the objective of which is to obtain the loss profile of a portfolio protected by a put option, i.e. to obtain non-symmetrical results according to the Stock Market evolution:

- the D.H. portfolio profits of Stock Market increases
- but, if the Market declines, the decrease of the D.H. portfolio is limited to a predetermined level (the "floor").

This is equivalent to hold a call option (with an exercise price equal to the floor) and cash

This technique is directly derived from the Black & Scholes model (1973) which demonstrated that any option can be replicated by the continuous adjustment of a portfolio comprising only stocks and the risk-free asset. D.H. strategies have been put into practice in the U.S. since many years and have been quite successful although their development now suffers from the 1987 crash.

Several remarks have to be made.

First a terminological one: strictly speaking, "portfolio insurance" should only refer to the direct purchase of a put on the risky asset, which gives full protection. "Dynamic Hedging", on the reverse, should refer to the replication of such a purchase of a put. It follows that the results of D.H. strategies (limiting the downwards movements of the portfolio to the floor level) cannot be formally guaranteed.

1 For a detailed analysis of the direct portfolio insurance strategy and of the impossibility of implementing it, see our paper JB & GGH (1989)
Second, D.H. is based on a predetermined maturity. It implies that its performance should only be assessed on that particular date and not before. It also implies the use of European options - non callable before maturity - instead of the more common American ones.

Finally, the cost of implementing a D.H. strategy which theoretically is the cost of buying the put of the direct insurance strategy, is only known ex post.

Content of the paper

This paper is organized in two parts. The first part recalls the essentials of Dynamic Hedging in order to show why the estimates of volatility and rate of interest are theoretically important for the D.H. technique.

The second part analyses via sets of simulations over 27 years how the results of the D.H. strategies implementations differ from the theoretical expected ones when one uses historical estimates instead of the "true" unknown ones.

The basic and overall result is that D.H. is extraordinary robust.

I THE ESSENTIALS OF DYNAMIC HEDGING

D.H. is based on the property of options replication which leads to an iterative procedure. As we have just explained, the objective of D.H. is to replicate a put.

Property of options replication

Among the different option parameters, one is of special importance for D.H. : the option • delta.

The option • delta measures the expected change of the premium (in $ terms) if the stock • price changes by 1 $. The delta is measured by the first partial derivative of the BLACK & SCHOLENS formula relative to the stock-price. For a put P:

$$\delta_P = \frac{dP}{dS} = N(d_1) - 1 < 0$$

The use of the Delta is important because it makes possible to create a stock • portfolio without risk, the change in premium being offset by the change in stock • price:

$$\text{Delta • Neutral Portfolio} = \text{Put} \cdot \delta_P S$$

If M is the money • value of this delta • neutral portfolio:

$$M = \text{Put} - \delta_P S$$

and one obtains two identities:

2The reader who is familiar with the Futures and options theory is invited to jump directly to Section II.

3The reader should notice that the Put-delta is negative. It explains why owning the Stock will appear with a negative sign.
106 ROBUSTNESS OF DYNAMIC HEDGING STRATEGIES

(4) IDENT I:
Put $\cdot$ \(\delta P S\) = loan at the risk - rate of interest of the amount \(M\)

(5) IDENT II:
Put = loan at the risk - \textit{free} rate of \(M\)
PLUS
shorting the related stock proportionately to \(\delta P\)

The "synthetic call" replication

From (4) and (5), it is possible to replicate the return of any asset (Call, Put, Stock, Treasury Bill...) when one has two of them.

For D.H., the objective is to replicate a portfolio consisting of stocks and their protective Puts, i.e., to replicate \(S + P\).

This is obtained in the following manner:

From (5):

(6) \(S + P = S + M + \delta pS\)
factoring:

(7) \(S + P = M + (1 + \delta p)S\)
Using Identity I, it gives a new identity III:

(8) \(S + P = \text{Loan at the risk - free rate for an amount } P - \delta pS\)
PLUS
Investment in stocks for an amount \((1 + \delta p)S\)
which can be expressed as:

(8 bis) \((S + P) \text{ EQUI } (1 + \delta p)S + (P - \delta pS)\)

Dynamic hedging in practice

D.H. is based on Identity III and equation 8 bis. The portfolio (with value \(S + P\)) has to be distributed, according to the theoretical Put delta, among actual stocks and investment in T. Bills.

At the beginning of the insurance program, one has to estimate the theoretical value of an European Put according to the Black & Scholes formula and to derive its delta.

Since this put is notional, the manager is completely free in its choices:

- for the insurance maturity: \textit{one}, two or three...years
- for the floor (which is the strike - price of the Put): 100%, 95% of \(S + P\); note that it is possible to increase the floor if the Market has \textit{sufficiently} gone up, giving the D.H. program a ratchet effect
- but not for the volatility and the rate of interest levels which should be equal to their \textit{ex post} values (see below).

\footnote{Some authors prefer to use a floor equal to 100 (or 95\%) of \(S\), putting apart the cost of the theoretical Put.}
The *second* phase of the D.H. is the portfolio revision because of the changes in delta. One should re-estimate the theoretical Put and re-distribute the portfolio according to the new delta.

As the "hedge ratio" $\delta_p$ inversely varies with the stock prices, the portfolio will be the more invested in stocks as the Market goes up and the more invested in T. Bills as the Market goes down.\footnote{Such a strategy is just the reverse of the traditional "orthodox" strategy : "buy low, sell high". Which answers the question : who is the seller of this insurance strategy? Of course, the market as a whole.}

In the Black & Scholes world, these portfolio revisions are made *continuously* and without transaction costs. But in the real world we have to face these two problems.

*First*, stock prices are *discontinuous* for two reasons : the quotations are made at discrete time intervals and moreover the markets are closed daily.

Second, transaction costs do exist.

Boyle & Emanuel (1980), and Leland (1985) have tried to *analyse* and measure the effects of these discrete adjustments and of the transaction costs on the results obtained by replicating strategies.

The Leland's answer to *both questions* - but only on the basis of a given adjustment period - is to increase the volatility used for estimating the theoretical Put and its delta.

Finally it should be noted that most of the D.H. programs use the Futures markets for buying or selling index futures contracts instead of buying or selling on the physical Stock Market. The revision policy is much less costly and much easier to implement thanks to the larger liquidity.

**D.H. conditions of success**

The success of D.H. is based on the quality of the estimate of the notional Put. If the theoretical Put is not the right one, the actual replicated portfolio will not behave as expected and, at maturity, there will be a discrepancy between the actual return and the expected theoretical one. Second, as mentioned above, the value of the Put amounts to the *ex-ante* cost of the D.H. program ; if the Put is miscalculated, the *ex-post* will either be less or larger than expected.

For estimating this Put, three of its components are given : the stock price or the stock index level, the maturity and the strike - price (these two latter being chosen by the investor). Thus the estimate will rely on the two other parameters : volatility and interest rate. The stock - volatility and the interest rate should be the *ex-post* ones, i.e those which will prevail during the D.H. period.

Of course, these two components are unknown at the time a D.H. program is launched although they are crucial for estimating the value of the theoretical Put.

The remaining part of the paper analyses in which way and how much prediction errors on these two parameters lead to results different than those expected. In other word, we aim to know if the D.H. procedure is robust or not at mis-estimations of the theoretical Put.
II ANALYSIS OF ROBUSTNESS OF D.H.

Our analysis of robustness of D.H. to misestimations of the theoretical put is based on a set of simulations comparing the results obtained with a perfect prediction versus those with imperfect ones.

The set of simulations

A D.H. program is started each month over a 27 years time span, (12.29.61 to 1231.88)
- for a one year period (AN) : 278 cases
- for a three years period (TH) : 182 cases

The D.H. program has a floor of 95% of starting value and portfolio revisions take place each month (because of the monthly data)\(^6\).

Per program period (AN\(^*\) or TH\(^*\)), four set of simulations are conducted:
- AN1 and TH1 are based on perfect predictions and will serve as reference standard: the volatility and the risk free rate are the ex post ones over the D.H. period
- AN2 and TH2 analyse the volatility effect, the volatility used is the historical standard deviation over the preceding period while the interest rate is the ex post one
- AN3 and TH3 analyse the interest rate effect: the interest rate used is the historical one over the preceding period while the volatility is the ex post one
- AN4 and TH4 analyse the actual situation (with no predictions): both volatility and interest rate are the historical ones over the preceding period.

The data are the French General Stock Index (monthly data) and the monthly average of the Call money rate?

The measure of D.H. programs accuracy

The robustness (or lack of) of each D.H. program is measured by the "Error" between the value of the D.H. portfolio at the end of the program relative to the theoretical objective

Namely:

\[
\text{Error} (\%) = \left( \frac{\text{Terminal Value of D.H. Portfolio}}{\text{Terminal Value of Theoretical Portfolio}} - 1 \right) \times 100
\]

Results of one year insurance programs

Table I shows the characteristics of the distribution of "Error" for each set of simulations.

\(^6\) All the volatilities are corrected to take into account the transaction costs (1% for a round trip) in the way proposed by Leland (1985, p. 1289). To give an order of magnitude the Leland's formula increases the actual volatility by around 10%.

\(^7\) The data are described in the appendix.
TABLE I(%)  

<table>
<thead>
<tr>
<th></th>
<th>AN1 Perfect Prediction</th>
<th>AN2 Volatility Effect</th>
<th>AN3 Interest rate Effect</th>
<th>AN4 Historical Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MEAN</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.D.</td>
<td>-0.47</td>
<td>-0.59</td>
<td>-0.28</td>
<td>-0.45</td>
</tr>
<tr>
<td>S.D.</td>
<td>0.74</td>
<td>2.79</td>
<td>0.94</td>
<td>2.53</td>
</tr>
<tr>
<td><strong>MODE</strong></td>
<td>-0.47</td>
<td>-0.58</td>
<td>-0.44</td>
<td>-0.45</td>
</tr>
<tr>
<td>(number of cases)</td>
<td>(42)</td>
<td>(40)</td>
<td>(38)</td>
<td>(44)</td>
</tr>
<tr>
<td><strong>MINIMUM</strong></td>
<td>-4.41</td>
<td>-9.16</td>
<td>-4.22</td>
<td>-8.83</td>
</tr>
<tr>
<td><strong>MAXIMUM</strong></td>
<td>4.09</td>
<td>8.81</td>
<td>3.90</td>
<td>8.35</td>
</tr>
</tbody>
</table>

a) D.H. with perfect prediction  
The AN1 simulation is used as "standard", since it is based on a perfect prediction of both parameters, volatility and rate of interest. One should expect the mean of "Error" to be zero.  
This is not the case and this seems to be mainly due to the revision process.  
First, the revision policy should be based on a variable frequency depending on the time to maturity and on the path followed by the Market: the nearer one is to maturity and the closer the Market price is to the strike price, the more the delta of the replicated portfolio (S + P) varies with the Market moves, consequently one has to adjust the actual portfolio more frequently.  
Second, due to the data availability, we were obliged to use a monthly revision period, which clearly violates the continuous trading assumption of the Back & Scholes model.  
On the graph I, the four lines drawn aim to show which results are "abnormal", i.e those which are outside the mean ± 2σ band: for AN 1, the results having a value greater than 1% or lesser than -1.95%; for AN IV those with a value greater than 4.6% or lesser than -5.5%.  
These points (numbered in the order of the D.H programs) corroborate the reasons just mentioned: these "abnormal" results are largely due to last moments moves which prevent corrective revisions to take place.

8 278 cases  
9 Classes interval : σ / 6  
10 Leland (1985) found also difference in D.H performances due to the choice of different revisions periods.
- 95 & 96 (D.H. program Nov 70 - Oct 71 & Dec 70 - Nov 71): decrease of 6.8% in October 1971

- 110 (D.H. Jan 74 - Dec 74): fall of 10.7% in September 74

- 187 (D.H. Jun 80 - May 81): this program terminates with the largest negative "Error" (-4.42%) because of a 14% drop which did occur in May 81 following President Mitterand's election and after the last monthly revision.

- 275 (D.H. Oct 87 - Sep 88): the 87 Krach (-19.76%) did occur at the beginning of the program and the subsequent revisions did not completely correct for it.

- 276 (D.H. Nov 87 - Oct 88): this program terminates with the largest positive "Em" (4.1%) because of a market increase of 6.6% occurred in the last month (exactly the reverse of case 187).

This analysis leads to the following conclusion: without the limitations due to the data used in the simulations, our reference standard would have had a mean much closer to zero and a much smaller standard deviation. On the whole, our standard simulation seems satisfactory.

b) D.H. based on historical data (i.e. without prediction)

The complete absence of predictive abilities and the systematic use of historical parameters for the succeeding insurance periods gives outstanding results.

Our simulations show that Dynamic Hedging is extraordinary robust.

The "Error" mean over the 278 one-year simulations is only -0.45% (inclusive of transactions costs) although the constant monthly adjustment period is unsatisfactory.

Of course the standard deviation is somewhat high (2.53%), thus letting the results to be generally comprised between -5.5% and 4.5%.

16 simulations are outside the mean ± 2α limits. 3 of them are due to brutal moves at the end of the insurance period:

- 149 (D.H. Apr 77 - May 78): drop of -7% in May 77


The 13 other cases are more interesting because they clearly show the influence of the misestimation of the volatility:

- 202 to 209 (D.H. Sep 81 - Aug 82 & subsequent programs) & 262 (D.H. Sep 86 - Aug 87): the very high positive values reached by "Error" come from the use of historical volatilities which are twice the ex post ones.

- 272 to 275 (D.H. Jun 87 - May 88 & subsequent programs): they correspond to the largest negative results (down to -8.8%) for the reverse reason; because of the effect of the Krach, the volatilities used are half the ex post volatilities.

11 The figures are monthly variations.
c) The volatility effect

The **AN2 simulation confirms** the theory.

If the anticipated volatility used is smaller than the ex post volatility, the theoretical put will be underestimated, leading to negative values "Error". At the reverse, if the anticipated volatility is greater than the ex post one, the theoretical put will be overestimated and "Error" becomes **positive**.

This volatility effect is very strong in terms of dispersion: the standard deviation is four times that of the **AN1** simulation (2.79 versus 0.74) with a mean of -0.59 compared to **AN1**'s mean of -0.47.

Note that the AN2 simulation on graph II and the AN4 simulation on graph I give very similar results but that the absolute values of the AN2 figures are higher because of the absence of the interest rate effect (see below).

d) The interest rate effect

The Black & Scholes option pricing theory predicts that the value of a put is a decreasing function of the rate of interest, which implies that if the interest rate used is larger than the ex post one, the theoretical put will be underestimated, leading to a negative value of "Error" and vice versa.

This is confirmed by the AN3 simulation: an interest rate effect does exist but its influence is rather small. The "Error" mean is 0.28 with a standard deviation of 0.94.

Graph III shows this impact: the scale of variations is much reduced as compared to that of **AN4** (errors on both parameters) and of AN2 (volatility effect).

One could notice that the misestimation of the volatility and of the interest rate tend to compensate each other. This is clearly visible on graphs II & III for the D.H programs 187,199 and 209 where the volatility error goes one way and the interest rate error goes the other way.

Results of three-year insurance programs

From an "insurance" point of view, a three-year period seems more attractive than a one-year period. **The put, i.e. the "cost" of the "insurance", is proportionately** very less expensive and the long time span "smoothes" the impact of the Market fluctuations on the accuracy of the Dynamic Hedging replication process.

On the other hand, the Black & Scholes option pricing model is based on the assumptions that the volatility and the interest rate are either constant or **predictable** with certainty over the life of the option. It is obvious that the weakness of these assumptions increases with the option maturity which could jeopardize the validity of the D.H. process. Therefore it is of **great interest** to inquire in that direction.

The results are as follows:
TABLE II(\%)

<table>
<thead>
<tr>
<th>TH1 Prediction</th>
<th>TH2 Volatility Effect</th>
<th>TH3 Interest Effect</th>
<th>TH4 Historical Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEAN(^{12})</td>
<td>-0.12</td>
<td>-0.80</td>
<td>0.56</td>
</tr>
<tr>
<td>S.D.</td>
<td>1.18</td>
<td>1.05</td>
<td>2.03</td>
</tr>
<tr>
<td>MODE(^{13}) (number of cases)</td>
<td>-0.31</td>
<td>-0.80</td>
<td>-0.11</td>
</tr>
<tr>
<td>(32)</td>
<td>(20)</td>
<td>(39)</td>
<td>(30)</td>
</tr>
<tr>
<td>MINIMUM</td>
<td>-3.27</td>
<td>-3.46</td>
<td>-2.92</td>
</tr>
<tr>
<td>MAXIMUM</td>
<td>4.01</td>
<td>3.22</td>
<td>9.23</td>
</tr>
</tbody>
</table>

a) D.H. with perfect prediction

The TH1 set of simulations shows a much better performance than the AN1 one: "Error" as a mean of only -0.12\% and a standard deviation of 1.18, the extreme values being -3.27\% and 4.01\%.

This betterment of the results is largely due to a much more adequate revision procedure. Indeed a monthly rebalancing of the portfolio fits much better over a three-years period, except near the end \(^{14}\).

We found only 16 cases where "Error" is outside the mean + 2 \(\sigma\) limit (\(-2.48, 2.24\)). They correspond, as for AN1, to large Market moves occurring at the end of the insurance period.

The positive values of "Error" (D.H. programs 6 & 12 - 15 which are over 1965 - 1968 and 109 - 117 over 1982 - 1985) results from Market increases near the end of the insurance period. The negative ones - D.H. 54 (Jun 69 - May 72) & D.H. 94 (Sep 78 - Aug 81) - come from a drop of -3.5\% in May 72 and from the Mitterand's effect respectively.

Most of the "abnormal" results of the AN1 simulations have disappeared, because they were due to the first situation mentioned above, namely the fact that the theoretical put was "at the money" near the end of the program and that a large Market move occurred at that time.

\(^{12}\) 182 cases

\(^{13}\) Class interval : \(\sigma/6\)

\(^{14}\) The reader should beware that the fluctuation of TH1 on graph IV are not more pronounced than in the case of AN1 on graph I: it only seem so because of a larger left-hand scale!
b) D.H. based on historical data (i.e. without prediction)

The TH4 simulation corroborates the extraordinary robustness of D.H.: the mean (-0.24%) and standard deviation (1.12%) are half those obtained from the one-year period simulations (AN4).

The extremes are only -3.6% and +6.3%. The former case relates to an insurance period over 1978-1981 (D.H. 95) ending with the large Market drop following President Mitterand's election. The latter case correspond to a D.H. program over 1979-1982 (D.H. 107) finishing with the sharp 1982 Market increase.

An extraordinary feature should be noticed: the right-hand side end of Graph IV does not show any sign of either the 1987 krach or either the 1988 rally! This confirms the influence of the situation mentioned above: if, near maturity, the Market stands very far from the strike price of the theoretical put (either deep in or far out the money) the delta of the D.H. portfolio becomes much less sensible to large market moves.

c) The volatility effect

TH2 on graph V shows that misestimations of the volatility have a much less pronounced effect over a three-years period than over a one-year period. The mean of the results is slightly larger in absolute value (-0.8% versus -0.6% for AN2) but the standard deviation is divided by three (1.05% versus 2.8% for AN2)\(^{15}\).

Among the greatest mismatches, only five of the largest negative ones (D.H. 105-106 and 119-121) are due to misestimates of the volatility over the period, while the six largest positive ones (D.H. 3,5-6 & 14) as well as the two remaining negative ones (D.H. 94-95) are mainly due to the inadequate revision procedure, like in TH1.

d) The interest rate

On the reverse and contrarily to the results obtained for the one-year period simulations (AN3), TH3 on Graph VI shows that misestimations of the rate of interest have a large impact.

The standard deviation is doubled relative to TH1 2.03% versus 1.18%) and, on these 27 years, these interest rate misestimations have led to a rather strong positive effect, since the mean becomes positive (0.56%/versus -0.12%/ for TH1).

The ten out of the taints points (D.H. 104-112 & 116) are related to the use of interest values much lower than the ex post rates, leading to the theoretical put much more higher as it should have been with perfect foresight.

Of course, this positive impact coupled with a smaller negative effect explain the extraordinary good results obtained in the TH4 simulation.

\(^{15}\) On graph V, one may notice that the historical volatilities used have been too small compared to the ex post ones, thus resulting in generally negative errors.
CONCLUSION AND SUMMARY

This paper brings many very important results.

- It empirically proves the **robustness** of the **dynamic hedging** procedure either on a one year period or even on a three - years period.

This latter result was largely unexpected. Indeed, one may have some theoretical doubts on the validity of the Black & Scholes model over so long a time period.

Our simulations might prove these doubts to be ill - founded: in practice, the model appears stronger than expected from its theoretical premises.

- The misestimations of the two parameters - ex post volatility and rate of interest - does not jeopardize the robustness of the method.

More work remains to be done in order to assess more precisely why these two parameters behave in such a way that they statistically compensate each other.

APPENDIX

DATA used:

a. Stock prices

- 12/29/61 - 12/31/83: INSEE
  General Stock Index: Monthly data
  (Average of weekly indices)
  Two sub - periods:
  a: 1962 - 1972 = basis 100 = 12/29/61
  b: 1973 - 1983 = basis 100 = 12/29/72

- 12/31/77 - 12/31/88: C.A.C. (S.B.F)
  General Stock Index: Monthly data
  (Average of end of weekly indices)
  (Basis 100 = 12/31/81)

Note:
All insurance programs begin and end on the same index or sub-index in order not to introduce bias due to the changes of basis.

b. Riskless interest rate

1961 - 1972: T4M: monthly average of the Call money rate ("effets publics")
1973 - 1988: T4M: idem ("effets privés")
Source: Banque de France.
BIBLIOGRAPHY

AFTALION F. & PORTAIL R.
La technique de "Portfolio Insurance"
R. Banque, oct. 1988, pp. 987 - 992

BERTHON J. & GALAIS-HAMONNO G.
L'assurance portefeuille
Analyse Financière 77 1989.

BIERMAN H. Jr.
Defining and Evaluating Portfolio Insurance Strategies

BLACK F. & JONES R.
Simplifying Portfolio Insurance

BOYLE P.P. & EMANUEL D.
Discretely Adjusted Option Hedges

BRIYS E. & CROUHY M.
Assurance de portefeuille et Program Trading

BOOKSTABER R. & LANGSAM J. A
Portfolio Insurance Trading Rules

CLARKER G. & ARNOTT R. D
The Cast of Portfolio Insurance : Trade-offs and Choices

FERGUSON R.
How to beat the S & P 500 (without losing sleep)

FERGUSON R.
A Comparison of the Mean-Variance and Long-Term Return Characteristics of three Investment Strategies

FIGLEWSKI S. & KON S. J.
Portfolio Management with Stock Index Futures

GARCIA C. B. & GOULD F.J.
An Empirical Study of Portfolio Insurance
LELAND H.E.
Who should buy Portfolio Insurance?
*J. Of Finance, May 1980.*

LELAND H.E.
Option Pricing and Replication with Transactions Costs
*J. Of Finance, Dec 1985.*

MERRICK J.
Portfolio Insurance with Stock Index Futures

RENDLEMAN M. & McENALLY R.W.
Assessing the Costs of Portfolio Insurance
*Fin Analyst J., May 1987.*

RUBINSTEIN M. & LELAND H. E.
Replicating Options with Positions in Stock and Cash

RUBINSTEIN M.
Alternative Path to Portfolio Insurance

ZURACK M.A.
*The Many Forms of Portfolio Insurance*