

Asset Allocation Optimization for German Life Insurers

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Abstract

The asset allocation model outlined in this contribution provides an explicit way to derive optimal portfolio structures for different constellations of company-specific parameters. This approach incorporates the institutional peculiarities of the investment problem that German life insurers have to face. Risk constraints are defined in terms of shortfall probabilities with respect to individual target returns. The empirical deviations of return distributions from the normal distribution were taken into account by use of the bootstrapping methodology. The put-hedge strategies considered in the optimization demonstrate their usefulness in a striking way.

Résumé

Dans l'exposé présent, une méthode pour la déduction des structures de portefeuille optimales est développée en différentes constellations des paramètres caractérisant le contexte individuel des compagnies d'assurance sur la vie. Ce procédé incorpore les particularités du problème d'investissement pour ces compagnies en Allemagne. Restrictions du risque sont définies par le concept de "shortfall risk". La déviation des distributions des rendements de la distribution normale entre en considération utilisant la méthode de "bootstrapping". Les stratégies de "put-hedge" qui sont tenues compte dans le procédé des optimisations prouvent très avantageuses dans ce contexte.

Keywords

Asset allocation, life insurance, option strategies, simulation.

A. Introduction and overview

The purpose of this contribution is to present an asset allocation model which is tailored to the specific needs of German life insurers and to outline some of the results which can be derived with such a model. Stock index option strategies are an important point of interest in the asset allocation decisions that are investigated in this article. Among other reasons this leads us to the use of a simulation model of asset class returns, since these option strategies imply the rolling-over of medium-term options for achieving risk-return profiles that are desirable in the longer term¹.

The whole process of deriving the asset allocation structures according to my model can be summed up as in the following diagram:

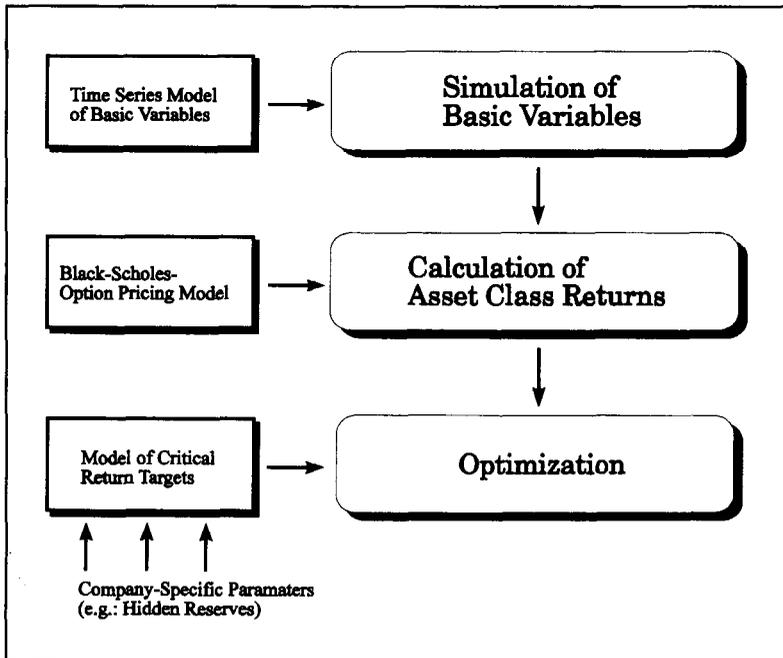


Diagram 1: Asset Allocation Optimization

For the process of the basic variables that are needed for the calculation of asset class returns I use the two estimated models that are presented in Stephan (1996). Model I is a pure vectorautoregressive model and Model II incorporates a cointegration relationship between bond yields and inflation rates. The asset classes considered here are German stocks, German government bonds, "Schuldscheindarlehen"² and stock index options.

An important feature of every time series model is the distribution of the disturbance term. Usually, a multivariate normal distribution according to the estimated covariance matrix is assumed. When dealing with financial time series, this assumption often proves to be heroic: Extreme values are more frequent than it would be consistent with the normal distribution. The following diagram shows the empirical distribution of the first component of the residual vector that is calculated as the difference between the observed and the predicted values:

$$\begin{pmatrix} \hat{\varepsilon}_{1t} \\ \hat{\varepsilon}_{2t} \\ \hat{\varepsilon}_{3t} \\ \hat{\varepsilon}_{4t} \end{pmatrix} := \begin{pmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \\ y_{4t} \end{pmatrix} - \begin{pmatrix} \hat{y}_{1t} \\ \hat{y}_{2t} \\ \hat{y}_{3t} \\ \hat{y}_{4t} \end{pmatrix} \quad (t = 1, 2 \dots T). \quad (1)$$

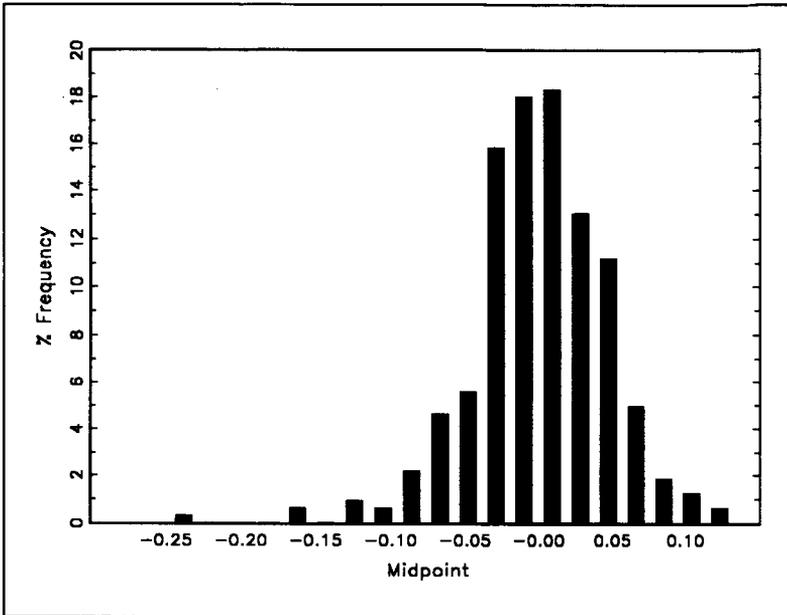


Diagram 2: Distribution of the first component of the residual vector in Model I

Please note that the first component of the residual vector is relevant for the stock market return.

The result of the visual inspection is confirmed by the *Anderson-Darling-test*³ for normal distribution:

Table 1

residuals tested	value of the test statistic
first component	1,3432801
second component	1,2234423
third component	1,9465604
fourth component	1,1634210

The critical value for rejection of the normal distribution hypothesis at the 1% level is 1,035 and 1,159 at the 0,5% level. The residuals in Model II are

distributed in a very similar way, so the hypothesis of normal distribution can also clearly be rejected.

As in our optimization approach special emphasis is put on the distribution of returns at the extreme negative end, it would be quite unsatisfactory to assume a normal distribution. Therefore in the simulations that lead to the optimal asset allocation the bootstrapping methodology is used in order to replicate the empirical distribution of the residuals.

B. Description of the simulation model

In each simulation run the basic variables of the monthly models I and II are calculated for 20 years with individual draws out of the empirical distribution of monthly residual vectors. This procedure is repeated 5000 times. Using the first three components of the vector Y_t according to Model I or II, the returns of the asset classes equities, bonds and "Schuldscheindarlehen" are calculated. Whereas the case of stocks and bonds is straightforward, concerning the "Schuldscheindarlehen" the following is assumed: At the end of the year t an amount of N_t is invested into that asset class. In the years past ($t < 0$) N_t is assumed to equal N_{t-1} and in the simulation period ($t = 0$ to 19) N_0 is invested. The yearly returns of the asset class "Schuldscheindarlehen" are now calculated as the weighted average of initial yields in each term-to-maturity class of "Schuldscheindarlehen"⁴.

The limitation of shortfall risks, which is highly relevant for insurance companies, hints at the use of option strategies that influence the return distribution in an asymmetric way. Therefore I have included two synthetic assets in the simulation/optimization procedure: These assets are a 1:1 put hedge for the equity index and a 1:1 covered short call. Option prices are calculated using the *Black-Scholes* formula, where the implied volatility is set equal to the average

volatility in the period 1967 to 1993, that was chosen for the estimation of the two time series models.

As longer-term index options are not very common in the German capital market, the rolling-over of one-year options was considered (this is a bit longer than the maximum time-to-maturity for DAX-options, but should lead to similar results). For the exercise prices of the options two types of strategies are investigated. One straightforward version is the "fixed percentage strategy": At each roll-over date options are bought or sold with exercise prices according to an fixed percentage p^P (for puts) or p^C (for calls) of the current price of the underlying. Therefore we have (for the puts on index I_1):

$$X^P(I_{1t'}) = p^P \cdot I_{1t'} \quad t' = 0, 12, 24, \dots, 240 \quad . \quad (2)$$

Typically $p^P < 1$ and $p^C > 1$ are chosen (out-of-the-money-options), but for the puts a strategy might prove interesting that involves the buying of in-the-money puts under certain circumstances.

In *Figlewski/Chidambaran/Kaplan* (1993) the so-called "ratchet"-strategy is proposed: The exercise prices basically follow the fixed percentage rule but never are allowed to fall short of a once reached exercise price. This is a rather intuitive method to reduce shortfall risks in a multi-period horizon. The disadvantage of the original version that is described in the article referred to above is that after a "crash" this strategy might stipulate the buying of puts with unrealistic exercise prices. In order to counter this drawback I used a modified version for the simulations:

$$X^P(I_{1t'}) = \max[p^P \cdot I_{1t'}, \min(X^P(I_{1t'-12}), q \cdot I_{1t'})] \quad . \quad (3)$$

The effect of these two distinct strategies in terms of risk reduction depends on the time horizon. First, it should be noted that the ratchet strategy involves on average the buying of puts with higher exercise prices than the fixed-percentage strategy with the same p^P . Therefore, if we want to make meaningful comparisons, we have to find out which values p^P and q for the ratchet strategy lead to the same mean return as the equivalent p^P fixed-percentage strategy. For ex-

ample, the 105% fixed-percentage strategy produced about the same mean return in the simulations as the ($p^p = 101\%$; $q = 120\%$) ratchet strategy. Whereas in the longer horizons the ratchet strategy proved superior in terms of risk reduction, for one year periods the opposite holds true. In the following the simulated distributions for the returns of the put-hedge in the tenth year of each simulation run are shown:

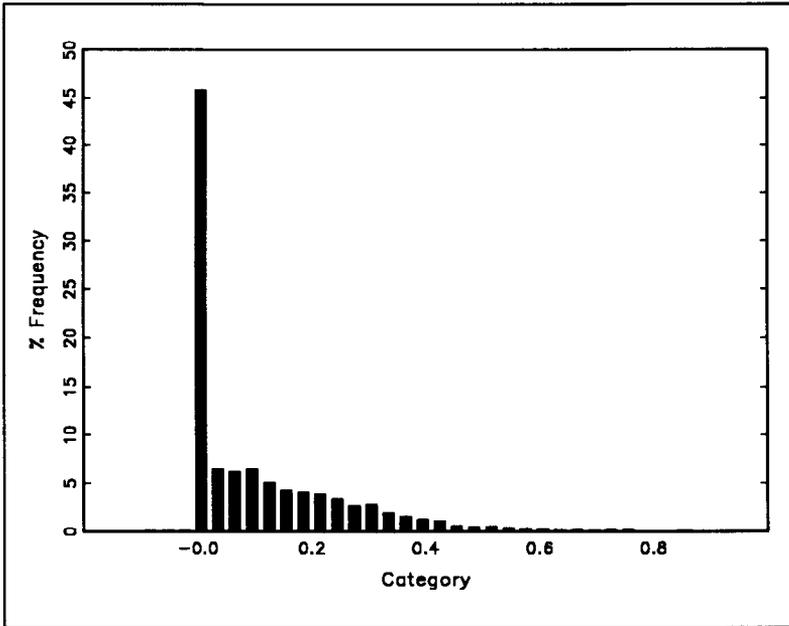


Diagram 3: Distribution of one-year returns of put-hedge with 105% fixed-percentage strategy

The analogous diagram for the comparable ratchet strategy is:

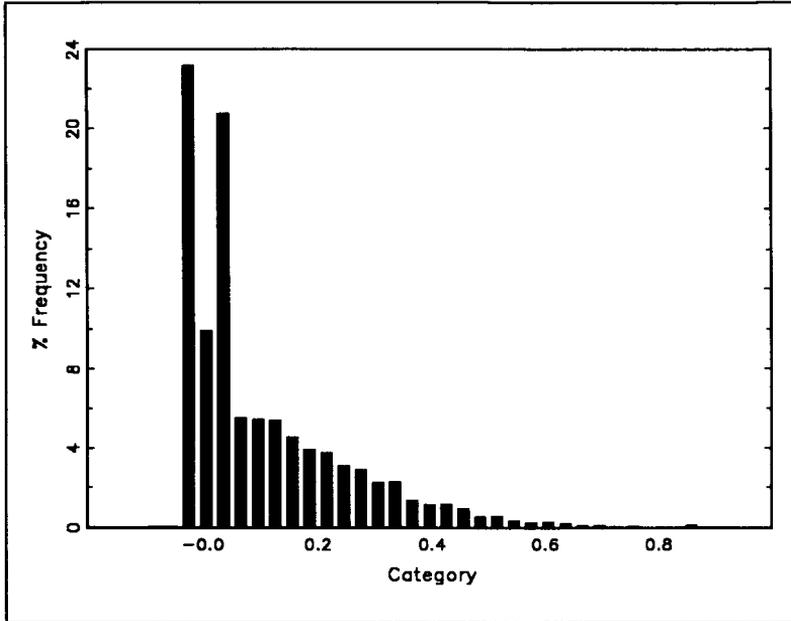


Diagram 4: Distribution of one-year returns of put-hedge with ($p^p = 101\%$; $q = 120\%$) ratchet strategy

The twin-peak distribution results from the path dependent exercise price according to this strategy. As can easily be seen, the fixed-percentage strategy limits the shortfall risks in a clearer way for the one-year horizon.

For 20-year average returns the distributions look quite the same for both types of strategies; therefore it is sufficient to show the diagram for the ratchet strategy:

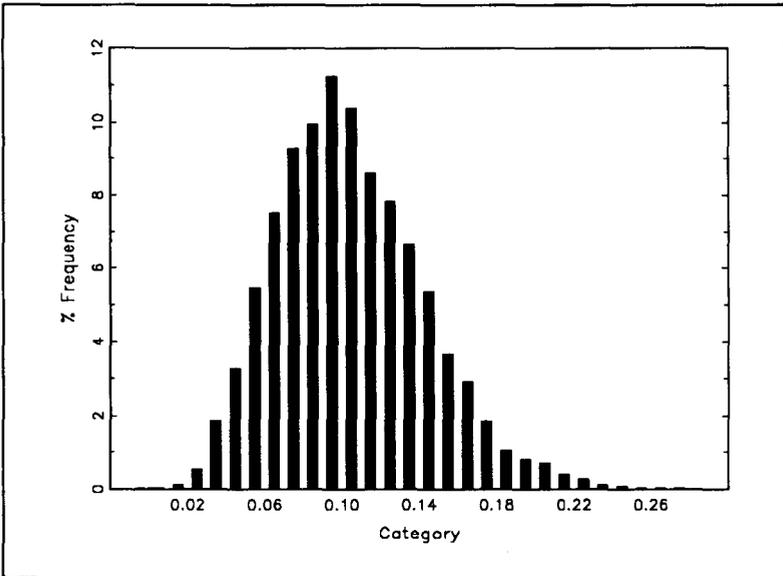


Diagram 5: Distribution of 20-year average returns of put-hedge with ($p^p=101\%$; $q = 120\%$) ratchet strategy

The next table shows the respective shortfall probabilities for the 20-year horizon:

Table 2

target average return	Shortfall probabilities with comparable strategies	
	Fixed-Percentage	Ratchet
- 0.01	0 %	0 %
0.00	0 %	0 %
+ 0.01	0.04 %	0 %
+ 0.02	0.26 %	0.10 %
+ 0.03	0.88 %	0.64 %
+ 0.04	2.58 %	2.52 %

As already mentioned, for the longer horizons the ratchet strategy is slightly superior in terms of shortfall risk reduction. Since in our investment problem

superior in terms of shortfall risk reduction. Since in our investment problem stated below restrictions of the one- to three-year shortfall-risks are stipulated, it can't be determined a priori which type of strategy will prove superior.

C. The investment problem

Let $r_i(s)$ denote the one-year returns of the whole portfolio in simulation run s ($s = 1, 2, \dots, S$). Furthermore, the average return over T years according to simulation run s is represented as:

$$\bar{r}_{1..T}(s) := \sum_{i=1}^T r_i(s) \quad (s = 1, 2, \dots, S) \quad . \quad (4)$$

In the following, critical minimum returns for investment horizons of one to three consecutive years are used. The actual levels of these minimum returns are company-specific; a methodology for deriving these levels is developed in Stephan (1995), pp. 98 - 140. Let $L(\text{ER})$, $L(\text{D})$, $L(\text{F}_1)$, $L(\text{F}_2)$ and $L(\text{F}_3)$ denote these critical levels. The event, that the simulation run s produces a shortfall with respect to one of these restrictions, is described by binary variables. Specifically we have:

$$sf^{ER}(s) := \begin{cases} 1 & \text{if } r_1(s) < L(\text{ER}) \\ 0 & \text{else} \end{cases}$$

$$sf^D(s) := \begin{cases} 1 & \text{if } r_1(s) < L(\text{D}) \\ 0 & \text{else} \end{cases} \quad (5)$$

as well as

$$sf^{F_i}(s) := \begin{cases} 1 & \text{if } \bar{r}_{1..i}(s) < L(\text{F}_i) \\ 0 & \text{else} \end{cases} \quad (i = 1, 2, 3) \quad .$$

The optimization problem is now to determine the asset allocation structure that

leads to:

$$\begin{aligned} & \frac{1}{S} \sum \bar{r}_{1..T}(s) \rightarrow \max ! \\ & \text{under the constraints} \\ & \frac{1}{S} \sum sf^{ER}(s) \leq 0,002 \quad (6) \\ & \frac{1}{S} \sum sf^D(s) \leq 0,01 \\ & \frac{1}{S} \sum sf^{F^i}(s) \leq 0,05 \quad (i=1,2,3) . \end{aligned}$$

(Note, that the VAG-restrictions confine the feasible asset allocation structures. These restrictions are left out in this representation for the sake of simplicity but are observed when deriving the optimal asset allocation structures.)

D. Results of the optimizations

In the following, the most important results of the optimizations within the simulation model are presented. The optimal asset allocation depends on the parameters used for calculating the critical return levels. Nevertheless, some general conclusions concerning the optimal shares of the asset classes considered can be drawn:

- 1) The optimal asset allocation according to the simulations with **Model I** and **Model II** is nearly identical for most parameter constellations.
- 2) The **covered-short-call** is almost never included in the optimal asset allocation structure.
- 3) In contrast, the **put-hedge** is for each parameter variant part of the optimal asset allocation and supersedes in many cases the unsecured equity asset class.
- 4) **Schuldscheindarlehen** dominate in every parameter variant; interestingly, their optimal share is mostly nearby 50% of the total portfolio,

which confirms the current investment practice of German insurers.

- 5) **Bonds** are included in every variant. This is quite remarkable, since the average returns of the asset classes "Bonds" and "Schuldscheindarlehen" are almost identical and the volatility of the "Schuldscheindarlehen" returns is clearly smaller (due to the different accounting treatment). The share of bonds in the optimal asset allocation therefore is attributable to diversification effects; specifically, there exist negative correlations between bond returns and "Schuldscheindarlehen" returns in a balance-sheet view.
- 6) The **unsecured equity portion** is minor in most variants.

Concerning the effects of a variation of input parameters for the critical minimum returns, the following general tendencies hold:

- 7) The influence of the percentage of **hidden reserves** on the optimal asset allocation and the average returns is tremendous. In the case of higher reserves (hidden or open) a more aggressive asset allocation can be chosen with higher mean returns. An increase of the actuarial target yield has a similar effect as a decrease in the percentage of hidden reserves.
- 8) The constraint of a maximum equity (and similar positions) share of 30% by regulation proves to be binding when there are hidden reserves. This result stems from the use of the put-hedge (which in value terms mostly consists of equities). Without considering option strategies the 30% rule is not restrictive.

Concerning the option strategies the following general conclusions can be drawn:

- 9) The results (achievable returns and asset allocation structure) for the **ratchet** and **fixed-percentage strategies** are very similar. In the variants with relatively high hidden reserves and smaller actuarial target return the ratchet strategies seem slightly superior.
- 10) In the most variants the best results are achieved with higher **strike**

prices. Only in variants with high hidden reserves and under consideration of the 30% rule, the lower strike price percentages are optimal.

Let us exemplify the major points with some diagrams. The first diagram shows the optimal asset allocation for a variant with 2% hidden reserves, when option strategies are excluded:

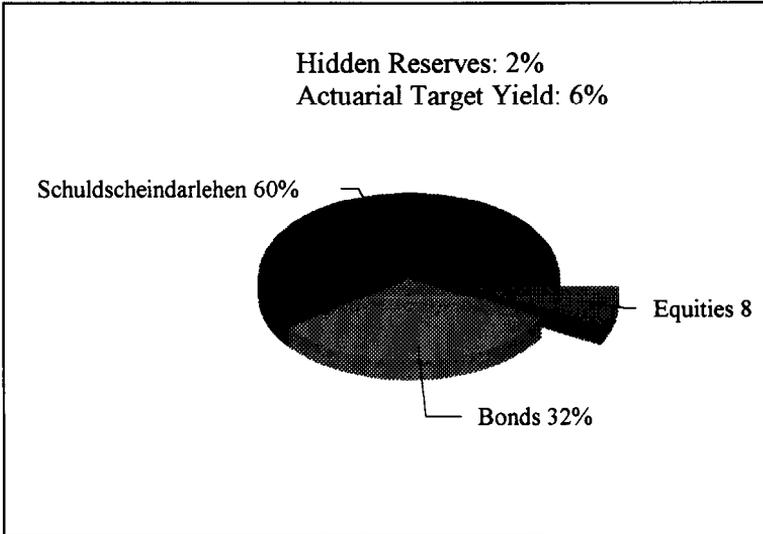


Diagram 6: Optimal asset allocation without option strategies

The result is a structure not uncommon in the portfolios of German life insurers. The low equity share is a consequence of the rather drastic constraints concerning the shortfall risks. The picture changes dramatically when we account for option strategies:

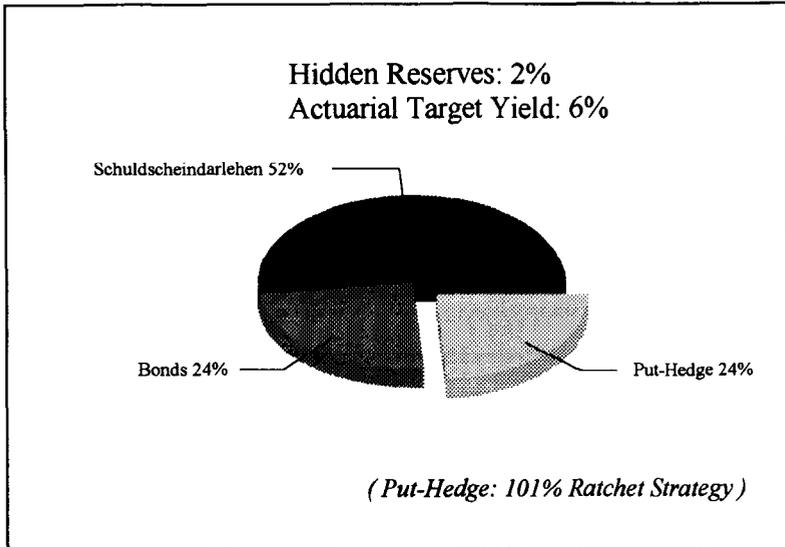


Diagram 7: Optimal asset allocation with option strategies

Here the optimal equity share has nearly tripled via the put-hedge, whereas the covered-short-call doesn't enter into the solution. If an insurance company has a bigger share of hidden reserves in its portfolio, an even more aggressive structure is optimal under the same constraints:

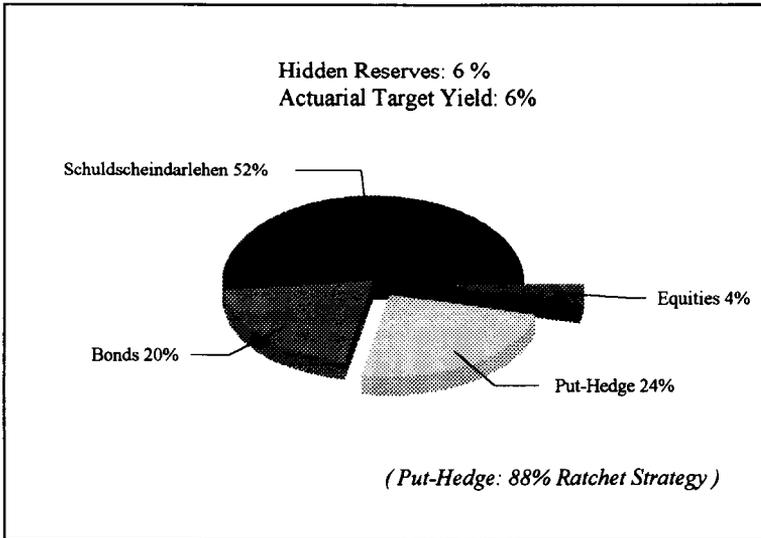


Diagram 8: Optimal asset allocation with 6% hidden reserves

Here the total equity share, which consists of the 4% unsecured equities and the large equity portion in the put-hedge, is again increased. Note, that now a $p^p = 88\%$ ratchet strategy (the lowest considered percentage) provided better results as the 101% ratchet strategy. The reason for this is the increased amount of implicit safety margin in form of higher hidden reserves.

The next chart shows the influence of the two parameters that were varied in the optimizations (hidden reserves and actuarial target yield) on the achievable returns:

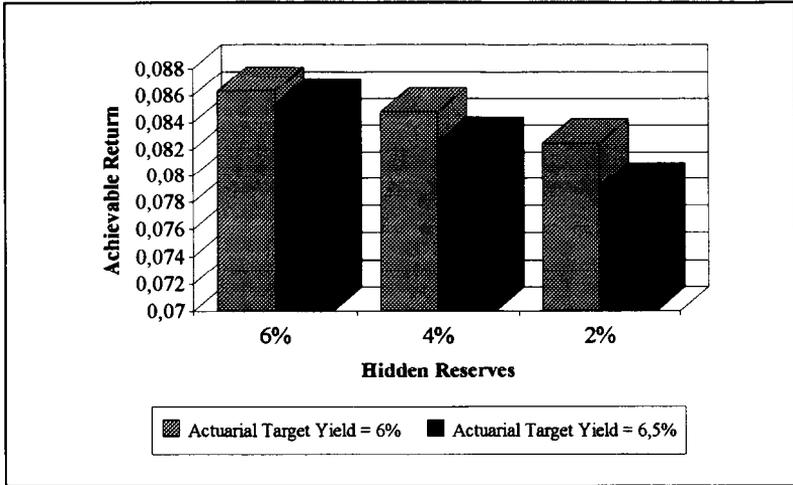


Diagram 9: Influence of company-specific parameters on achievable return

We see that the optimal asset allocation for hidden reserves of 6% leads to a clearly higher mean return than for the variant with 2% hidden reserves. The last diagram exemplifies the additional level of mean return that can be achieved by including option strategies into the optimization:

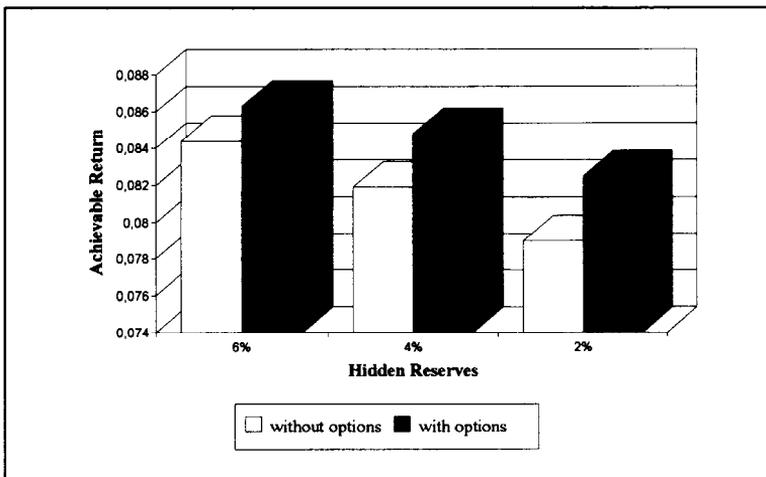


Diagram 10: Effect of including option strategies in the optimization

E. Conclusion

The asset allocation model outlined in this contribution provides an explicit way to derive optimal portfolio structures for different constellations of company-specific parameters. Variations of these parameters have an measurable effect on the optimal asset allocation and on the achievable mean return. Special emphasis was put on the investigation of option strategies. In particular, put-hedge strategies proved their usefulness for the investment problem considered here.

1. Option theory gives little insight in these dynamic problems, see Figlewski et al. (1993) and Dybvig (1988).
2. German accounting standards necessitate the distinct treatment of this asset class.
3. According to D'Agostino (1986), pp. 370 - 374, the *Anderson-Darling*-test is to be preferred versus alternative tests like the *Kolmogorow-Smirnow*-test or the *Kuiper*-test.
4. For details please see Stephan (1995), S. 278 - 281.

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