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RELIABILITY OF DYNAMIC HEDGING

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ROBUSTESSE DES TECHNIQUES DE DYNAMIC HEDGING

JEAN BERTHON AND GEORGES GALLAIS - HAMONNO

ABSTRACT

The basic technique used to cover portfolio risk is dynamic hedging. In the first part of this paper, the authors set out the theoretical basis of the technique, stressing the fundamental importance of choices regarding volatility and interest rate levels in the design of a hedging package.

The second part is devoted to the statistical appraisal of the impact of mistaken forecasts concerning these two parameters. 278 one - year hedging plans and 182 3 - year programs (obtained by moving forward systematically one month at a time) based on the CAC 240 index are simulated for the 27 years from 1962 to 1988.

For each duration (one year or three years), four different simulations are made :

- one reference simulation based on observed volatility and interest rates
- two simulations analyzing the respective impact of volatility and interest rates, on the basis of historical levels of the parameter concerned
- one simulation analyzing the combined effects of errors concerning interest rates and volatility.

Overall, results show that dynamic hedging is remarkably reliable, both over one year and over three years. It also appears that the relative importance of volatility and interest rates is inverted over time volatility is the crucial factor in one - year hedges, but not over three years. Finally errors regarding volatility and interest rates apparently balance each other out.

00C ROBUSTESSE DES TECHNIQUES DE "DYNAMIC HEDGING"

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RÉSUMÉ

" L'assurance de portefeuille" est fondée sur l'utilisation de la Technique du "Dynamic Hedging (DH) ".

Dans une première partie nous rappelons les bases théoriques du D.H. afin de montrer l'importance du choix des niveaux de volatilid et de taw d'intérêt dans la mise en place d'un programme d'assurance.

La seconde partie a pour objet de mesurer statistiquement les effets des erreurs de prévisions sur ces deux paramètres.

A cet égard, 278 programmes d'assurance à 1 an et 182 programmes à 3 ans (obtenus par décalage systématique d'un mois) portant sur le CAC 240 sont simulés sur la période globale de 27 ans couvrant 1%2 - 1988.

Pour chacune de ces périodes de programme (1 an ou 3 ans), 4 séries de simulations sont effectuées :

- une simulation de référence utilisant la valeurs ex post de la volatilité et du taux d'intérêt,

- deux simulations analysant les effets respectifs de la volatilité et du taux d'intérêt, en utilisant comme valeur du paramètre étudié sa valeur historique,

- une simulation anlysant les effets conjugués des erreurs sur ces deux paramètres.

Les résultats obtenus font apparaître globalement une robustesse remarquable du D.H., que ce soit sur des périodes d'un an ou de trois ans.

I1 semblerait que l'importance relative de la volatilid et du taux d'intérêt s'inverse en fonction de la longueur du programme d'assurance : sur un an l'impact de la volatilité est prédominant alors qu'il ne le paraît plus sur trois ans.

D'autre part est apparu un phénomène de compensation entre les erreurs de volatilité et celles de taux d'intérêt.

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INTRODUCTION

The financial futures and options markets are completely changing the management of stocks or bonds portfolios and they lead to the creation of new management techniques. Presently, the more popular is represented by "Index Funds", either "tilted" or not.

According to our belief, a more promising technique - even if it is presently less popular- is "Dynamic Hedging", also called "Portfolio Insurance".

Definition

To be more precise, Dynamic Hedging (D.H.) is a systematic management technique, the objective of which is to obtain the loss - profile of a portfolio protected by a put option, i. e. to obtain non -symetrical results according to the Stock Market evolution :

- the D.H. portfolio profits of Stock Market increases
- but, if the Market declines, the *decrease* of the D.H. portfolio is limited to a pre-determined level (the "floor").

This is equivalent to hold a call option (with an exercice price equal to the floor) and cash

This technique is directly derived from the Black & Scholes model (1973) which demonstrated that any option can be replicated by the continuous adjustment of a portfolio comprising only stocks and the risk - free asset. D.H. strategies have been put into practice in the U.S. since many years and have been *quite* successful although their development now suffers from the 1987 krach.

Several remarks have to be made.

First a terminological one : strictly speaking, "portfolio insurance" should only refer to the direct purchase of a put on the risky asset, which gives full protection. "Dynamic Hedging", on the reverse, should refer to the replication of such a purchase of a put. It follows that the results of D.H. strategies (limiting the downwards movements of the portfolio to the foor level) cannot be formally guaranteed[1].

[1] ***For a detailed analysis of the direct portfolio insurance strategy and of the impossibility of implementing it, see ow paper JB & GGH (1989)***

Second, D.H. is based on a predetermined maturity. It implies that its performance should only be assessed on that particular date and not before. It also implies the use of European options - non callable before maturity - instead of the more common American ones.

Finally, the cost of implementing a D.H. strategy which theoretically is the cost of buying the put of the direct insurance strategy, is **only known** ex post.

Content of the paper

This paper is organized in two parts. The **first** part recalls the **essentials** of Dynamic Hedging in order to show why the estimates of volatility and rate of interest are theoretically important for the D.H. technique.

The second part analyses via sets of simulations over 27 years how the results of the D.H. strategies implementations differ from the theoretical expected ones when one uses historical estimates instead of the "true" unknown ones.

The basic and overall result is that D. H is extraordinary robust.

I THE ESSENTIALS OF DYNAMIC HEDGING[2]

D.H. is based on the property of options replication which leads to an iterative procedure. As we have just explained, the objective of D.H. is to replicate a put.

Property of options replication

Among the different option parameters, one is of special importance for D.H. : the option - delta.

The option - delta measures the expected change of the premium (in $ terms) if the stock - price changes by 1 $. The delta is measured by the **first partial derivative** of the BLACK & SCHOLES formula relative to the **stock-price.** For a put P :

$$(1)\ \delta_P = \frac{dP}{dS} = N(d_1) - 1 < 0$$ [3]

The use of the Delta is important because it makes **possible** to create a stock - portfolio without risk, the change in premium being offset by the change in stock - price :

(2) Delta - Neutral **Portfolio** = Put - δ_{pS}

If M is the money - value of this delta - neutral portfolio :

$$(3)\ M = Put - \delta_{pS}$$

and one obtains two identities :

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[2]The reader who is familiar with the Futures and options theory is invited to jump directly to Section II.

[3]The reader should notice that the Put-delta is negative. It explains why owning the Stock will appear with a negative sign.

(4) IDENT I :

Put - δ PS ≡ loan at the risk - rate of interest of the amount M

(5) IDENT II :

Put ≡ loan at the risk - free rate of M

PLUS

shorting the related **stock proportionately to δ** P

The "synthetic call" replication

From (4) and (5), it is **possible** to replicate the return of any asset **(Call, Put,** Stock, Treasury Bill...) when one has two of them.

For D.H., the objective is to replicate a portfolio consisting of stocks and **their protective** Puts, **i.e.,** to replicate S + P?

This is obtained in the following manner :

From (5):

(6) $S + P = S + M + \delta_p S$

factoring :

(7) $S + P = M + (1 + \delta_p)S$

Using Identity I, it gives a new identity III :

(8) S + P ≡ Loan at the risk - free rate for an amount $P - \delta_p S$

PLUS

Investment in stocks for an amount **$(1 + \delta_p)S$**

which can be expressed as :

(8 bis) (S + P) EQUI $(1: \delta_p)S + (P - \delta_p S)$

Dynamic hedging in practice

D.H. is based on Identity III and **equation** 8 bis. The portfolio (with value S + P) has to be distributed, according to the theoretical Put delta, among actual stocks and investment in T. Bills.

At the beginning of the insurance program, **one** has to **estimate** the theoretical value of an European Put according to the Black & Scholes formula and to derive its delta.

Since this put is notional, the **manager** is **completely free** in its choices :

- f a the insurance maturity : **ane,** two or three ...years
- **for the floor (which is the strike - price of the Put) : 100%, 95% of S+ P[4] ; note that** it is possible to **increase** the floor if the Market has **sufficiently** *gone* up, **giving** the D.H. program a ratchet effect
- but not for the ***volatility*** and the ***rate of interest*** levels which should be equal ***to*** their ***expost*** values(see below).

[4] ***Some authors prefer to use a floor equal to 100 (or 95*** %) ***of S, putting apart the cost of the*** theoretical ***Put.***

The **second** phase of the D.H. is the portfolio revision because of the changes in delta **One** should re - estimate the theoretical Put and re - **distribute** the portfolio according to the new **delta.**

As the "hedge ratio" δ_p inversely varies with the stock prices, the portfolio **will** be the more invested in stocks as the Market ***goes*** up and **the** more invested in T. Bills as the Market goes down [5].

In the Black & Scholes world, these portfolio revisions are made **continously** and without transaction costs. But in the real world we have to face these two problems.

First, stock **prices** are **discontinous** for **two** reasons : the quotations are made at discrete time intervals and moreover the markets are closed daily.

Second, transaction costs do exist

Boyle & **Emanuel (1980),** and Leland (1985) have tried to **analyse** and measure the effects of these discrete adjustments and of the transaction costs on **the** results obtained by replicating strategies.

The Leland's answer to **both questions** -but only on the basis of a given adjustment period - is to increase **the** volatility used for estimating the theoretical Put and its delta.

Finally it should be noted that most of the D.H. programs use the Futures markets fa buying **or** selling **index** futures **contracts** instead of buying **or** selling ***on*** the physical Stock **Market.** The revision policy is much less costly and much easier to implement thanks to the larger liquidity.

D.H. conditions of success

The success of D.H. is based on the quality of the estimate of the notional Put If the theoretical Put is not the right one, the actual replicated portfolio will not behave as **expected** and, at maturity, there **will** be a discrepancy between **the** actual return and the expected **theoritical** one. Second, as **mentioned above,** the value of the Put amounts to the ***ex-ante*** cost of the D.H. program ; if the Put is miscalculated,the ***ex - post will*** either ***be*** less or larger than expected.

For estimating this Put, three of its components are given : the stock **price or** the stock index level, the maturity and the strike - price (these two **latter** being chosen by the investor). Thus the estimate **will** rely on the two other parameters : **volatility** and interest rate. The stock - volatility and the interest **rate should** be ***the ex - post ones, i..e those which will prevail during the D.H. period.***

Of course, these two components are unknown at the time a **D.H.program is** launched although they are crucial f a estimating the value of the theoretical Put

The remaining part of **the** paper analyses in which way and how much prediction errors on these two **parameters** lead **to** results different than **those** expected. In other **word,** we aim to know if the D.H. **procedure is robust or** not at **mis** - estimations of the theoretical Put

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[5] ***Such a strategy is just the reverse of the traditional "orthodox" strategy : "buy low. sell high". Which answers the question : who is the seller of this insurance strategy ? Of course, the market as a whole.***

II ANALYSIS OF ROBUSTNESS OF D.H.

Our analysis of robustness of D.H. to misestimations of the theoretical put is based on a set of simulations comparing the results obtained with a perfect prediction versus those with imperfect ones.

The set of simulations

A D.H. program is started each month over a 27 years time - span, (12.29.61 to 12.31.88)

- for a one year period (AN) : 278 cases
- for a three years period (TH) : 182 cases

The D.H. program has a floor of 95% of starting value and portfolio revisions take place each month (because of the monthly data)[6].

Per program period (AN* or TH*), four set of simulations are conducted :

- AN1 and TH1 are based on perfect predictions and will serve as reference standard ; the volatility and the risk free rate are the ex - post ones over the D.H. period
- AN2 and TH2 analyse the volatility effect, the volatility used is the historical standard deviation over the preceding period while the interest rate is the ex post one
- AN3 and TH3 analyse the interest rate effect ; the interest rate used is the historical one over the preceding period while the volatility is the ex - post one
- AN4 and TH4 analyse the actual situation (with no predictions) : both volatility and interest rate are the historical ones over the preceding period.

The data are the French General Stock Index (monthly data) and the monthly average of the Call money rate?

The measure of D.H. programs accuracy

The robustness (or lack of) of each D.H. program is measured by the "Error" between the value of the D.H. portfolio at the end of the program relative to the theoretical objective

Namely :

$$\text{Error (\%)} = \left(\frac{\text{TERMINAL VALUE OF D.H. PORTFOLIO}}{\text{TERMINAL VALUE OF THEORETICAL PORTFOLIO}} - 1 \right) * 100$$

Results of one - year insurance programs

Table I shows the characteristics of the distribution of "Error" for each set of simulations.

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[6] AN the volatilities are corrected to take into account the transaction costs (1% for a round - trip) in the way proposed by Leland (1985, p. 1289). To give an order of magnitude the Leland' s formula increases the actual volatility by around 10%.

[7] The data are described in the appendix.

TABLE I(%)

	AN1 Perfect Prediction	AN2 Volatility Effect	AN3 Interest rate Effect	AN4 Historical Data
MEAN[8]	-0.47	-0.59	-0.28	-0.45
S.D.	0.74	2.79	0.94	2.53
MODE[9]	-0.47	-0.58	-0.44	-0.45
(number of cases)	(42)	(40)	(38)	(44)
MINIMUM	-4.41	-9.16	-4.22	-8.83
MAXIMUM	4.09	8.81	3.90	8.35

a) D.H. with perfect prediction

The AN1 simulation is used as "standard", since it is based on a perfect prediction of both parameters, volatility and rate of interest. One should expect the mean of "Error" to be zero.

This is not the case and this **seems** to be **mainly** due to the revision process.

First, the revision policy should be based on a variable frequency depending on the time to maturity and on the path followed by the Market : the nearer one is to maturity and the closer the Market price is to the strike - price, the more the **delta** of the replicated portfolio (S + P) varies with the Market moves, **consequently** one has to adjust the **actual** portfolio more frequently.

Second, due to the data availability, we were obliged to use a *monthly* revision period, which clearly violates the continuous trading assumption of the Back & **Scholes** **model**[10].

On the **graph** I, the four lines drawn aim to show which results are **"abnormal", i.e** those **wich** are **outside** the mean $\pm 2\sigma$ band : for AN 1, the results having a value greater than 1% or lesser than -195 % ; for AN IV those with a value greater than 4,6% or lesser than **-5,5%.**

These points (numbered in the order of the D.H programs) corroborate the **reasons** just mentioned : these "abnormal" results are largely due **to** last moments moves which prevent corrective revisions to take place :

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[8] *278 cases*

[9] *Classes interval* : σ / 6

[10] ***Leland*** (1985) ***found also difference in D.H performances due to the choice of different revisions periods.***

- **95** & 96 (D.H. program **Nov 70 - Oct 71 & Dec 70** - Nov **71)** : **decrease of 6,8%** in October **1971** [1]

- **110 (D.H. Jan 74 - Dec74)** : fall of **10.7%** in September **74**

-**187 (D.H.** Jun **80** - May **81)** : this program terminates with the largest negative "Error" **(- 4 42%)** because of a **14% drop** which did **occured** in May **81** following President Mitterand's **election** and after the last **monthly** revision.

- **275 (D.H. Oct 87 - Sep 88)** : the **87 Krach (-19.76%) did** occur **at** the beginning of the **program** and the subsequent **revisions** did not completely correct far it.

- **276 (D.H.** Nov **87-Oct 88)** : this **program** terminates with the largest positive " E m " **(4.1%)** because of a market increase of **6.6% occured** in **the** last month (exactly the reverse of case **187).**

This analysis leads to **the** following conclusion : without the limitations due to the **data** used in the simulations, our reference standard would have had a mean much closer to zero and a much smaller standard deviation. **On** the whole, our standard simulation **seems satisfactory.**

b) D.H. based on historical data (i.e without prediction)

The complete absence of predictive abilities and the systematic use of historical parameters for the succeding insurance **periods** gives outstanding results.

Our **simulations** show that Dynamic ***Hedging is extraordinary robust.***

The **"Error"** mean over the **278** *one* **-** year simulations is only **- 0.45%** (inclusive of transactions **costs) although** the constant monthly **adjustement period** is unsatisfactory.

Of course the standard deviation is somewhat high **(2.53** %), thus **letting the** results to be generally **comprised** between **- 5.5% and 4.5%.**

16 simulations **are outside** the mean + **- 2 a** limits. **3** of them are due to brutal moves at **the end of the insurance period :**

- **149 (D.H. Apr** 77 **-** May **78)** : **drop** of - 7% in May 77

- **187 & 188 (D.H. Jun 80 -** May **81 &** Jul **80 - Jun 81)** : fall in May **81** due to the **Mitterand's** effect already mentionned

The 13 other cases **are** more interesting because they clearly **show** the influence of the misestimation of the volatility :

- **202 to 209 (D.H. Sep 81 -** Aug **82 &** subsequent **programs) &** 262 **(D.H. Sep 86 -** Aug **87)** : the very high ***positive*** values reached by "Error" come from the use of historical volatilities which are **twice** the ex post ones.

- **272 to 275** (D.H. **Jun 87 -** May **88** & **subsequents** programs) : they *correspond* ***to*** the largest ***negative*** results (down ***to*** **- 8.8%)** for the **reverse** reason ; because of the effect *of* the **Krach,** the volatilities used are half the ex post volatilities.

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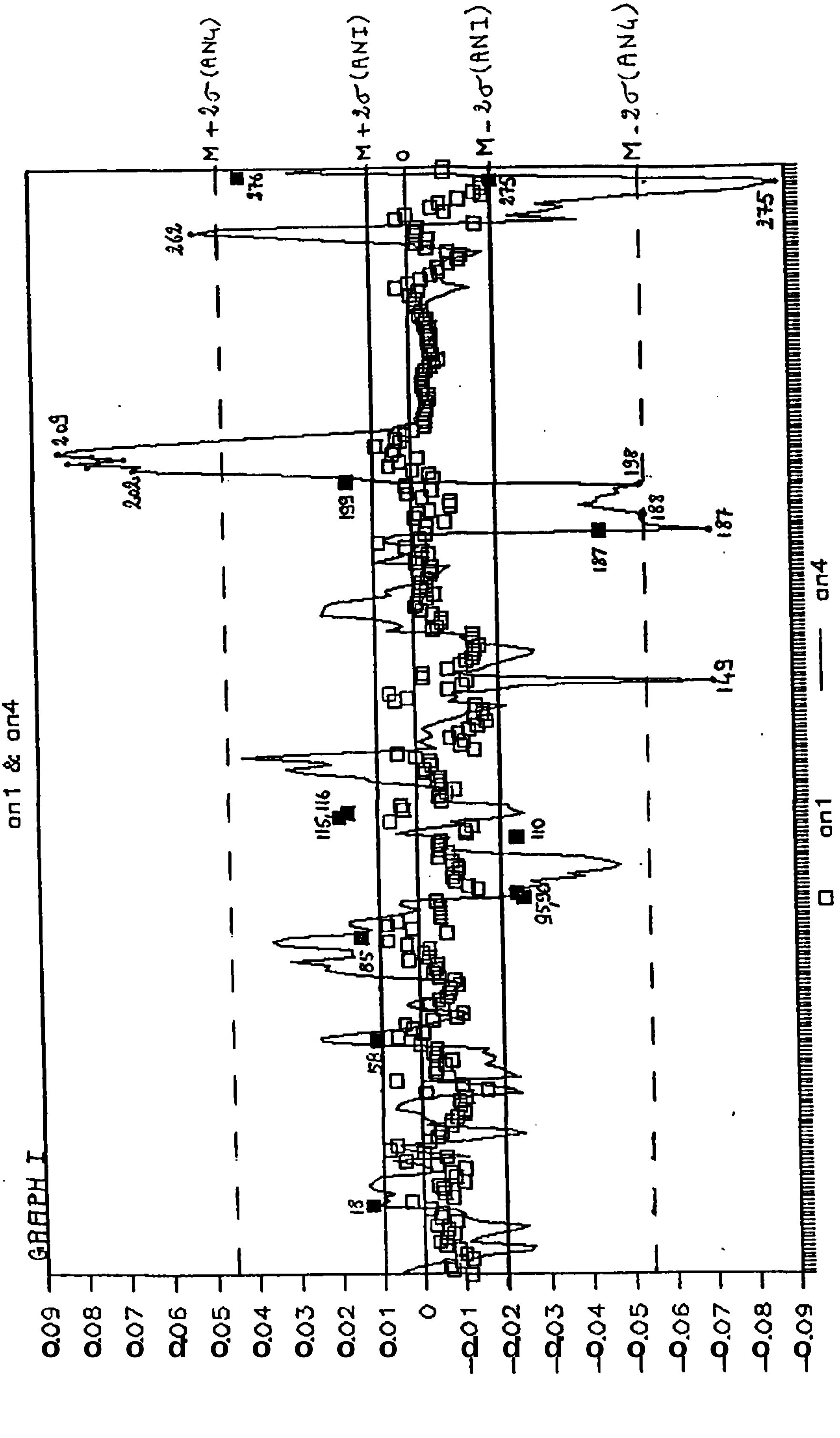
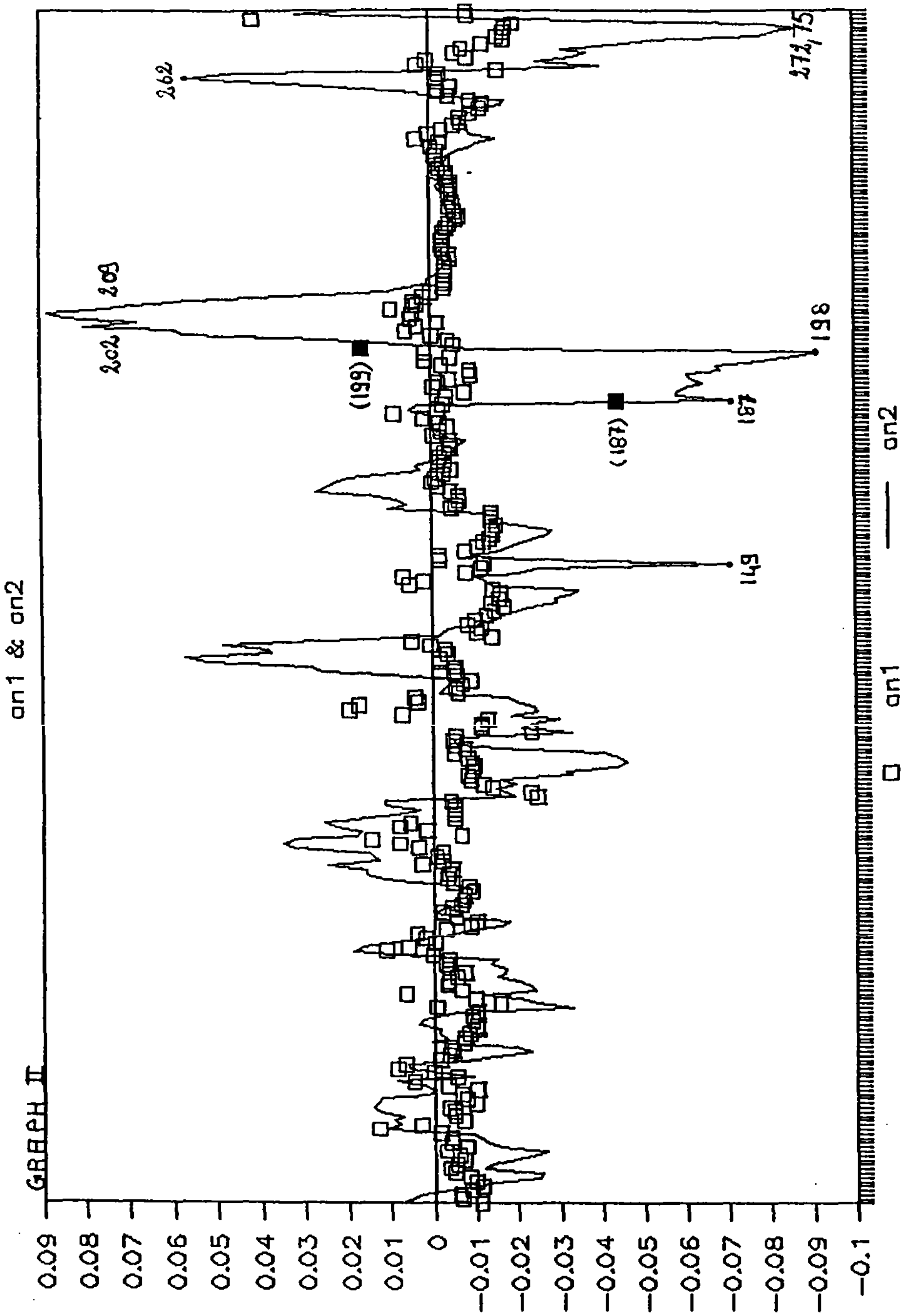
[1] ***The figures are monthly variations.***


GRAPH I
an1 & an4
M + 2σ (AN4)
M + 2σ (AN1)
M - 2σ (AN1)
M - 2σ (AN4)
an1
an4
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09


GRAPH II
an1 & an2
0.09
0.08
0.07
0.06
0.05
0.04
0.03
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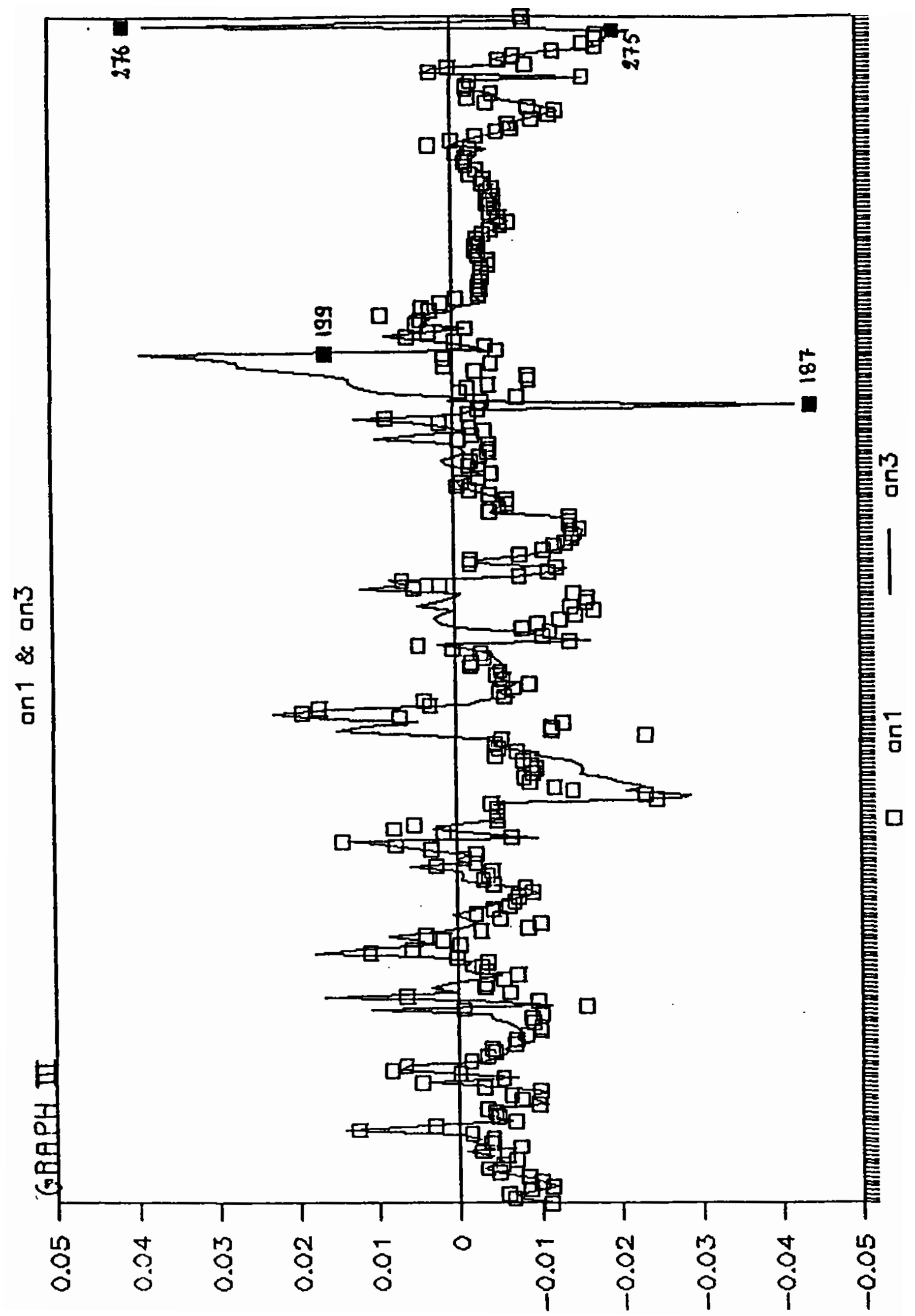
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
262
274,75
202
203
(199)
198
187
(187)
149
an1
an2


GRAPH III
an1 & an3
0.05
0.04
0.03
0.02
0.01
0
−0.01
−0.02
−0.03
−0.04
−0.05
176
175
199
187
an1
an3

c) The volatility effect

The AN2 simulation confirms the theory.

If the anticipated volatility used is smaller than the ex post volatility, the theoretical put will be underestimated, leading to negative values "Error". At the reverse, if the anticipated volatility is greater than the ex post one, the theoretical put will be overestimated and "Error" becomes positive.

This volatility effect is very strong in terms of dispersion : the standard deviation is four times that of the AN1 simulation (2.79 versus 0.74) with a mean of - 0.59 compared to AN1's mean of - 0.47.

Note that the AN2 simulation on graph II and the AN4 simulation on graph I give very similar results but that the absolute values of the AN2 figures are higher because of the absence of the interest rate effect (see below).

d) The interest rate effect

The Black & Scholes option pricing theory predicts that the value of a put is a decreasing function of the rate of interest, which implies that if the interest rate used is larger than the ex post one, the theoretical put will be underestimated, leading to a negative value of "Error" and vice - versa

This is confirmed by the AN3 simulation: an interest rate effect does exist but its influence is rather small. The "Error" mean is - 0.28 with a standard deviation of 0.94.

Graph III shows this impact : the scale of variations is much reduced relative to that of AN4 (errors on both parameters) and of AN2 (volatility effect).

One could notice that the misestimation of the volatility and of the interest rate tend to compensate each other. This is clearly visible on graphs II & III for the D.H programs 187,199 and 209 where the volatility error goes one way and the interest rate error goes the other way.

Results of three - years insurance programs

From an "insurance" point of view, a three-year period seems more attractive than a one-year period The put, i.e. the "cost" of the "insurance", is proportionately very less expensive and the long time - span "smoothes" the impact of the Market fluctuations on the accuracy of the Dynamic Hedging replication process.

On the other hand, the Black & Scholes option pricing model is based on the assumptions that the volatility and the interest rate are either constant or predictable with certainty over the life of the option. It is obvious that the weakness of these assumptions increases with the option maturity which could jeopardize the validity of the D.H. process. Therefore it is of great interest to inquire in that direction.

The results are as follows :

TABLE II(%)

	TH1 Perfect Prediction	TH2 Volatility Effect	TH3 Interest rate Effect	TH4 Historical Data
MEAN[12]	-0.12	-0.80	0.56	-0.24
S.D.	1.18	1.05	2.03	1.12
MODE[13]	-0.31	-0.80	-0.11	-0.05
(number of cases)	(32)	(20)	(39)	(30)
MINIMUM	-3.27	-3.46	-2.92	-3.63
MAXIMUM	4.01	3.22	9.23	6.26

a) D.H. with perfect prediction

The TH1 set of simulations shows a much better performance than the AN1 one : "Error" as a mean of only - 0.12% and a standard deviation of 1. 18, the extreme values being - 3.27% and 4.01%.

This betterment of the results is largely due to a much more adequate revision procedure. Indeed a monthly rebalancing of the portfolio fits much better over a three-years period, except near the end [14].

We found only 16 cases where "Error" is outside the mean + - 2 σ limit (- 2 48, 2.24). They correspond, as for AN1, to large Market moves occuring at the end of the insurance period.

The positive values of "Error"(D.H.programs 6 & 12 - 15 which are over 1965 - 1968 and 109 - 117 over 1982 - 1985) results from Market increases near the end of the insurance period. The negative ones - D.H.54 (Jun 69 - May 72) & D.H. 94 (Sep 78 - Aug 81) - come from a drop of - 3.5% in May 72 and from the Mitterand's effect respectively.

Most of the "abnormal" results of the AN1 simulations have disappeared, because they were due to the first situation mentionned above, namely the fact that the theoretical put was "at the money" near the end of the program and that a large Market move occured at that time.

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[12] *182 cases*

[13] *Class interval : σ / 6*

[14] *The reader should beware that the fluctuation of TH1 on graph IV are not more pronounced than in the case of AN1 on graph I : it only seem so because of a larger left - hand scale !*

b) D.H. based on historical **data (i.e without prediction)**

The TH4 simulation corroborates the extraordinary robutness of **D.H.** : the **mean (- 0.24%) and** standard deviation **(1.12%)** are half **those** obtained f a the one - **year period** *simulations* **(AN4).**

The extremes are only **-3.6% and + 6.3%.** The former case relates to an insurance **period** over **1978-1981 (D.H. 95)** ending **with the** large Market drop following President **Mitterand's** election. **The** latter **case** correspond to a **D.H. program** over **1979 - 1982 (D.H. 107) finishing** with **the** sharp **1982 Market** increase.

An extraordinary feature **should** be noticed : the right-hand side end of Graph IV does not show any sign of either the **1987 krach or** either the **1988** rally **! This confirm** the influence of the situation **mentionned** above : if, near **maturity,** the Market stands very far **from** the strike price of the theoretical **put** (either deep in or far out the money) the delta of the **D.H.** portfolio **becomes** much less sensible to large market mwes.

c) The volatibility effect

TH2 on graph **V** shows that misestimations of the volatility have a ***much less pronounced effect*** **over** a ***three*** - years period **than** over a ***one*** **year** period. The **mean** of the results is slightly larger in absolute value **(- 0,8%** versus **- 0.6%** for **AN2)** but **the** standard deviation is divided by three **(1.05%** versus **2.8%** for **AN2)**[15].

Among the greatest **mismatchs,** only five of the largest negative **ones (D.H. 105 - 106** and **119 - 121)** are due **to misestimates** of **the** volatility **over** the period, **while** the six largest positive ones **(DH1 - 3, 5 - 6 & 14) as** well **as** the **two** remaining negative ones **(D.H.** 94 **- 95) are** mainly due to the **inadequate** revision **procedure,** like in TH1.

d) The interest rate

On the reverse and contrarily to the results obtained for the one - year period **simulations (AN3), TH3** on **Graph** VI shows **that** misestimations of the rate of interest have a ***large impact.***

The standard deviation is doubled relative to **TH1 2.03%** versus **1.18%/) and,** ***on*** **these** 27 years, these interest rate **misestimations** have led to a rather strong positive effect, since the mean **becomes** positive **(0.56%/. versus - 0.12%/.** for **TH1).**

The ten out - of - the **limits** points **(D.H. 104 - 112 & 116) are** related **to** the use of interest values much lower than the ex post rates, **leading** to a **theoretical** put much more higher **as** it should have been with perfect **foresight.**

Of course, **this** positive impact coupled with a **smaller** negative effect explain the extraordinary **good** results obtained in **the TH4** simulation.

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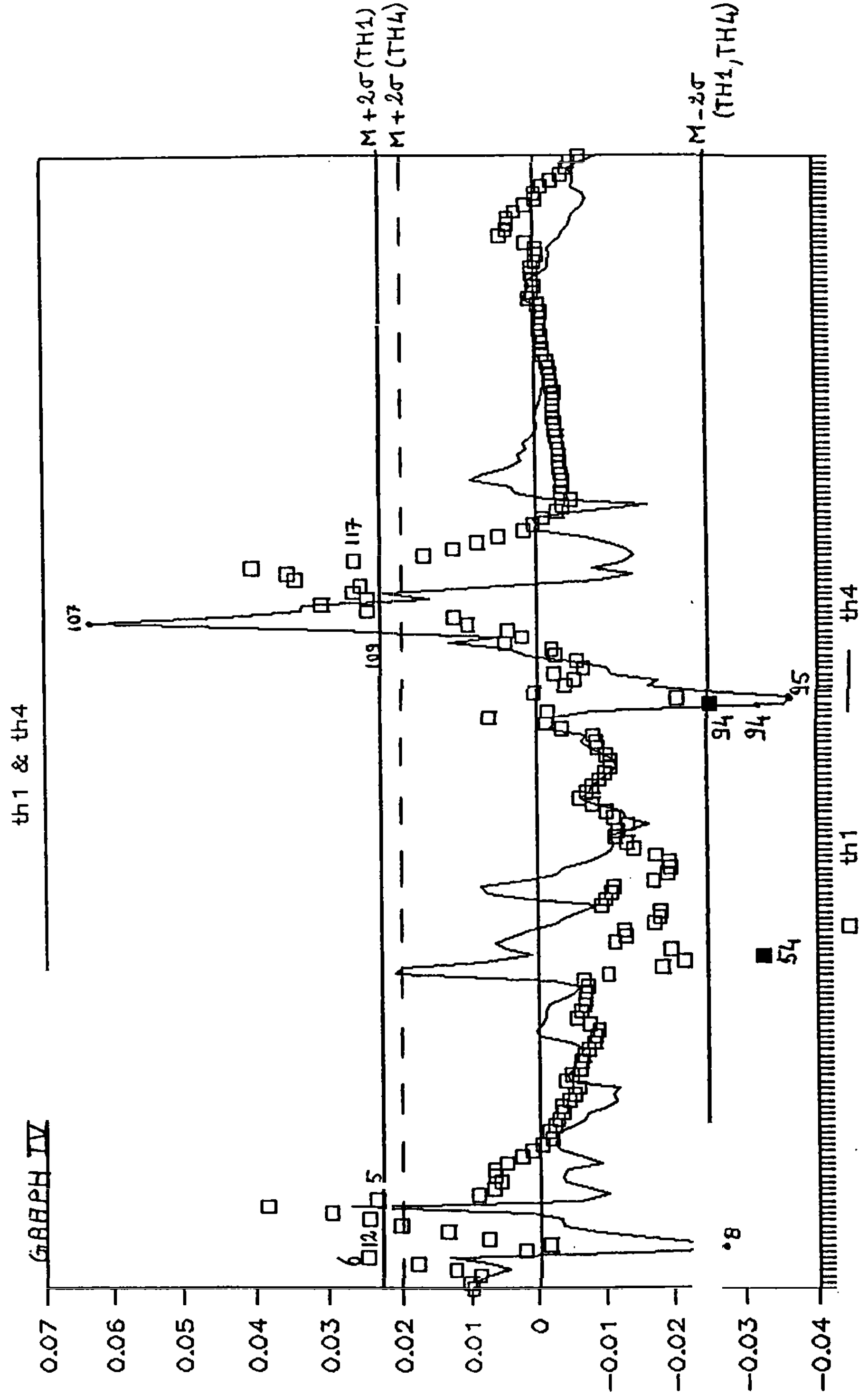
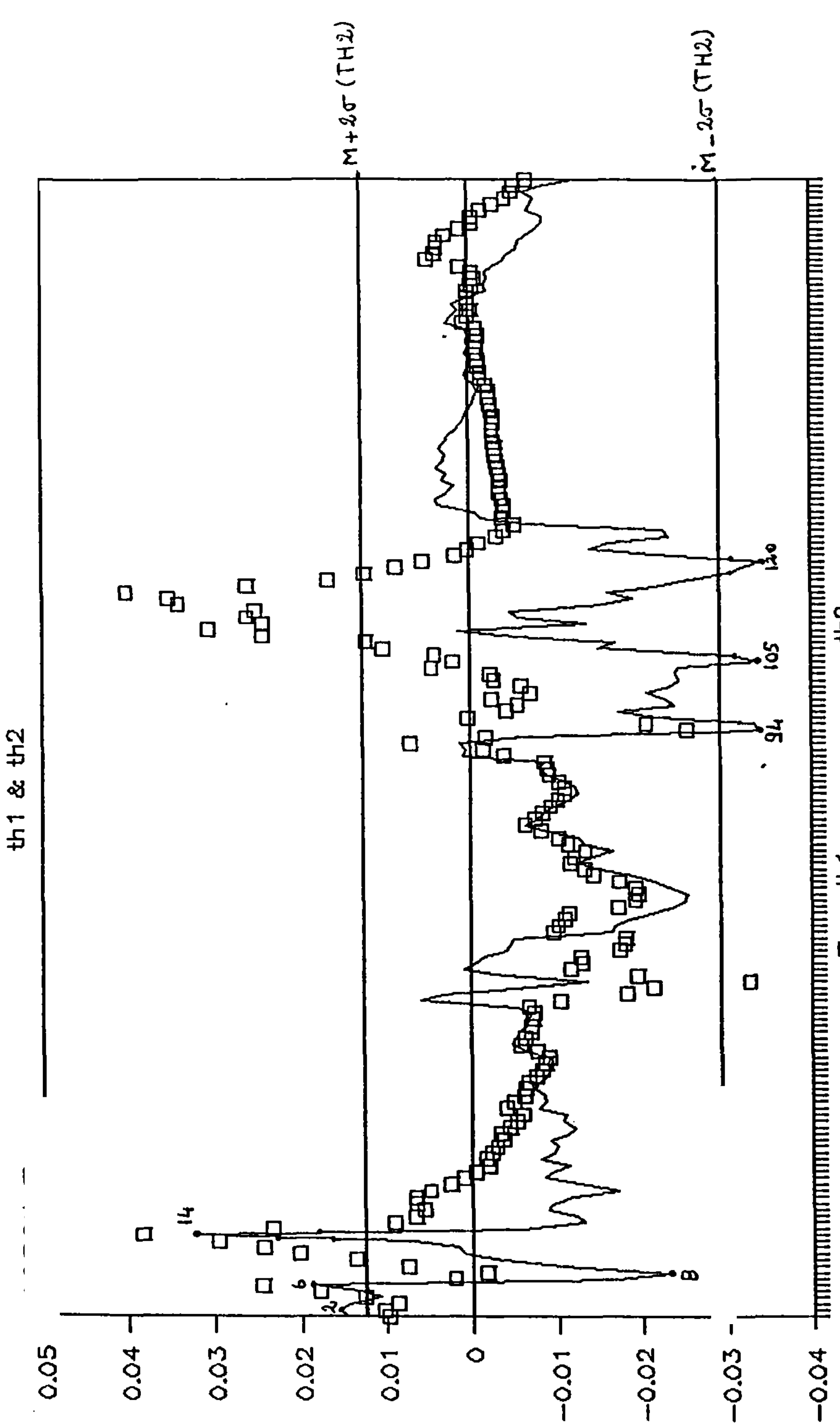
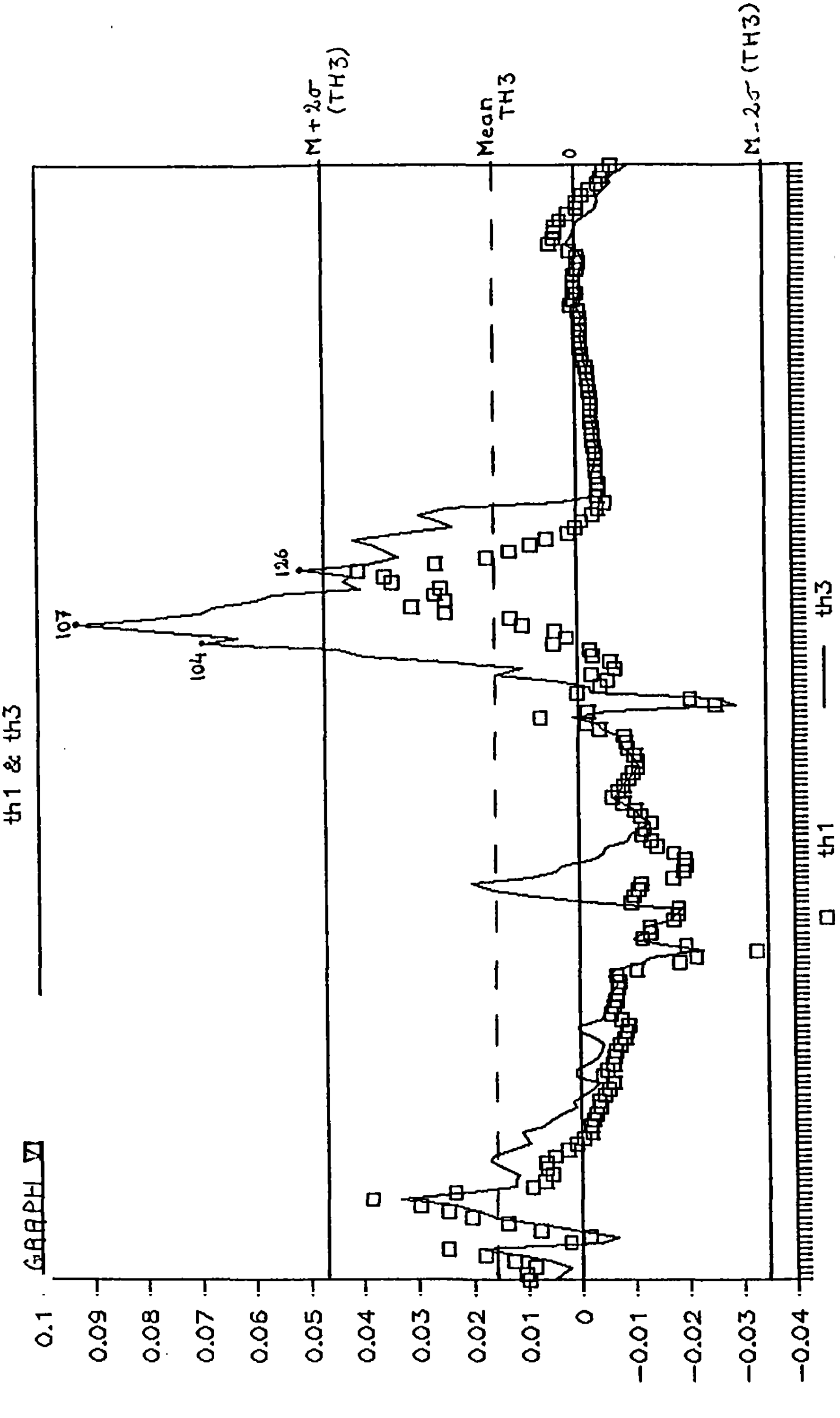
[15] On graph V, one may notice that the historical volatilities used have been too small compared to the ex post ones, thus resulting in generally negative errors.


GRAPH IV
th1 & th4
M + 2σ (TH1)
M + 2σ (TH4)
M - 2σ
(TH1, TH4)
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
th1
th4
107
109
117
95
94
94
54
5
6
12
8


th1 & th2
M+2σ (TH2)
M-2σ (TH2)
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
2
6
8
14
94
105
120
th1
th2


GRAPH VI
th1 & th3
M + 2σ (TH3)
Mean TH3
0
M - 2σ (TH3)
107
104
126
th1
th3
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04

CONCLUSION AND SUMMARY

This paper brings many very important results.

- It empirically proves the ROBUTNESS of the DYNAMIC HEDGING procedure either on a one year period or even on a three - years period

This latter result was largely unexpected. Indeed, one may have some theoretical doubts on the validity of the Black & Scholes model over so long a time period

Our simulations might prove these doubts to be ill - founded : in practice, the model appears stronger than expected from its theoretical premisses.

- The misestimations of the two parameters - ex post volatility and rate of interest - does not jeopardize the robutness of the method.

More work remains to be done in order to assess more precisely why these two parameters behave in such a way that they statistically compensate each other.

APPENDIX

DATA used :

a. Stock prices

- 12/29/61 - 12/31/83 : INSEE
  General Stock Index : Monthly data
  (Average of weekly indices)
  Two sub - periods :
  a : 1962 - 1972 =basis 100 = 12/29/61
  b : 1973 - 1983 =basis 100 = 12/29/72
- 12/31/77 - 12/31/88 : C.A.C. (S.B.F)
  General Stock Index : Monthly data
  (Average of end of weekly indices)
  (Basis 100 = 12131 /81)

Note:
All insurance programs begin and end on the same index or sub-index in order not to introduce bias due to the changes of basis.

b. Riskless interest rate

1961 - 1972 : T4M : monthly average of the Call money rate ("effets publics")

1973 - 1988 : T4M : idem ("effets privés")

Source : Banque de France.

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