Asset/Liability-Modelling for Life Offices
Allowing for Dated Fixed Interest Investments
and Performance Related Changes in Liability

Martin J Muir and Richard J Squires

Abstract

The paper considers the requirements of an asset/liability-model for a life office, including provision for dated fixed interest stocks, and for the feed-back from investment conditions into valuation bases, bonus rates and surrender value scales. Methods of modelling these features are presented, and a model which uses unit price for the investment portfolio as the link between the asset and liability parts of the model is described. The benefits of examining particular scenarios before moving on to stochastic modelling are then considered, and a worked example for a hypothetical life office is presented to illustrate how an office might use this approach to develop a defensive investment strategy.

Résumé

Le document étudie les besoins d'une compagnie vie pour un modèle Actif/Passif contenant une provision pour des obligations à taux fixes et qui fait référence aux conditions d'investissement pour définir les bases d'évaluation, les taux de participations bénéficiaires et les taux de valeurs de rachat. Il présente des méthodes de modélisation de ces caractéristiques et un modèle qui utilise un prox unitaire pour le portefeuille d'investissement comme lien entre des parties actif et passif du modèle. Il considère ensuite les avantages d'examiner des scénarios particuliers avant de commencer la modélisation stochastique et il donne une exemple pour une compagnie vie hypothétique afin d'illustrer la manière dont compagnie peut utiliser cette approche pour développer une stratégie d'investissement.

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Asset/liability modelling for life offices allowing for dated fixed interest investments and performance related changes in liability

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1 Introduction

1.1 It is generally now accepted that the assumption of a uniform rate of interest is not adequate for the purposes of investigating the risks faced by a life office. Asset models are required that enable particular economic scenarios to be considered, and stochastic modelling of stock market behaviour to be undertaken. Stochastic modelling has become established as a standard technique for pension funds in considering their investment strategy, but has not been extensively employed by life offices so far, except in relation to specific products involving equity investments and guaranteed benefits.

1.2 The common feature of the established applications has been relative simplicity in the modelling of the liabilities side of the projection. When we turn to traditional life office products we find considerably more complexity in the specification of the liabilities. We may also find that the level of guaranteed benefit is such that it is not sufficient to specify the proportion of the fund invested in fixed interest stocks, but it is also necessary to consider the maturity dates of those stocks. Very often we will also find that the liability cash flows cannot be projected independently of the assumed investment returns. Rates of

bonus will be varied in line with investment conditions, and surrender value scales may not be guaranteed. The levels of new business and the rate of surrender of policies may be affected by investment conditions. The permitted basis for the statutory actuarial valuation will also be affected by changes in asset values and yields, and the asset mix may be changed in response to changes in investment conditions.

1.3   In this paper we describe the way in which we are developing modelling systems in order to cope with these complications, by designing a structure that will accommodate different requirements and differing levels of complexity according to the nature of the problem.

1.4   At the centre of this approach is the concept of a unitised fund. The price of a unit of the fund is the main means of communication between the asset and liability sides of the projection. This means that the elements of the asset model can be developed without consideration of the nature of the liabilities that are being projected. They may relate to life assurance policies, which we will consider in this paper, but they might equally relate to pension liabilities, or general insurance or long term care products. The performance of the unit price is itself a useful indicator of the success of the investment policy. On the liabilities side it can feed asset share calculations, and together with the projected investment income will drive the yield calculations required in considering the valuation basis, bonus rates and surrender value scales.

1.5 In the next two sections of the paper we describe in more detail our approach to modelling assets and liabilities. We then discuss the uses of the model in considering specific scenarios and stochastic modelling, and present a simplified example.

2 Modelling the assets

2.1 The heart of our system is a cash flow projection generator that enables the features of individual products to be defined. It was designed to produce cash flows and valuations for policies, but can be used equally well to project cash flows under fixed interest securities or other investments, as it does not in itself distinguish between assets and liabilities. There is a separate program for each class of asset and each class of policy, and a global program then pulls together the individual results to perform some global calculations and to allow for interaction between the separate programs.

2.2 A record is set up for each fixed interest stock assumed to be available. These may be real stocks, or hypothetical stocks with maturity dates over the required range. For the purpose of our examples we have assumed that stocks will be available maturing in each future period covered by the projection and as far beyond as is necessary to match liabilities. We have assumed that equity investment will be via a pooled fund that generates dividend payments uniformly throughout the year, and that cash flows will be to or from a deposit account paying an appropriate rate of interest. We have not made any

provision for investment in real property, but this could be accommodated if required.

2.3 We have chosen to work with quarterly accounting periods as a compromise between using annual periods, which would probably be too insensitive to give useful results, and monthly periods which would involve significantly more processing and a greater volume of output. We have assumed that each fixed interest stock pays dividends half-yearly, with the final dividend on the maturity date.

2.4 The fixed interest stocks are valued by reference to the dividends maturity proceeds due and an assumed term related yield. The yields could be specified for each term individually, but for the purpose of the illustration we have assumed that only the deposit rate and the yields on five year and twenty year stocks will be prescribed. Yields for intermediate terms are then calculated by interpolation. Similarly, the dividends on the equity portfolio are projected first, and market values are then calculated by applying an assumed dividend yield. This approach is convenient for considering particular scenarios, and is also consistent with the approach used in the Wilkie stochastic model.

2.5 The final part of the asset model consists of a set of decision rules for the investment of net cash inflow or the sale of investments to provide for a net outflow. This is only limited by the ability of the modeller to encapsulate ideas in mathematical formulae. One possibility would be to define standard proportions of equity and fixed interest investment, together with a term spectrum for the latter, and to

rebalance the fund to this pattern at the end of each period. Another possibility would be to deal with maturities, surrenders and new money separately.

3 Modelling the liabilities

3.1 The basic system can operate either from individual policy records, or from representative model points to cut down the volume of calculations required, by reference to a database and specifications of the policy details. The normal revenue items are projected, together with the net cash flow, the asset shares and values of the liabilities at the end of each period. Terminal bonuses are assumed to have been paid during the quarter year on the basis of a scale set at the beginning of the quarter reflecting the value of the assets at that time.

3.2 For unit-linked business both maturity and surrender payments will reflect the value of the asset share directly, and little more consideration is required. The more interesting problems arise with conventional with profits contracts on which reversionary bonuses are added each year with a terminal bonus payable on death or on maturity. The surrender values may also include an element of terminal bonus, and thus to some extent reflect the value of the asset shares. For the remainder of the paper we shall consider only such policies.

3.3 We have assumed that the terminal bonus on maturity will be calculated as the excess of 99% of the value of the asset share over the amount of the guaranteed payment, with the asset share calculated

using a smoothed unit value. This has been calculated quarterly using an exponential smoothing formula as:

New smoothed price = 0.8 x old smoothed price x 1.0235
                     + 0.2 x new unsmoothed price.

The 1% deduction is intended to provide a contribution to the free estate in consideration of the provision of a guaranteed minimum benefit. The unsmoothed price is used to measure the investment performance, and also the difference between asset shares released on claims and the corresponding payouts.

3.4 We have assumed that the rate of reversionary bonus will be changed more gradually than might be indicated by significantly changed investment conditions, and that there will be an element of smoothing involved. It would be possible to calculate the rate of reversionary bonus that could be supported by the current yield on the fund given the current asset share and the target proportion for the terminal bonus. We have illustrated below a simpler rule relating the reversionary bonus to the current dividend yield on equities and the redemption yield on bonds with a smoothing formula.

3.5 We have assumed that the statutory valuation basis will be constrained by the yield on the fund. This has been achieved by calculating reserves on three different interest bases for each product in the individual policy projections, and determining the overall reserve by interpolating between the resulting totals for the permissible valuation interest rate. We have also assumed that a resilience reserve will be

required on the basis currently prescribed by the Government Actuary's Department in the UK, which allows for future variance in asset values and yields. This has a significant effect on technical solvency in adverse investment conditions.

3.6 Most UK companies operate on the basis of surrender values that are not guaranteed, but in fact only change their surrender value scales in extreme conditions. We have modelled a theoretical surrender value scale as a percentage of the asset share which reaches 99% at maturity.

4 Dynamic solvency testing

4.1 The concept of dynamic solvency testing as an extension to the valuation process has already been introduced in North America, and is being actively considered in the UK. UK companies will need to be able to make cash flow projections of their funds on the basis of a number of specified scenarios within the next few years in order to comply with best practice requirements. Deciding what scenarios are appropriate for the purpose of demonstrating the ability of the company to survive likely variations in financial conditions will require a full understanding of the mechanics of the company's business, based on examination of a wider range of scenarios.

4.2 In the meantime we have considered some simple variations that it might be instructive to evaluate. The late Sydney Benjamin suggested many years ago that actuaries might adopt the concept of control theory as used by engineers, and adapt it to their own purposes, but as far as we are aware this has never been pursued. The idea behind

this is to define a number of standard changes to be applied to the "input signal" to a system, and record the changes to the "output signal". In actuarial terms, the input signal might be the yield on the fund, and the output signal the degree of solvency, or the level of the reversionary or terminal bonus rate.

4.3   The charts represented below show a set of changes that might be applied to an input variable. They are referred to as "spike", "jump, "ramp", and "wave". The first refers to a sudden change with a quick reversion to the original level, the second to a sudden change to a new permanent level, the third to a gradual change to a new level, and the last to cyclic variation around a central trend line. These can all be regarded as simplified pictures of how stock market yields have behaved at various times in the past and may be expected to behave at some time in the future.
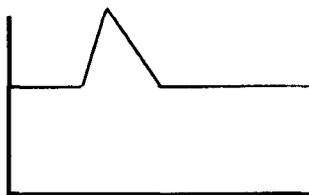
Chart 1: Spike   **Chart 2: Jump**
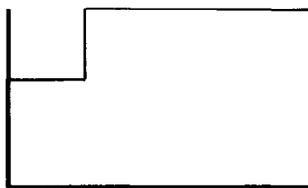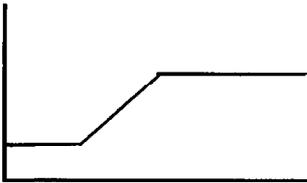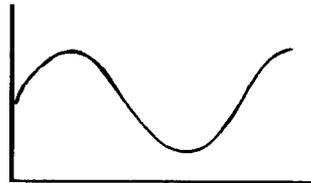
Chart 3: Ramp   **Chart 4: Wave**

4.4   In addition to their use on their own, we believe that examining particular scenarios is an important step in considering the defensive strategies that might be built into a model to be used in stochastic projections. For example, it is unrealistic to measure the risk of insolvency for a company using a stochastic model that assumes that surrender values will not be changed in unfavourable circumstances if the policy conditions allow them to be changed. Other examples of defensive strategies that could be adopted include changing bonus scales, or the asset mix, or restricting the writing of new business. If the scenarios that would lead to insolvency according to the stochastic model can be identified they can then be examined, and it may be possible to identify the circumstances that should trigger action that would avoid subsequent technical insolvency.

5   **Results for a Hypothetical Model Office**

5.1   We have constructed a hypothetical model office which is assumed to have issued a single class of with profit endowment assurance for many years, and projected results on various scenarios. A description of the policy modelled and the assumptions made in the projections

are set out in the Appendix A to this paper, and a set of charts, as described below, are reproduced in Appendix B.

5.2 In control theory terms, we have taken the yield on the fund as our input signal, assuming a constant relationship between the equity yield and the fixed interest investment yields, and have looked at a number of different output signals. The two cases we have looked at are a spike, with yields increasing in one quarter and then falling back to the previous level over the next two years, and a ramp, with yields falling steadily over a period of two years to a new plateau. These are likely to be critical scenarios for with profit funds.

5.3 We have shown on the same chart in each case the actual unit price that results, and the smoothed unit price. The smoothed price is important because it drives the calculation of the payment of policyholders, via the terminal bonus. We have assumed that the reversionary bonus will be affected by the smoothed prospective yield on the fund.

5.4 The levels of reversionary and terminal bonus are then shown on a separate chart, together with an index of the total pay-out, to avoid the confusion of presenting too much information on one chart. Levels of bonus are important considerations from the point of view of the marketability of the contract.

5.5 On a third chart we have then shown the free asset ratio (ie the excess of the value of the assets over the value of the liabilities expressed as a percentage of the value of the assets), and, for guaranteed surrender

values, a surrender value indicator. This shows the ratio of the total of the guaranteed surrender values on the policies in force to the total of the reserves. It has been assumed that the reserve would not be less than the surrender value payable for any policy. If the office chooses to hold its surrender value scale when asset values are depressed, this will impact on the free asset ratio through the higher reserves, and where surrender values are guaranteed the effect will be greater as a result of the resilience reserve.

5.6   There are many scenarios that one could examine. As an example we have shown the effect of a fall in market values coupled with a sharp increase in the number of policies surrendered in the context of guaranteed surrender values. Other combinations of changes in the variables could produce critical scenarios, and in a real situation a large number of such would need to be considered before deciding on those which should be brought to the attention of management.

5.7   It is not our intention to try to consider all the aspects of dynamic solvency testing in this paper, but one point is perhaps worth making. We have observed that it can be dangerous to try to draw conclusions about the longer term from the shape of the graphs over the relatively short term we have considered. For example, the graph of policy payouts for the reducing yield scenario shows that this increases over the first five years. Clearly over the longer term it will fall to below the initial level.

5.8   We should also make the point that in our examples the only feed-back we have allowed for is from investment performance into

bonus rates and the valuation basis. In some cases it would make sense to alter the asset mix, for example where the combination of a sharply increased yield, the provision of a resilience reserve, and a high percentage of the fund in equity investments would lead to technical insolvency.

5.9   We believe that we now have a practical tool that will enable a wide variety of scenarios to be investigated, and valid conclusions drawn. The next stage will be to use it in conjunction with a stochastic model of asset performance, and possible rules for rebalancing the asset portfolio

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Appendix A

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Assumptions made in the projections

1   Asset model

1.1   Central economic assumptions

Return on cash   : 7% pa
Income on bonds   : 7.5% pa
Redemption yield on bonds   : 7% pa
Dividend yield on equities : 4% pa
Dividend growth on equities : 5.5% pa

The market value of bonds is calculated from the redemption yield, term and coupon rate. The market value of equities is calculated from the dividend yield and the level of the dividends.

1.2   Investment rebalancing

Fixed investment mix: 70% equities; 25% bonds; 5% cash.

If bonds need to be sold to maintain the fixed investment mix, then the existing holdings are sold in equal proportions. If bonds need to be purchased to maintain the fixed investment mix, then a bond with an outstanding term of 20 years is purchased.

2 Liability model

2.1 Policy details

Product	:	Conventional with-profit endowment assurance
Term	:	20 years
Age at entry	:	35
Data	:	A portfolio of existing policies with maturity dates spread over each future quarter and new business incepting in each future quarter
Annual premium	:	5% sum assured (payable monthly in advance)
Sum assured	:	£12,000 for entry in year one of projection. Sums for other years adjusted for inflation

2.2 Bonus rates

Unsmoothed reversionary bonus rate in step t, URB_t

= 75% (cash yield in step t - 3%)

Smoothed reversionary bonus rate in step t, SRB_t

= $0.8 \, SRB_{t-1} + 0.2 \, URB_t$

Terminal bonus	:	Calculated to equate the maturity value to 99% of the asset share, but subject to a minimum of zero

2.3   Asset shares

Asset shares are calculated by reference to the smoothed unit price, SP_t , at the end of the step.

The unsmoothed price, UP_t, is based on the market value of the assets, but reflects also the difference between asset shares released on claims and the corresponding claim amounts paid out.

SP_t is then calculated as   : $SP_t = 0.8 \, SP_{t-1} \times 1.0235 + 0.2 \, UP_t$

(1.0235 is the long term growth assumption per step)

The asset share is then calculated for each policy as:

$$AS_t = AS_{t-1} \, SP_t/SP_{t-1} + (P_t - C_t - E_t)\left(SP_t/SP_{t-1}\right)^{\frac{1}{2}}$$

where

AS_t = asset share at end of step t

P_t = premiums received during the step

C_t = commissions paid during the step

E_t = expenses incurred during the step

2.4   Surrender values

Basic surrender values are calculated as x% of the asset share where

x% = 99% - 2% (policy term - in force duration)

Guaranteed surrender values (where applicable) are based on a net premium reserve on the following basis:

Interest   : 6%

Mortality   : 80% A67/70

Zillmer   : 4.5% sum assured

2.5   Reserves

Basic policy reserves: net premium reserves calculated on the following basis:

Interest   : 97.5% of current yield on fund*

Mortality   : 100% A67/70

Zillmer   : 3.5% sum assured

*   Market value weighted average of dividend yield on equities, yield on cash and redemption yield on bonds

Solvency margin: 4% (basic policy reserves + mismatching reserve)

Mismatching reserves: assets and reserves recalculated for the following scenarios:

A   -   yields on cash and bonds fall by 3% in conjunction with a 25% fall in the market value of equities (equity dividends maintained leading to an increase in dividend yields)

B   -   yields on cash and bonds rise by 3% in conjunction with a 25% fall in the market value of equities (equity dividends maintained leading to an increase in dividend yields)

The mismatching reserve is then:

$$\max \left[(A_O - R_O) - (A_A - R_A), (A_O - R_O) - (A_B - R_B) \right]$$

where:

A = assets   subscript O = original

R = reserves   subscript A = scenario A

subscript B = scenario B

2.6   Demographic experience

Mortality	:	80% AM(80)
Withdrawal rates	:	12% in year 1
		8% in year 2
		6% in year 3
		5% pa year 4 and thereafter

2.7   Expenses and commission

Initial expenses	:	£100 per policy at start of projection
Renewal expenses	:	£20 per policy pa at start of projection
Inflation	:	5% pa
Initial commission	:	78% annual premium in month 1
Clawback of commission in month t (t<31)	:	initial commission x ä $\overline{_{31-t}}$ /ä$\overline{_{31}}$ calculated at 1% per month
Renewal commission	:	2.5% premium from month 32 onwards

3   **Interactions between assets and liabilities**

3.1   **Feedback into asset program**

- Cash flow from liabilities affect asset value.

- Cash flow from liabilities and difference between asset shares released on claims and corresponding payouts affect calculation of unit price.

3.2   **Feedback into liability program**

- Unit price affects asset share calculation and hence terminal bonus for next step.

- Maximum valuation interest rate affects net premium reserve.

3.3   **Global calculation**

- Mismatching reserve
- Solvency

Unit prices
Yields – 30% gradually over 2 yrs

Unsmoothed price — ■ — Smoothed price — ♦ — Composite yield — ▲ —

Bonus rates

Yields −30% gradually over 2 yrs

Legend: Reversionary — Terminal — Payout index
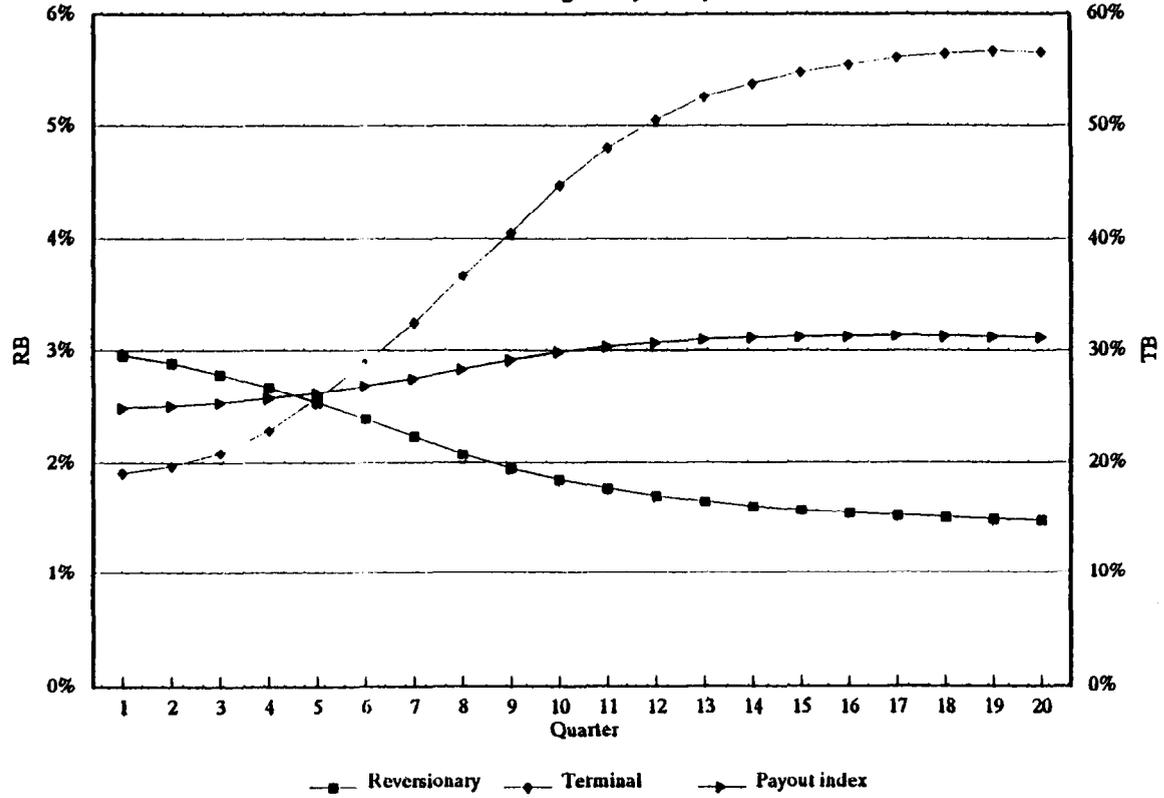
Asset – liability ratios
Gtd SVs + jump in yields + inc surr rates

Legend: Assets/Reserves   Assets/Asset shares   GSV/NPR
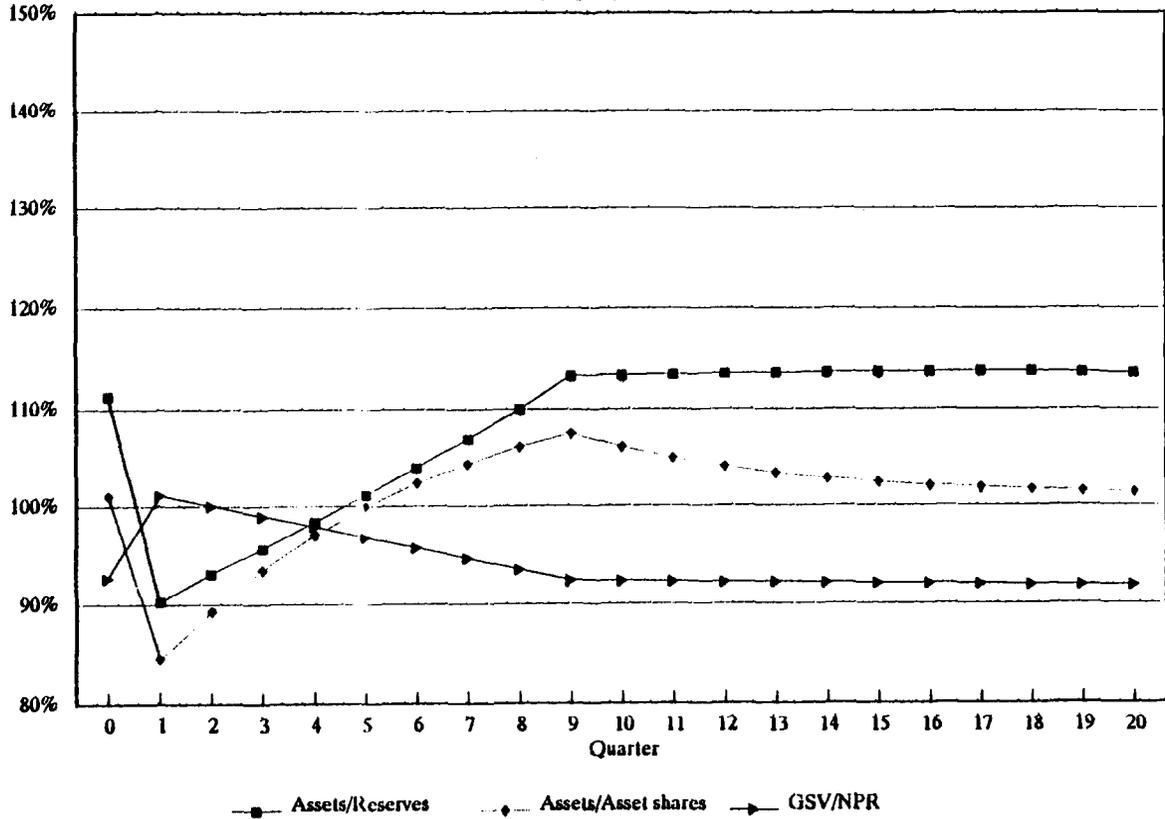
Asset−liability ratios

Yields −30% gradually over 2 yrs

Asset–liability ratios
Yields +30%, gradual return

Quarter

— ■ — Assets/Reserves   ⋯ ◆ ⋯ Assets/Asset shares

Bonus rates

Yields +30%, gradual return

Legend: Reversionary — Terminal — Payout index
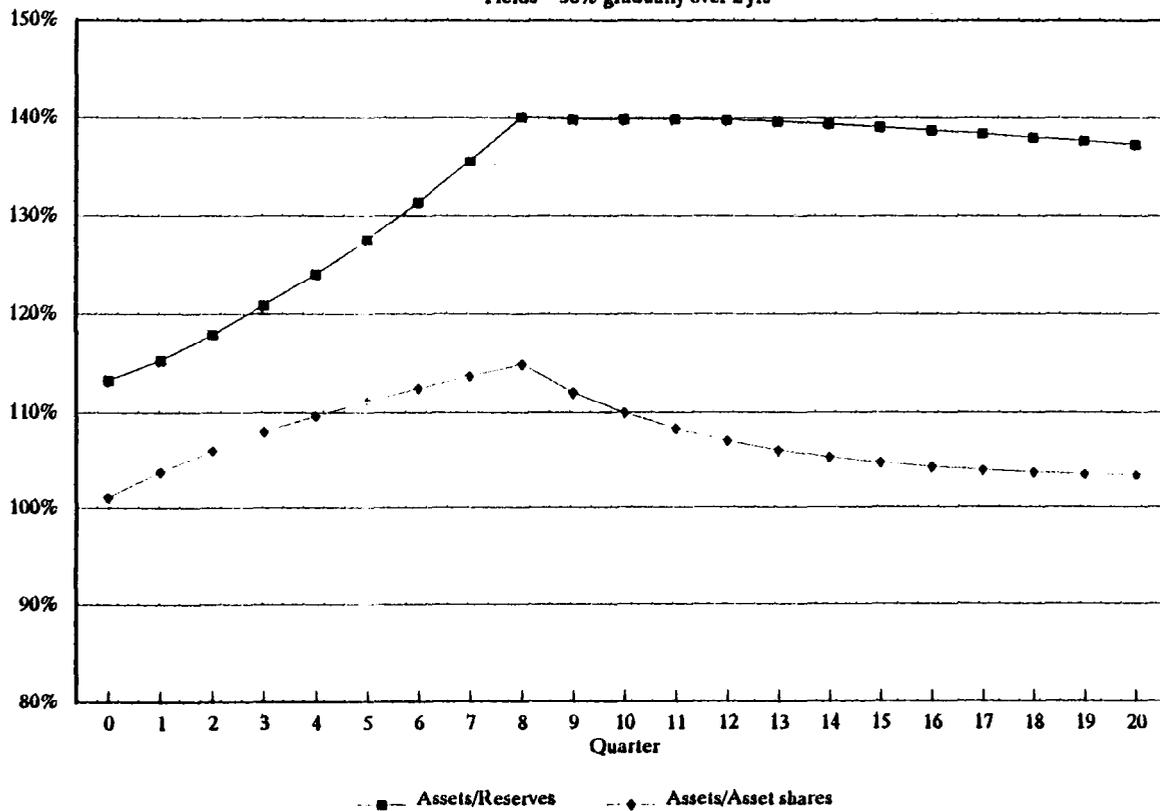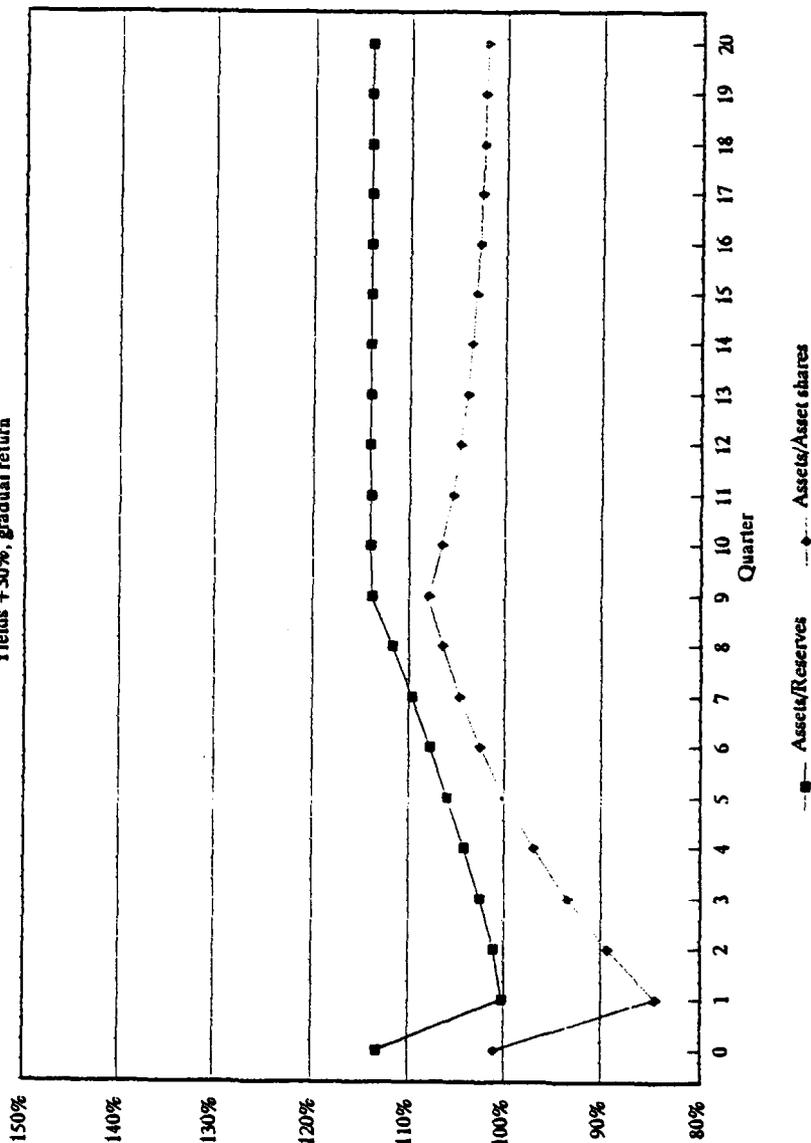
Unit prices
Yields + 30%, gradual return

Unsmoothed price • Smoothed price ▲ Composite yield
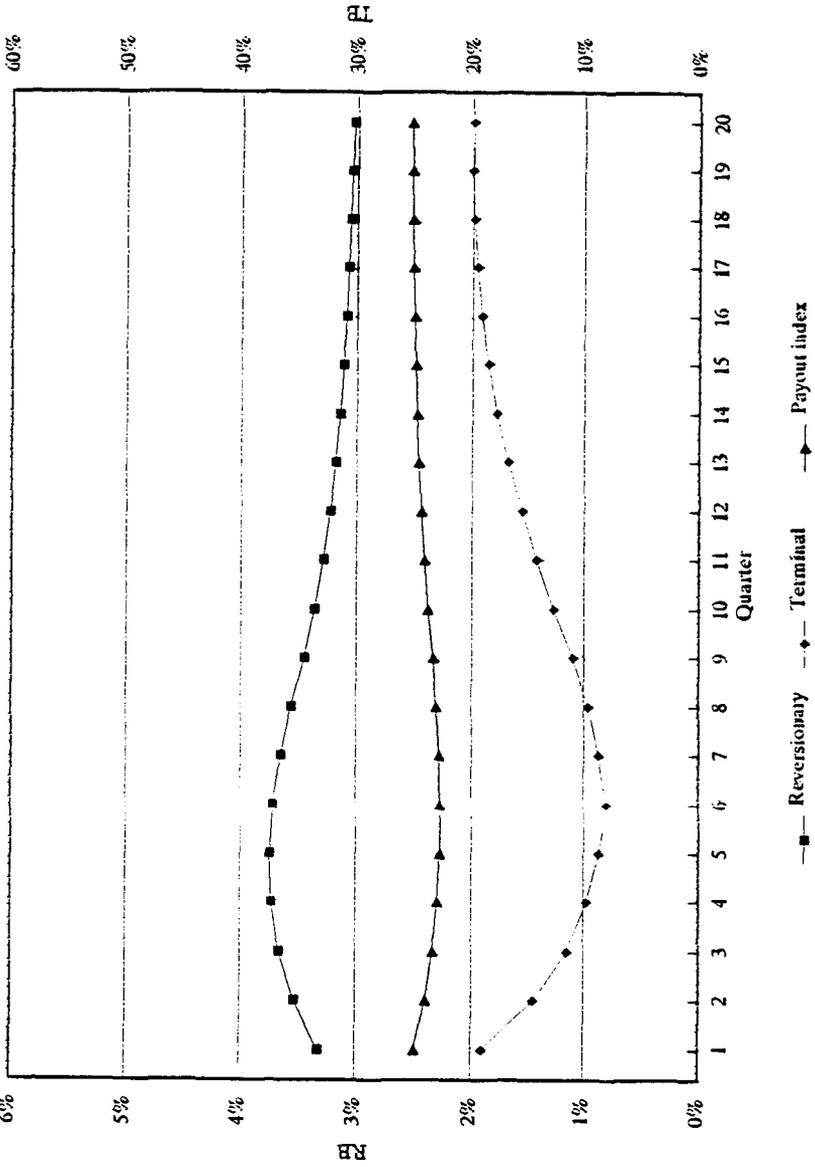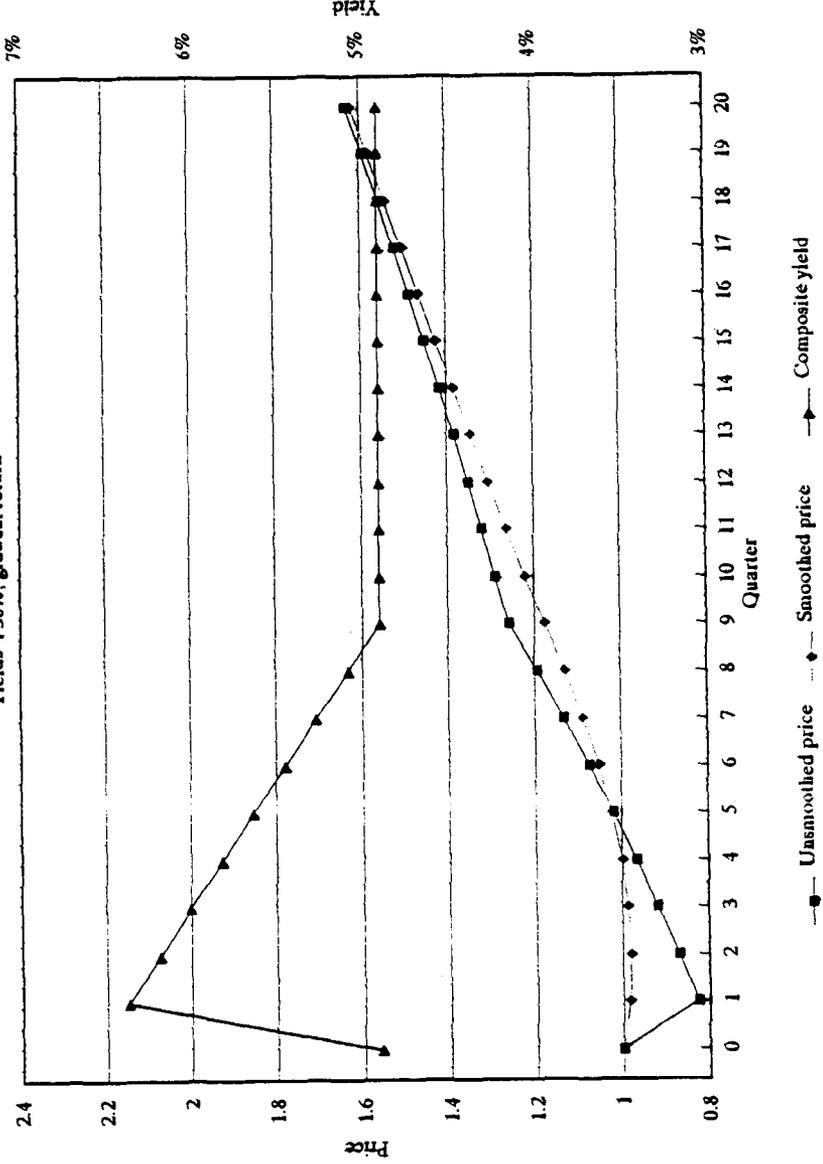
Asset–liability ratios

Central assumptions

Legend: ■ Assets/(Res+SM)   ● Assets/Reserves   ▶ Assets/Asset shares
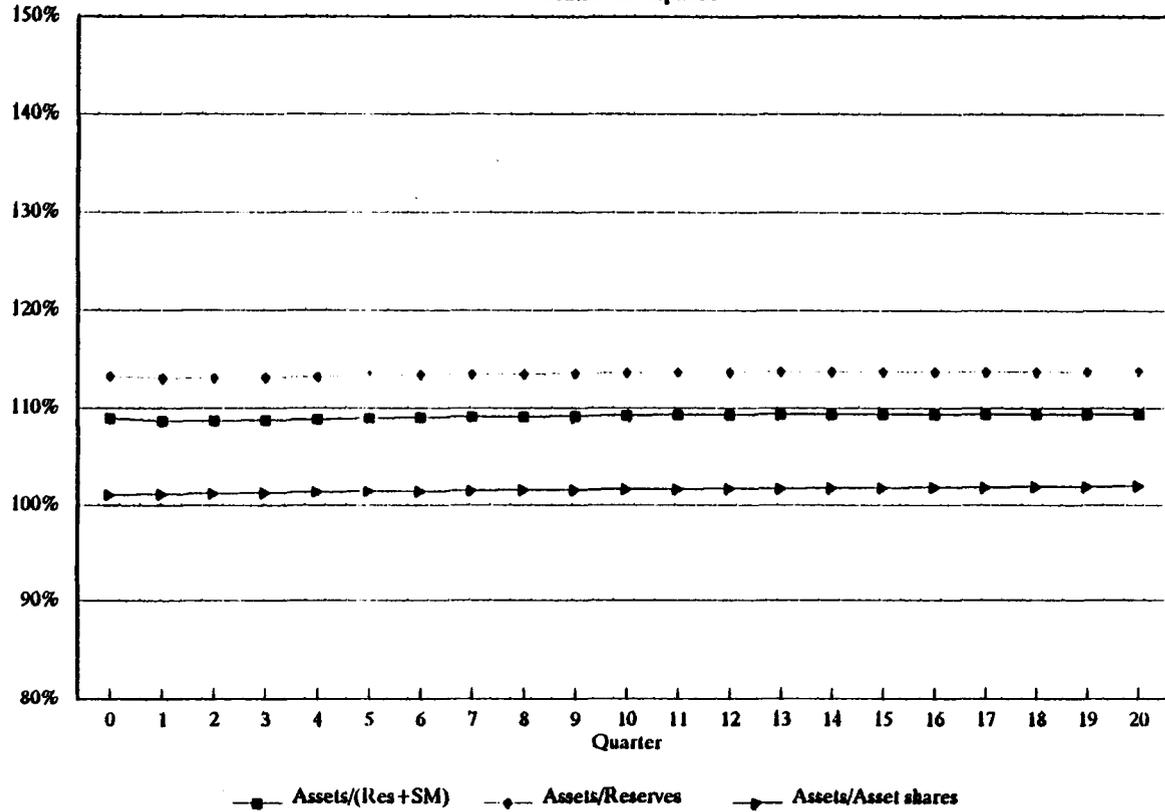
Bonus rates

Central assumptions
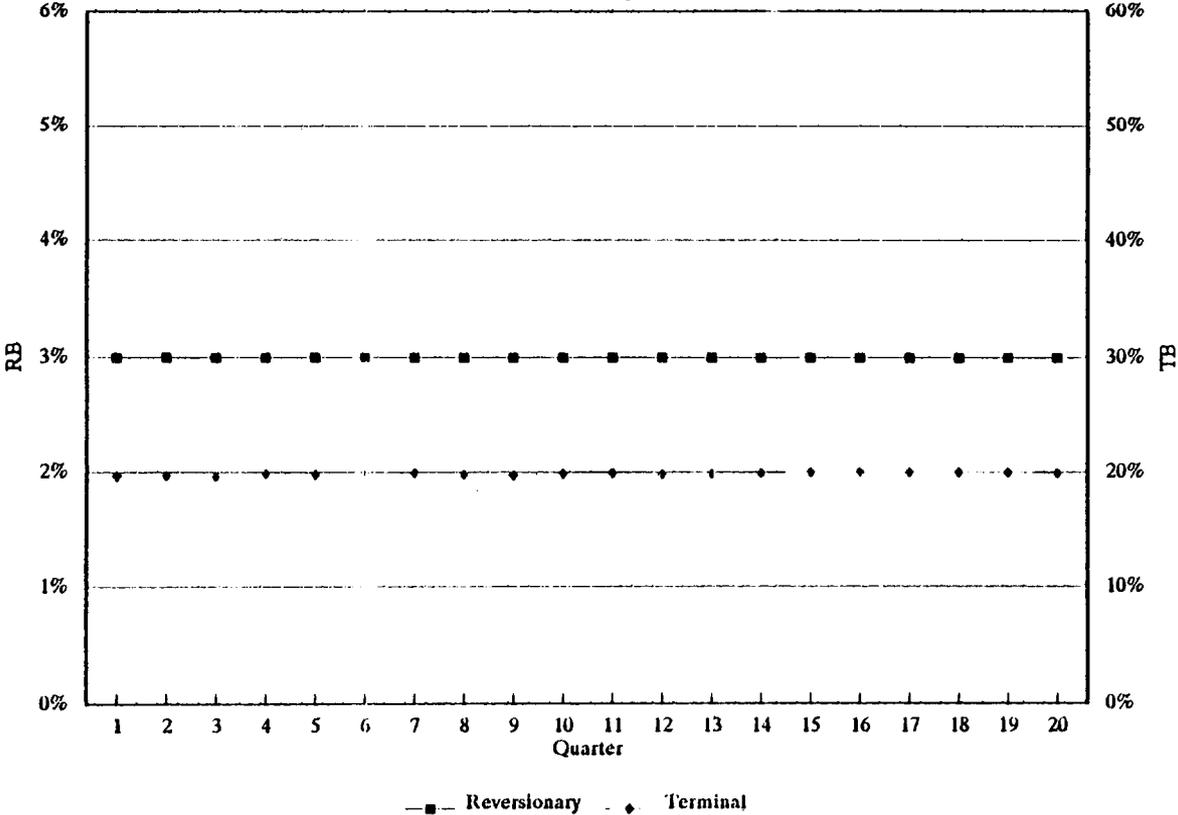
Unit prices
Central assumptions