CONTRIBUTION N° 19

TERM STRUCTURE
OF VOLATILITY
IN FINANCIAL MARKETS

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Australie / Australia

STRUCTURE DES TERMES
DE LA VOLATILITE
DES MARCHES

STRUCTURE DES TERMES DE LA VOLATILITE DES MARCHES FINANCIERS

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RESUME

Une quantification précise de la volatilité des marchés financiers devient de plus en plus importante pour la gestion du risque portefeuille et pour la prise. Le besoin se manifeste de quantifier la relation entre la volatilité et la durée de la période d'intérêt. Cet article montre que l'hypothèse classique du modèle de promenade aléatoire conduit à des estimations non bornées de la volatilité, quand la période d'intérêt augmente, ce qui ne correspond pas au comportement observé sur la plupart des marchés financiers. On propose un autre modèle stochastique, dont on dérive une relation entre la volatilité et la durée. On montre que ce modèle est borné, et on le teste sur une période de dix ans, sur des obligations d'Etat australiennes. Ce test limité donne une confirmation préliminaire de la validité du modèle.

TERM STRUCTURE OF VOLATILITY IN FINANCIAL MARKETS

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SUMMARY

Precise quantification of the volatility of financial markets is becoming increasingly important ti, both portfolio risk management and to position taking. A need exists for quantifying the way in which volatility and the duration of the period of interest are related. This paper shows that the usual assumption of a random walk model leads to unbounded estimates of volatility as the period of interest increases. This docs not correspond to observed behaviour in most financial markets. An alternative stochastic model is proposed and its volatility - time period relationship derived. The model is shown to be bounded and is tested on 10 year Australian Government bonds. This limited testing gives preliminary confirmation of the validity of the model.

1 - INTRODUCTION

The events of recent years in financial markets, especially those surrounding the October 1987 stock market crash, demonstrate the increasing importance of financial risk management. No longer can institutional investors, superannuation funds, corporations and other serious investors continue without formalised and accurate analysis in the management of their portfolio of financial assets and their attendant financial risks. Consequently, the actuary or financial mathematician working in this area needs not only empirical observations, but also a soundly based theory supporting those practical observations.

This paper presents a preliminary report on a theoretical model of wide generality which establishes a framework for analysing the term structure of volatility in financial markets. By this, we mean the way in which the volatility a standard deviation varies with the time period under consideration. Preliminary testing of the model is performed using a limited amount of daily data on the market yields for 10 year Australian Government bonds. Further testing will be available by the time of the conference.

2 - NOTATION

c, c_1, c_2	constants
f	Frequency, cycles per time unit
$H(s)$	Transfer function
j	$\sqrt{-1}$
s	Laplace transform parameter
T	Arbitrary point in time a duration
T_1	Time constant
t	Time
$w(t)$	White noise

$y(t)$	Measurement or price in a financial market
$\Phi_{yy}(\omega)$	Power spectral density of $y(t)$
$\phi_{yy}(\tau)$	Autocorrelation function of $y(t)$
$\sigma_y(T)$	Stantard deviation of $y(t)$ during period T
w	Circular frequency, radians per time unit

3 - DEFINITIONS AND INTERPRETATIONS

The statistical and stochastic properties of a random variable, such as the on - market yield rate for a financial instrument, are neatly and powerfully specified using the power spectral density (PSD). Readers unfamiliar with the concept are referred to Newland (5) or any of the better books on stochastic processes.

Consider a measurement, $y(t)$, made on a financial market at time, t. The measurement might be the on - market yield of fixed interest instrument. Firstly, define an autocorrelation function,

$$\phi_{yy}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} y(t) \quad y(t+\tau) \quad dt \qquad (1)$$

The autocorrelation function is simply the average value of the measurement, $y(t)$, multiplied by itself τ time units later. Note that when the time separation, τ, is zero, the autocorrelation function becomes simply the mean square value of y.

Next, the power spectral density (PSD), Φ_{yy} (w), is defined as the Fourier, or bitateral Laplace, transform of the autocorrelation function.

$$\Phi_{yy}(\omega) = \int_{-\infty}^{\infty} \phi_{yy}(\tau) \quad e^{-j\omega\tau} \quad d\tau \qquad (2)$$

This may seem an unnecessarily complex process to the uninitiated reader. In its defence, however, it can be said that it leads to some of the most powerful relationships avaitable f a linear stochastic systems. Furthermore, its physical interpretation which is explained below is quite simple and straight forward.

The PSD represents the frequency content of the measurement. PSD is usually plotted as a function of frequency, f, in cycles per time unit (day, week, month, year, hour, etc), rather than natural frequency, w, in radians per time unit. The two are related by $W = 2\Pi f$. A commonly occurring PSD is shown in Figure 1.

The area under the PSD curve is proportional to the variance of the original measurement Furthermore, the area under the curve between any two frequencies, say f_1 and f_2 in Fig. 1, represent the contribution of that frequency band to the overall variance. In Fig. 1, mast of the variance arises from low frequencies, that is the longer term variations, while the higher frequencies, representing say the day to day fluctuations, are less significant.

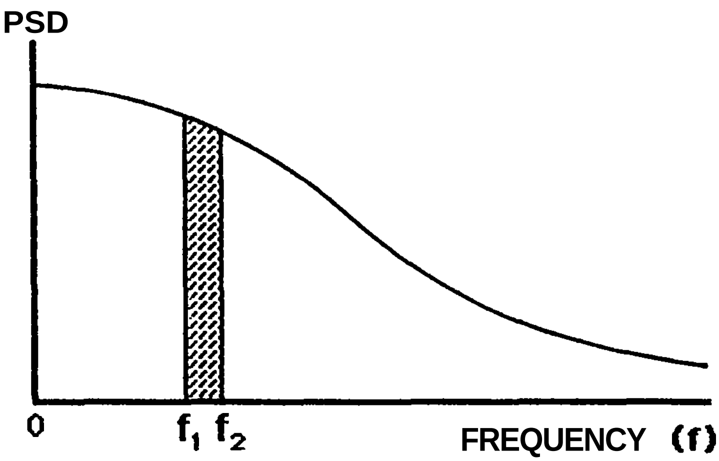

Figure 1 - A commonly Observed PSD

4 - UNSUITABILITY OF A RANDOM WALK MODEL

Random walk models are widely used in the financial markets and are assumed in the celebrated Black and Scholes option pricing model. Random walks, however, suffer from a serious deficiency. Whilst their short term standard deviation is finite, their infinite time horizon standard deviation is infinite. Such behaviour is not descriptive of most financial markets.

For the sceptics, the PSD of a random walk is c / f^2, where c is a constant. Clearly, the area under this curve is infinite. See also Section 7.

5 - DYNAMICAL SYSTEMS THEORY PRELIMINARIES

Those familiar with stochastic processes will realise that a random walk is generated by integrating white noise. This can be stated in dynamical systems theory terms as follows. If an integrator is driven by white noise at its input, then a random walk results at its output. This is represented in Figure 2.

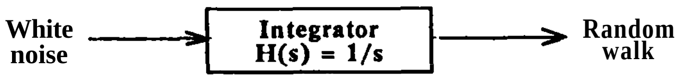

Figure 2 - Production of a Random Walk

White noise is a term used to describe a purely random variable (with no serial correlation). It has equal energy or variance at all frequencies and hence has a flat PSD.

In Figure 2, the integrator is denoted by its transfer function H(s). The transfer function is the ratio of the Laplace transform of the output to that of the input. This might seem a strange ratio to actuarial readers but its significance to control systems theory is immense. It is discussed in some detail in an actuarial context in Balzer and Benjamin (3) and Balzer (4).

6 - PROPOSED SPECTRAL MODEL

The random walk model has been proposed in many other areas of scientific modelling. In particular, it was proposed as a model for track roughness and misalignment for high speed ground transport systems. In that context Balzer (2) showed that lagged white noise, an almost equally simple stochastic process, produced a very much more satisfactory model than a random walk. (In fact the model was so successful that is highlighted a systematic surveying error at 23m intervals associated with gaps in the track, and hand trowelling difficulties at 1.5m intervals due to the way the concrete formwork was supported during pouring).

Lagged white noise is produced, as shown in Figure 3, by passing white noise through a first order lag rather than the integrator associated with a random walk. In Figure 3, T_1 is the time constant and governs the speed of response of the system.

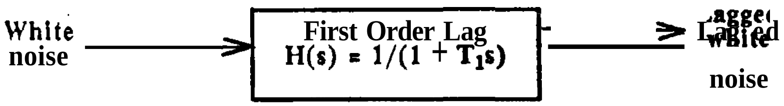

Figure 3 - Production of Lagged White Noise

The PSD of lagged white noise is of the form

$$\Phi_{yy}(\omega) = \frac{c_1}{1 + c_2^2 \omega^2} \qquad (3)$$

where c_1 and c_2 are constants.

Equation (3) is now proposed as an alternative and better stochastic model for financial markets.

Implicit in Equation (3) and Figure 3, is a specification of the dynamic responses to various changes in the input. If it is assumed that Wall Street has a significant driving influence on the Australian stock market, then the Australian response to the October 1987 crash should be consistent with the proposed model. In dynamical systems theory terms, the Wall Street crash was a negative step change input to the Australian market. The expected response, if the model in Equation (3) were accurate, would be as shown in Figure 4.

Figure 4 - Australian Response to Wall Street Crash

The predicted response was in fact observed, giving further credence to the model.

[Not only does Equation (3) model welded train track, aircraft runways, highways, cow pastures and Belgium cobbled paving but also atmospheric turbulence at ground level, Balzer (1) & (2)].

7 - TERM STRUCTURE OF VARIANCE

The properties of the PSD are such that expected value of the variance observed over the time interval T is given by

$$\sigma_y^2(T) = \frac{1}{2\pi}\int_{\frac{2\pi}{T}}^{\infty} \Phi_{yy}(\omega)\,d\omega + \frac{1}{2\pi}\int_{-\infty}^{-\frac{2\pi}{T}} \Phi_{yy}(\omega)\,d\omega$$

where $\sigma_y(T)$ is the expected standard deviation for a period of duration T.

For the proposed model, Equation (4) leads to

$$\sigma_y^2(T) = \frac{c_1}{2c_2} - \frac{c_1}{\pi c_2}\tan^{-1}\left(\frac{2\pi c_2}{T}\right) \tag{5}$$

Equation (5) embodies the model for the term structure of volatility. It allows the calculation of the standard deviation, and hence volatility for any given time period T.

It should be noted that $\sigma^2_y(T)$ in equation (5) is bounded and approaches the finite limit of $c_1/2c_2$ as T approaches infinity. On the other hand, if a random walk model

$$\Phi_{yy}(\omega) = \frac{c_1}{\omega^2} \tag{6}$$

is used, then Equation (4) results in

$$\sigma_y^2(T) = c_1\frac{T}{2\pi^2} \tag{7}$$

which is clearly unbounded as T increases. Such unbounded behaviour is unrealistic and unsatisfactory. The model embodied in Equations (3) and (5) is clearly superior.

8 - EVALUATION OF PARAMETERS

At least two approaches are possible for the determination of the parameters c_1 and c_2 in the proposed model. Firstly, the PSD of the measurement or price of interest can be calculated and Equation (3) fitted in some optimal way to gives values for c_1 and c_2, Alternatively the standard deviation over various time periods can be calculated and Equation (5) is fitted to the data.

9 - AUSTRALIAN 10 YEAR BOND VOLATILITY

Under normal circumstances, both PSD and standard deviation calculations would be carried out on extensive data sets. In the present case, the fundamental theoretical research work was brought to fruition only days before the deadline for this paper. Consequently, only a limited data set has been used for testing and no spectral density calculations have been performed. It is expected that the results of more extensive testing will be availablef a presentation at the conference.

Testing was performed on daily data for 10 year Australian Government bonds covering the 17 month period from January 1988 to late April 1989. This period includes 345 trading days. Figure 5 shows the standard deviation actually observed and the predictions from Equation (5), with the coefficients chosen by limited trial and error. The limited nature of the data (see below) did not warrant a more sophisticated approach to chosing c_1 and c_2.

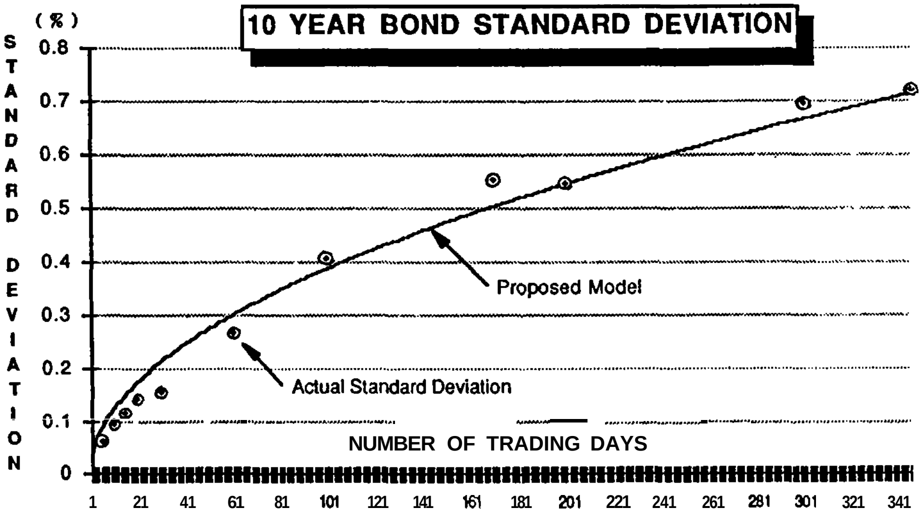

Figure 5 - Standard Deviation of Australian 10yr Bonds

The uneveness of the observed data shows that the trial period was insufficiently long for precise estimation of the standard deviations. However, longer periods will be analysed in the near future and additional results should be available at the conference. In the meantime, the predictive capability of the proposed stochastic model is clearly shown by Figure 5.

10 - CONCLUSIONS

The normal assumption of a random walk for financial markets leads to an unbounded estimate for the standard deviation, variance and volatility of prices when calculated for increasing periods of time. Such behaviour is not observed in most financial markets. On the other hand, the 'lagged white noise" stochastic model proposed in this paper approximates a random walk over short time periods but leads to a finite limiting value for standard deviation and volatility for increasingly long periods. Preliminary testing on limited 10 year Australian Government bond data gives reasonable goodness of fit and tends to confirm the validity of the model.

REFERENCES

(1) BALZER, L.A. (1977), Atmospheric Turbulance Encountered by High Speed Ground Transport Vehicles, J. Mech. Eng. Sc., London, 19(5), 227 - 235.

(2) BALZER, L.A. (1978), HSGT - A Stochastic Model of Track Roughness and Misalignment, J. Mech Eng. Sc., London, 20(3), 143 - 148.

(3) BALZER, L.A. & BENJAMIN, S. (1980), Dynamic Response of Insurance Systems with Delayed Profit/Loss Sharing Feedback to Isolated Unpredicted Claims, J. Inst. Actuaries, London, 107 (4), 513 - 528.

(4) BALZER, L.A. (1982), Control of Insurance Systems Delayed Profit/Loss Sharing Feedback and Persisting Unpredicted Claims J. Inst. Actuaries, London, 109(2), 285 - 316.

(5) NEWLAND, D.E. (1975), An Introduction to Random Vibrations and Spectral Analysis, Longman, London.