Optimal Control Strategies versus Supervisory Rules Suggestions from Bond Portfolio Selection Theory

Carlo Mottura[1] and Marco S. Ristuccia[2]

Abstract

The control of interest rate risk has become a relevant problem for banking regulation. Since the first proposal by the Basle Committee on Banking Supervision of the BIS (1993), regulators and bankers are paying increasing attention to measuring precisely the riskiness of financial portfolios. The debate on the proper method for setting capital requirement covering interest rate risk is heated. Standard formulas and rules proposed by supervisors are considered to be outdated compared with banks´ own risk-management techniques. In the paper, a comparison between regulators´ approach and optimal control strategies for bond portfolios is carried out, in order to test standard requirements against banks´ internal models. In particular, an application to Italian Government securities market has been considered: the minimum risk/maximum return portfolio selection problems suggested by stochastic immunization theory, assuming the single-factor Cox, Ingersoll and Ross term structure model, has been performed and capital requirements on the efficient portfolios have been calculated both by using the BIS (1996) and the Bank of Italy (1994) standard formulas. The consistency between the risk measures used by supervisors and the ones derived by pricing models with uncertainty is considered. An analysis of capital requirements with respect to stochastic duration is also proposed.

Keywords

Stochastic immunization theory, bond portfolio management, optimal control schemes, supervisory capital requirements.

[1] Department of Actuarial and Financial Sciences, University of Rome "La Sapienza", Via Nomentana 41, I - 00161 Rome (Italy); Tel: + 39-6-44250297, Fax: + 39-6-44250289, E-mail: ecofin@itcaspur.bitnet.it

[2] Finance Department, CREDIOP, Via XX Settembre 30, I - 00187 Rome (Italy); Tel: + 39-6-47713925, Fax: + 39-6-47715958, E-mail: ecofin@itcaspur.bitnet.it

1. Introduction

The control of interest rate risk has become a relevant problem for banking regulation. The debate on the proper method for setting the amount of capital to hold in reserve, to defend against the risk that the price of on or off-balance sheet positions may fall because of interest rates dynamics before the positions can be liquidated or offset with other positions, is heated. The driving force developing international standards is the Basle Committee for Banking Supervision of the BIS (Bank for International Settlements) [1]. The first proposal of the Committee, to determine capital requirements on banks' trading portfolio, was issued on April 1993. The Committee introduced a standard scheme, based on a sensitivity approach, to quantify the amount that the bank may reasonably expect to lose because of markets' volatilities [2]. On January 1996 the Committee issued a new proposal, that represents a very important step forward in recognizing that banks should have the flexibility to use more advanced own risk-management techniques, under the control of domestic regulators [3]. The new rules will become applicable at the end of 1997, and even if they addressed a number of issues that were raised after the first proposal, further debate is expected.

In the paper, we analyze the BIS and the Bank of Italy approaches by performing a risk-management model based on the results of stochastic immunization theory. In section 2, the optimal control strategies for bond portfolios are defined. The regulators' standard schemes are considered in section 3. In section 4, referring to the Italian Government securities market, a comparison between capital requirements and the results from bond portfolios selection theory is proposed, assuming the single-factor Cox, Ingersoll and Ross term structure model. The consistency between the risk measures used by supervisors and the ones derived by pricing models with uncertainty is considered. An analysis of capital requirements with respect to stochastic duration is also proposed. The conclusions are reported in section 5.

2. Optimal strategies for the control of interest rate risk

We will refer to the stochastic immunization valuation framework, as in [De Felice, 1995]. The market is described, at the valuation time t, by the spot rate $r(t)$, single state variable. The stochastic process $\{r(t)\}$ is a continuous-time markov process. Each default-free interest rate sensitive (irs) contract, traded on the market at time t and described by its vector of contractual parameters $\mathbf{c}$, can be characterized in terms of its price: $P(t) = P(r(t), t, \mathbf{c})$, and its basis risk (semi-elasticity of the price with respect to the state variable): $\Omega(r(t), t; \mathbf{c}) = -P_r(r(t), t; \mathbf{c})/P(r(t), t; \mathbf{c})$. Operationally, the price of each irs instrument can be calculated by solving the general valuation equation under the proper boundary condition. In many cases it is possible to express the basis risk in units of time by defining the stochastic duration: $D(r(t), t, \mathbf{c}) = \varphi^{-1}\{\Omega(r(t), t, \mathbf{c})\} - t$, where $\varphi^{-1}(\cdot)$ is the inverse function, with respect to the variable s, of $\varphi(r(t), t, s) := -v_r(r(t), t, s)/v(r(t), t, s)$, being $v(\cdot)$ the equilibrium discount function (it determines the term structure of interest rates prevailing on the market at time t).

The stochastic immunization theorem is the methodological reference to construct interest rate risk control strategies [4]. The portfolio selection problems can be formally stated by considering the matrix $\mathbf{C} = \{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_n\}$ of the n bonds parameters vectors traded on the market at time t, the time structure $\mathbf{t} = \{t_1, t_2, \ldots, t_m\}$ and the vector $\mathbf{q} = \{q_1, q_2, \ldots, q_n\}$ of the securities' market prices (quotations). The portfolio is described by the vector $\vec{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_n)$, that represents the shares of the contracts of basket $\mathbf{C}$ held by the bank. The market price of the portfolio at time t is, obviously: $C(r(t), t, \vec{\alpha}) = \sum_{i=1}^{n} \alpha_i q_i$. The model price of the portfolio at a future time T, $P(T, r(T), \vec{\alpha})$, is an r_T-measurable random variable. The pair $(\overline{\alpha}_i, \overline{\overline{\alpha}}_i)$, $i = 1, 2, \ldots, n$ can be used to indicate portfolio's composition constraints (if, for example, they are defined to be non-negative it means that short selling is ruled-out [5]).

If L represents the strategic target at a future time H $(t \leq T \leq H \leq t_m)$, i.e. the planned value at the end of the holding period, the minimum cost and the minimum risk portfolio selection problems can be written as follows:

$$\min C(t, \vec{\alpha}), \tag{1}$$

and:

$$\min \mathbf{E}_t \left[P_N^2(r(T), T, \vec{\alpha})\right], \tag{2}$$

under constraints:

$$\begin{aligned} P(r(t), t, \vec{\alpha}) &= P(r(t), t, L) \\ \Omega(r(t), t, \vec{\alpha}) &= \Omega(r(t), t, L) \\ \overline{\alpha}_i \leq \alpha_i &\leq \overline{\overline{\alpha}}_i, \quad i = 1, 2, \ldots, n, \end{aligned}$$

The portfolio selection problems take the form, respectively, of linear programming and quadratic programming problems. The solution of problem (1) is the vector $\vec{\alpha}$ that represents the mimimun cost benchmark portfolio, that can be selected from the basket of market's opportunities to cover the target L at time H, given the portfolio composition constraints. It is, in other terms, the maximum holding period return immunized portfolio on the holding period (t, H). In problem (2), $P_N(r(T), T, \vec{\alpha}) = P(r(T), T, \vec{\alpha}) - P(r(T), T, L)$ is the random net price at time $T = t+\tau$, $\mathbf{E}_t[\,\cdot\,]$ is the conditional expectation operator, $\mathbf{E}_t[P_N^2(r(T), T, \vec{\alpha})]$ is the second order moment of $P_N(T)$. The variable τ represents the size of the rebalancing horizon [6]. Minimizing the second order moment of the random variable $P_N(T)$ means to minimize the probability (at the evaluation time t) that the net absolute value of the portfolio at the next rebalancing time T, will be greater than an assigned level (fixed at time t) [7]. The vector $\vec{\alpha}$, solution of problem (2), hence represents the minimum risk benchmark portfolio, that can be selected from basket $\mathbf{C}$ at time t, carrying minimum second order moment at time T (among all the instantaneously riskless portfolios), given the portfolio composition constraints. The balance-sheet and the basis risk (stochastic duration) constraints represent the conditions of stochastic immunization theorem. To set a correct risk/return ordinal criterion, the constraint: $\mathbf{E}_t[P_N(T)] \geq 0$, must also be considered in the selection problems [8].

The portfolio selection problems can be solved only after the market stochastic model has been defined and the contractual features of the bonds have been considered. A specification of the programming problems on the Italian Government securities market, assuming the single-factor Cox, Ingersoll and Ross term structure model, have been characterized [Mottura, 1992] [Castellani, De Angelis, 1996].

3. Regulators standard schemes

We will consider the BIS [1996] and the Bank of Italy [1994] standard formulas to determine capital requirements against market risk. In particular, we will analyze capital requirements for general market risk, that are designed to capture the risk of loss arising from changes in market interest rates [9].

In the standardised measurement method proposed by the Basle Committee (maturity method), long or short positions in debt securities and other sources of interest rate exposures including derivative instruments are slotted into a maturity ladder comprising thirteen time-bands (or fifteen time-bands in case of instruments having coupons less than or equal to 3% [10]). Fixed rate instruments should be allocated according to the residual term to maturity and floating-rate instruments according to the residual term to repricing date [11]. The first step in the calculation is to weight the positions in each time-band by a factor designed to reflect the price sensitivity of those positions to assumed changes in interest rates. The next step is to offset the weighted longs and shorts in each time-band, resulting in a single short or long position for each band. A 10% capital charge is levied on the smaller of the offsetting positions, be it long or short (so called vertical disallowance), since each band would include different instruments and different maturities. In addition, banks are allowed to conduct two rounds of "horizontal offsetting", first between the net positions in each of the three zones and subsequently between the net positions in the three different zones [12].

Formally, let S be the set of contracts, $\mathbf{s} = \{s_k\}$ and $\mathbf{p} = \{p_k\}$, $(k = 1, 2, \ldots, 13)$, the vectors that identify, respectively, the time-bands and the sensitivity of the price in each time-band. Let us denote with $\Im_k$ the generic sub-set of S having instruments with residual term to maturity (or to the next repricing date, if floating rate instruments) between s_{k-1} and s_k ($\Im_k$ is a partition of S).

If $Q_k = \sum_i \alpha_i q_i$ (being the sum referred to all the instruments $\mathbf{c}_i$ in $\Im_k$), the capital requirement for general market risk on bond portfolios is simply given by: $\Upsilon = \sum_{k=1}^{13} p_k Q_k$.

Always in the framework of the standardised methodology, the BIS also considers the alternative duration method: banks with the necessary capability may, under their supervisors' consent, use a more accurate method by calculating the price sensitivity of each position separately. The Bank of Italy [1994] standard scheme is similar to the BIS maturity method [13].

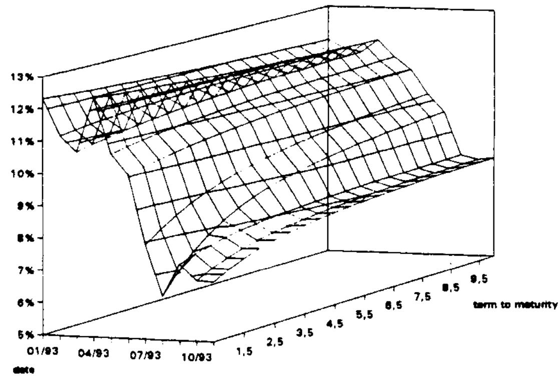
4. Empirical evidences

A comparison between the optimal control strategies and the regulators approach is carried out, in order to test the standard schemes proposed by supervisors with respect to the results from bond portfolio selection theory. The analysis, in particular, has been referred to the Italian Government securities market.

The stochastic immunization framework has been specified assuming the single-factor Cox, Ingersoll and Ross term structure model: the model has been estimated daily from January to November 1993 on the prices of the Italian Treasury bonds (*BTPs*) and of Italian Treasury Bills (*BOT*) quoted on the screen-based market, applying an estimation technique based on linear and non-linear regression procedures performed on historical data and on daily *BOTs* and *BTPs* prices. In particular, a "three step" estimation procedure has been used: it allows to separately identify the values of the model's parameters so that expected prices, as characterized, for example, in problem (2), can be calculated [14]. The evolution of the term stuctures of spot rates, considering the first days of each month in the sample period, is illustrated in table 1.

Referring to the bond portfolio selection schemes as defined in section 2, the matrix **C** has been defined, on each quoted day in the sample period, considering all securities traded on the market: *BOTs*, *BTPs*, Italian Treasury Credit Certificate (*CCTs*) and Italian Treasury Certificate with embedded Option (*CTOs*), on a daily time structure [15].

Table 1



Case 1: first order vs second order risk measures. The linear and the quadratic portfolio selection problems have been solved, for each day in the sample period, having assumed a target $L = 1,000$ ITL, a holding period of 1 year and a rebalancing horizon of ten days ($T - t = 10$). A portfolio composition constraint, $\overline{\overline{\alpha}}_i \leq 200$ ITL, has been set, to determine a liquidity restriction on each market's opportunity (maximum amount on daily trading for each security available in the market). Cash-flows with maturity in (t, T) have been reinvested (until time T) at the expected spot rate derived by the Cox, Ingersoll and Ross model. On the benchmark market portfolios, the BIS (maturity method) and the Bank of Italy (BOI) capital requirements have been calculated [16]. The results are reported in table 2, considering the asset standard deviation (at time T) of the two benchmark market portfolios and the corresponding BIS and BOI requirements. The risk/return trade-off analysis on 4/1/93, having considered a rebalancing period of 1 day ($T = 1$), is illustrated in table 3.

From table 2, the supervisory measures, calculated by the maturity method, seem to be quite stable in the sample period (as expected from theory, having all the portfolios the same stochastic duration), even if the average capital charge on the minimum risk benchmark portfolio is significantly different from the miminum cost one. This disalignement does not seem to support the BIS duration method as all the efficient portfolios (both carrying minimum risk and minimum cost) could have the same capital requirements. Table 3 suggests, as expected, that the dynamic

Table 2

	minimum risk			minimum cost		
date	BIS (itl)	BOI (itl)	stand. dev. (itl)	BIS (itl)	BOI (itl)	stand. dev. (itl)
04-gen-93	7,8754	5,2689	2,4791	9,7281	6,4820	3,0745
01-feb-93	8,9512	5,9597	2,4287	7,8869	5,2595	5,7896
01-mar-93	8,0768	5,3878	2,3788	8,4130	5,6239	5,7358
02-apr-93	8,3033	5,5304	2,5259	9,4959	6,3495	6,2341
03-mag-93	8,5009	5,6844	2,7576	11,5353	7,6844	5,8584
01-giu-93	8,7713	5,8477	2,3090	8,8689	5,9044	5,1989
01-lug-93	8,7534	5,8427	1,8728	8,6497	5,7681	2,9542
02-ago-93	7,8689	5,2674	1,7866	7,3466	4,9182	1,9675
01-set-93	8,6567	5,7870	1,6781	10,1814	6,8069	2,1370
01-ott-93	7,3873	4,9452	1,6124	8,2617	5,5232	2,0467
02-nov-93	9,2987	6,2197	1,6013	14,2563	9,5124	1,6109
average	8,4040	5,6125		9,5113	6,3484	

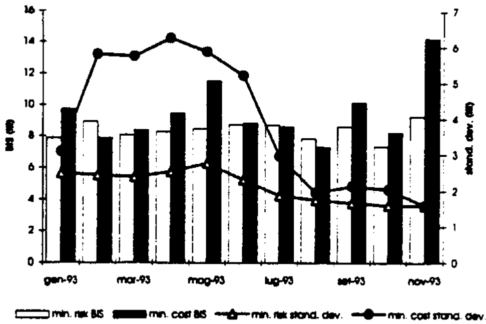
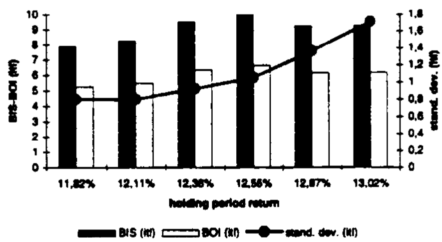


Table 3

04-gen-93

portfolio	holding period return	BIS (itl)	BOI (itl)	stand. dev. (itl)
min. risk	11,82%	7,8756	5,2660	0,8014
1st interm.	12,11%	8,2347	5,4903	0,8014
2nd interm.	12,36%	9,4920	6,3332	0,9215
3rd interm.	12,55%	9,9617	6,6412	1,0647
4th interm.	12,87%	9,2375	6,1537	1,3656
min. cost	13,02%	9,2711	6,1752	1,7152

omega CIR: 0,8494 stochastic duration: 1 year



ABI	instr.	min. risk	1st interm.	2nd interm.	3rd interm.	4th interm.	min. cost	market price (accrual)	model price	omega	duration
12658	BTP	200,0000	—	—	—	—	—	103,5674	102,7885	0,5671	0,6290
12671	BTP	200,0000	200,0000	60,2632	32,9950	—	—	101,9974	101,9504	1,1626	1,4749
12682	BTP	—	92,8285	—	—	—	—	99,2167	99,4948	1,7913	2,7520
12688	BTP	—	38,1407	200,0000	200,0000	200,0000	200,0000	98,4500	99,4422	2,0380	3,4610
13074	CCT	0,0047	0,0010	107,3012	200,0000	200,0000	200,0000	102,9903	103,3824	0,3390	0,3599
13097	CCT	—	—	—	—	12,1123	200,0000	98,7271	99,1814	0,2201	0,2286
13204	CCT	—	—	—	18,4976	200,0000	200,0000	100,1728	101,0763	0,4884	0,5334
13212	CTO	—	—	—	—	0,0057	—	101,8925	102,7698	1,4701	2,0306
36069	BOT3	9,0006	194,2291	200,0000	200,0000	200,0000	22,4833	97,5000	97,3317	0,2240	0,2329
36070	BOT6	200,0000	200,0000	200,0000	200,0000	16,6915	—	94,9100	94,5299	0,4476	0,4849
36071	BOT12	200,0000	200,0000	152,1582	—	—	—	89,5800	89,2128	0,8396	0,9863
36607	BTP	—	—	—	51,1914	63,1235	65,4616	95,0008	95,7783	2,1350	3,7953
36615	BTP	106,0474	—	—	—	—	—	99,4017	99,0152	1,5863	2,2714

of the standard measures does not seem to be related to the standard deviation of the portfolio.

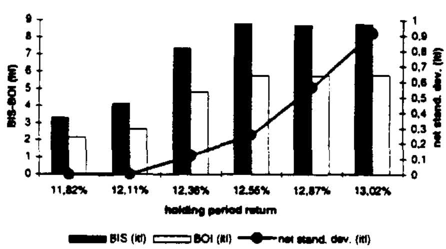
Case 2: bond portfolio selection and asset-liability management. On the efficient market portfolios of the quoted day 4/1/1993 the BIS and BOI capital requirements have been calculated considering each portfolio as an intermediation portfolio with a liability side made of the portfolio's planned value at the end of the holding period ($L = 1,000$ ITL at time $H = 1$ year). The results are reported in table 4, where the net standard deviation has been represented. Because of the offsetting of the weighted long and short positions and as the vertical and horizontal disallowance factors work, capital charges are different from the ones illustrated in table 3. They seem to capture the second order risk profile of the different benchmark portfolios. Do they possibly represent, in the holding period, the correct requirements against interest rate risk if an accurate dynamic risk-management is carried out ?

Table 4

04-gen-93

portfolio	BIS (itl)	BOI (itl)	net stand. dev. (itl)
min. risk	3,3233	2,1571	0,0001337
1st interm.	4,1175	2.6499	0,0002627
2nd interm.	7,3501	4,8138	0,1201031
3rd interm.	8,7726	5,7499	0,2532663
4th interm.	8,7039	5,7479	0,5641810
min. cost	8,7607	5,7881	0,9137835

stochastic duration: 1 year



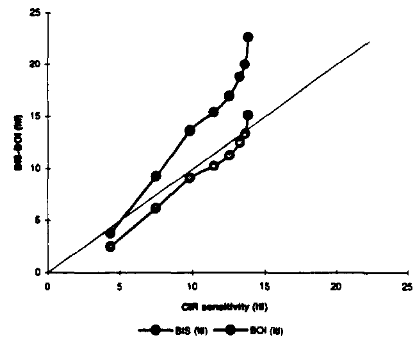
Case 3: capital requirements vs stochastic duration. The programming procedures (1) and (2) have been solved referring to different investment horizons (stochastic duration between 0.5 and 4 years), on the quoted day 4/1/1993, for $T = 1$ day. On the optimal stochastically immunized portfolios, BIS and BOI capital requirements have been calculated. The results are reported in table 5 [17]. The black and the grey points represent, respectively, the BIS and the BOI capital charges. The distance of the points from the bisecting line illustrates the "efficiency" of capital requirements in term of stochastic immunization. In particular, the BOI charges, in the sample day, appear to be quite significant with respect to the theoretical sensitivity values.

5. Conclusions

The BIS [1996] and the Bank of Italy [1994] capital requirements have been analyzed in the framework of stochastic immunization and of bond portfolio selec-

Table 5

				04-gen-93
portfolio	CIR sensitivity	standard dev. (lit)	BIS (lit)	BOI (lit)
1	4,3531023	0,7457	3,7662	2,5143
2	7,5154114	1,7152	9,2711	6,1752
3	9,8500286	1,3559	13,6689	9,1145
4	11,47335	1,5306	15,4010	10,2772
5	12,568237	1,6049	16,9289	11,2987
6	13,277015	1,4179	18,7707	12,5252
7	13,647377	1,4576	19,9998	13,3456
8	13,838497	1,4714	22,6511	15,1078



ABI	instr.	port. 1	port. 2	port. 3	port. 4	port. 5	port. 6	port 7	port. 8	market price (accrual)	model price	omega	duration
12658	BTP	165,5296	-	-	-	-	-	-	-	103,5674	102,7885	0,5671	0,6290
12682	BTP	-	-	-	-	-	81,0225	-	-	99,2167	99,4948	1,7913	2,7520
12683	BTP	-	-	-	-	-	-	-	44,2037	97,3367	97,1125	2,1614	3,8938
12686	BTP	-		20,3052	123,0504	200,0000	200,0000	200,0000	-	96,4467	96,8012	1,9150	3,0860
12687	BTP	-	-	-	-	-	-	78,9867	200,0000	93,6267	93,5359	2,2448	4,2275
12688	BTP	-	200,0000	200,0000	200,0000	200,0000	200,0000	200,0000	21,4817	98,4500	99,4422	2,0380	3,4610
13074	CCT	200,0000	200,0000	130,3894	-	-	-	-	-	102,9903	103,3824	0,3390	0,3599
13097	CCT	-	200,0000	-	-	-	-	-	-	98,7271	99,1814	0,2201	0,2286
13204	CCT	39,6356	200,0000	100,5630	191,9636	164,3220	0,0558	-	-	100,1728	101,0763	0,4884	0,5334
36069	BOT3	200,0000	22,4833	200,0000	94,4251		47,4862	-	-	97,5000	97,3317	0,2240	0,2329
36070	BOT6	200,0000	-	-	-	-	-	-	-	94,9100	94,5299	0,4476	0,4849
36071	BOT12	160,2647	-	-	-	-	-	-	-	89,5800	89,2128	0,8396	0,9863
36605	BTP	-	-	-	-	-	-	-	200,0000	95,3767	95,1809	2,2118	4,0908
36606	BTP	-	-	-	-	2,2268	-	17,3161	-	97,1267	97,3511	1,9351	3,1439
36607	BTP	-	65,4616	200,0000	200,0000	200,0000	200,0000	200,0000	200,0000	95,0008	95,7783	2,1350	3,7953
36608	CTO	-	-	-	-	-	-	0,0008	-	98,3417	99,0485	1,6296	2,3664

tion theory. An application to the Italian Government securities market, assuming the single-factor Cox, Ingersoll and Ross model has been performed. As expected from theory, the supervisory requirements appear to be significant in the sample period as first order risk measure (price sensitivity). Referring to the general portfolio's risk profile, they don't capture, as expected, the second order risk effect. On this line, capital requirements have also been calculated considering each portfolio as an intermediation portfolio, with the liability side made of the portfolio's planned value at the end of the holding period. Under this construction, the regulatory schemes apparently characterize the second order effect. With respect to the stochastic duration, BIS and BOI capital requirements are significant. The BOI charges, in particular, are quite aligned to the theoretical sensitivity values.

Endnotes

[1] Also the European Community, in 1993, issued a Capital Adequacy Directive (see [14]), to define standard rules to measure interest rate risk.

[2] Operationally, the main problems are concerned with the derivatives activities; many institutional studies have been proposed (see, for example, [8] and [9]).

[3] The Bank of Italy [1994] standard scheme already moves along these guidelines (see [2]). The risk-management techniques are referred, in particular, to the Value at Risk (VaR) approach (for an analyis of the peculiar aspects of this approach see [16]). The Italian Bankers Association (ABI) proposed, in 1995, an asset-liability management model to quantify the bank' exposure to interest rate risk, based on sensitivity analysis (see [1]).

[4] Semi-deterministic and stochastic schemes based on financial immunization theory for the control of interest rate risk are defined in [3]. For an empirical test see [5].

[5] In the case of contracts with deterministic cash-flows, $\mathbf{C}$ is a $(n \times m)$ matrix of non-negative elements, where the generic element c_{ik} represents the amount to be received at time t_k for the i^{th} asset. The composition constraints could also be used to manage outstanding portfolios.

[6] The quadratic programming problem is analyzed in [4] from the theoretical point of view and in its operational implications.

[7] In fact, by Cebicef inequality, it results:

$$Prob[\mid P_N(r(T),T) \mid \geq a] \leq \frac{\mathbf{E}_t[P_N^2(r(T),T)]}{a^2}.$$

The variance of the portfolio is given by: $\mathbf{E}_t[P_N^2(r(T),T,\vec{\alpha})] - \mathbf{E}_t[P_N(r(T),T,\vec{\alpha})]^2$.

[8] It could be also relevant to define the portfolio selection problem having the objective function: $\max\ \mathbf{E}_t[P_N(r(T),T,\vec{\alpha})]$. The selected portfolio is the one that maximises the expectation of the net value at time T, so that it will be in the "best condition as possible" (in terms of expectation) at the next rebalancing time (an application is analyzed in [15]). Together with (2), the two problems allow to identify, in the risk/return plane, the efficient portfolios' frontier. This leads to redefine the traditional "mean-variance" approach in terms of stochastic immunization. For details on the meanings of the non-negative constraint, see [4].

[9] See, respectevely, [7] and [2]. The capital charges for interest rate related instruments are applied to the current market value of items in banks' trading books (for the definition of trading portfolio see [7], p. 1).

[10] The time-bands are considered within three zones: zero to one year, one year to four years and four years and over (the zones for coupon less than 3% are 0 to 1 year, 1 to 3.6 years, 3.6 years and over).

[11] The derivatives (futures, forward contracts, FRAs and swaps) are converted into positions in the relevant underlying. Options are separately treated referring to three alternative approaches: the simplified approach, the delta-plus method and the scenario approach. Those banks which solely use purchased options, are free to use the simplified approach. Those banks which also write options are expected to

use one the two other approaches. The general criterion is that the more significant is options' trading, the more the bank is expected to use a sophisticated approach. Opposite positions of the same amount in the same issues can be omitted from interest rate maturity framework (see[7], p.16).

[12] The vertical disallowance factors are referred to the so called basis risk and the gap risk. The horizontal disallowance ones are considered to take into account the correlation effect (some considerations on correlation between financial variables implied by the BIS [1993] rules, are in [13]). Obviously, in the case of bond portfolios, both horizontal and vertical disallowances don't work being all positions of the same sign.

[13] For the duration method, see [6] (pp.13-14). In table A.1 are illustrated the risk weights, the horizontal and the vertical disallowance factors of the BIS and of the Bank of Italy.

[14] The 3-months *BOTs* yield has been used to "represent" the spot rate. The time series of the yields from january 1980 until november 1993 has been considered and the Cox, Ingersoll and Ross parameters have been estimated on a discrete time equivalent of the stochastic differential equation. For details on the "three steps" and the Brown and Dybvig estimation techniques see [3], pp. 124-128. The estimated parameters and the square mean root of the estimation, for the first days of each month in sample period, are reported in table A.2.

[15] An analysis of *CCTs* volatility with respect to the [1993] BIS proposal in [12]. See also [11] for an analysis of capital requirements on italian SIM. As far as *CTOs* are concerned, all the securities actually traded on the market are fixed rate bonds.

[16] The analysis of *CTOs* has been performed by the Cox, Ingersoll and Ross model (see [3], p. 179-182). To calculate capital requirements, the market value has been assigned to the time-band corresponding to the semi-elasticity of the price calculated by the model. The rebalancing period of a decade is in line with the BIS quantitative criteria for the use of internal models. The non-negative constraint on the first order moment of the variable $P_N(T)$ has also been considered. Computational programmes are developed in MS-DOS environment, using MATLAB, NAG and OSL libraries. The composition of the minimum risk benchmark portfolio and of the minimum cost one are reported, respectively, in table A.3. and in table A.4

[17] In the quoted day, for investment horizons over 4 years, the efficient portfolios cannot be identified (infeasible problems).

[18] In the single-factor Cox, Ingersoll and Ross term structure model the evolution of the spot rate, $r(t)$, is described by the Ito stochastic differential equation: $dr(t) = \alpha(\gamma - r(t))dt + \rho\sqrt{r}, \quad \alpha, \gamma, \rho > 0.$
The market price of risk is given by: $q(r,t) = \pi\sqrt{r}/\rho, \quad \pi$ constant. In the Brown and Dybvig estimation procedure the following aggregate parameters are considered: $d = \sqrt{(\alpha - \pi)^2 + 2\rho^2}, \quad d_1 = (\alpha - \pi + d)/2, \quad \nu = 2\alpha\gamma\rho^2.$

Table A.1

DATE	r	di	di1	ni	sqmr
04/01/93	0,11625	0,35083	0,34250	13,27190	0,51970
01/02/93	0,10529	0,35087	0,34246	13,25440	0,27851
01/03/93	0,10071	0,35091	0,34243	13,20320	0,37918
02/04/93	0,11716	0,35091	0,34244	13,04930	0,63382
03/05/93	0,10067	0,35093	0,34242	12,95260	0,27755
01/06/93	0,09465	0,35086	0,34249	12,86280	0,32967
01/07/93	0,07229	0,56469	0,55663	12,69610	0,68612
02/08/93	0,05405	0,77392	0,76621	12,66000	0,44404
01/09/93	0,06883	0,77301	0,76643	12,29200	0,46560
01/10/93	0,06353	0,77297	0,76643	12,29200	0,35794
02/11/93	0,06306	0,77304	0,76643	12,29000	0,37964

Table A.2 [18]

Time band		0 - 1	1 - 3	3 - 6	6 - 12	1 - 2	2 - 3	3 - 4	4 - 5	5 - 7	7 - 10	10 - 15	15 - 20	over 20
		Months				Years								
Weight (%)	BIS	0,00	0,20	0,40	0,70	1,25	1,75	2,25	2,75	3,25	3,75	4,50	5,25	6,00
	BOI	0,00	0,13	0,27	0,47	0,83	1,17	1,50	1,83	2,17	2,50	3,00	3,50	4,00
Vertical Disallowance Factor	BIS - BOI	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%
Horizontal Disallowance Factor within the same zone	BIS	40%				30%			30%					
	BOI	30%				20%			20%					
Horizontal Disallowance Factor beetween zones 1 - 2	BIS	40%												
	BOI	30%												
Horizontal Disallowance Factor beetween zones 2 - 3	BIS								40%					
	BOI								30%					
Horizontal Disallowance Factor beetween zones 1 - 3	BIS	100%												
	BOI	100%												

Table A.3

ABI	instr.	04/01/93	01/02/93	01/03/93	02/04/93	03/05/93	01/06/93	01/07/93	02/08/93	01/09/93	01/10/93	02/11/93
12658	BTP	200,00	--	--	--	--	--	--	--	--	--	--
12664	BTP	--	200,00	--	--	--	--	--	--	--	--	--
12666	BTP	--	200,00	--	--	--	--	--	--	--	--	--
12670	BTP	--	--	200,00	200,00	--	200,00	200,00	200,00	200,00	180,72	--
12671	BTP	200,00	146,58	200,00	200,00	--	--	200,00	--	200,00	200,00	--
12672	BTP	--	--	--	92,55	--	200,00	--	--	200,00	--	--
12676	BTP	--	--	--	--	--	--	--	65,71	--	--	--
12825	CCT	--	--	--	--	--	--	3,47	--	--	--	--
13074	CCT	0,01	--	--	--	124,42	0,03	--	200,00	2,58	1,13	--
13090	CCT	--	--	--	--	--	--	--	35,83	--	--	--
13098	CTO	--	--	--	--	--	0,04	--	--	--	--	--
36069	BOT3	9,00	--	--	--	--	--	--	--	--	--	--
36070	BOT6	200,00	--	--	--	--	--	--	--	--	--	--
36071	BOT12	200,00	--	--	--	--	--	--	--	--	--	--
36076	BOT6	--	161,91	--	--	--	--	--	--	--	--	--
36077	BOT12	--	200,00	--	--	--	--	--	--	--	--	--
36079	BOT6	--	--	96,88	--	--	--	--	--	--	--	--
36082	BOT6	--	--	200,00	--	--	--	--	--	--	--	--
36083	BOT12	--	--	200,00	--	--	--	--	--	--	--	--
36087	BOT3	--	--	--	27,85	--	--	--	--	--	--	--
36088	BOT6	--	--	--	200,00	--	--	--	--	--	--	--
36089	BOT12	--	--	--	200,00	--	--	--	--	--	--	--
36093	BOT3	--	--	--	--	200,00	--	--	--	--	--	--
36094	BOT6	--	--	--	--	200,00	--	--	--	--	--	--
36099	BOT3	--	--	--	--	--	37,98	--	--	--	--	--
36100	BOT6	--	--	--	--	--	200,00	--	--	--	--	--
36101	BOT12	--	--	--	--	--	200,00	--	--	--	--	--
36105	BOT3	--	--	--	--	--	--	145,49	--	--	--	--
36107	BOT12	--	--	--	--	--	--	200,00	--	--	--	--
36111	BOT3	--	--	--	--	--	--	--	--	12,07	--	--
36113	BOT12	--	--	--	--	--	--	--	200,00	--	--	--
36119	BOT12	--	--	--	--	--	--	--	--	200,00	--	--
36122	BOT12	--	--	--	--	--	--	--	--	--	200,00	--
36125	BOT12	--	--	--	--	--	--	--	--	--	200,00	--
36129	BOT3	--	--	--	--	--	--	--	--	--	--	159,02
36130	BOT6	--	--	--	--	--	--	--	--	--	--	200,00
36131	BOT12	--	--	--	--	--	--	--	--	--	--	200,00
36615	BTP	106,05	--	34,38	--	200,00	82,90	166,14	200,00	92,30	152,34	--
36621	BTP	--	--	--	--	185,45	--	--	--	--	--	--
36630	BTP	--	--	--	--	--	--	--	--	--	--	200,00
36634	BTP	--	--	--	--	--	--	--	--	--	--	172,61

Table A.4

ABI	instr.	04/01/93	01/02/93	01/03/93	02/04/93	03/05/93	01/06/93	01/07/93	02/08/93	01/09/93	01/10/93	02/11/93
12670	BTP	--	--	--	--	--	--	200,00	200,00	200,00	--	--
12671	BTP	125,95	--	--	--	--	--	141,72	--	--	--	--
12676	BTP	--	--	--	--	--	--	--	200,00	--	--	--
12677	BTP	--	--	--	--	--	200,00	--	17,47	--	--	--
12679	BTP	--	--	24,53	--	200,00	--	135,68	--	--	--	200,00
12683	BTP	--	--	--	--	59,50	--	--	--	--	--	--
12684	BTP	--	--	--	200,00	--	--	--	--	--	--	--
12685	BTP	--	--	--	--	--	24,16	--	--	--	--	--
12686	BTP	--	40,66	--	--	--	--	--	--	--	--	--
12688	BTP	200,00	--	200,00	--	--	--	--	--	--	--	--
13074	CCT	166,94	--	--	--	--	--	--	--	--	--	--
13084	CCT	--	49,52	64,62	--	--	200,00	--	--	--	200,00	--
13086	CTO	--	--	4,76	--	--	--	--	--	--	200,00	--
13096	CCT	--	200,00	--	--	--	46,15	--	--	--	--	--
13097	CCT	--	--	--	--	--	--	--	--	200,00	--	--
13098	CTO	--	--	--	--	--	--	2,82	--	--	--	--
13099	CCT	--	200,00	--	--	35,71	--	--	--	--	--	--
13202	CCT	--	--	--	--	200,00	200,00	--	--	--	--	--
13204	CCT	--	--	--	--	--	--	200,00	--	--	--	--
13205	CCT	--	--	--	--	200,00	--	--	--	--	--	--
13207	CCT	--	--	200,00	--	--	--	--	--	--	--	--
13208	CCT	--	--	--	--	--	--	--	--	--	45,80	--
13210	CCT	--	--	--	--	--	--	--	--	--	--	5,34
13211	CCT	--	--	200,00	60,25	--	--	--	--	--	--	--
13212	CTO	--	--	--	--	1,33	2,43	--	--	--	0,65	--
13213	CCT	--	--	--	200,00	200,00	--	200,00	--	--	--	--
36069	BOT3	200,00	--	--	--	--	--	--	--	--	--	--
36070	BOT6	200,00	--	--	--	--	--	--	--	--	--	--
36111	BOT3	--	--	--	--	--	--	--	--	41,72	--	--
36113	BOT12	--	--	--	--	--	--	--	200,00	--	--	--
36119	BOT12	--	--	--	--	--	--	--	--	200,00	--	--
36122	BOT12	--	--	--	--	--	--	--	--	--	200,00	--
36125	BOT12	--	--	--	--	--	--	--	--	--	200,00	--
36129	BOT3	--	--	--	--	--	--	--	--	--	--	200,00
36130	BOT6	--	--	--	--	--	--	--	--	--	--	200,00
36131	BOT12	--	--	--	--	--	--	--	--	--	--	200,00
36601	CCT	--	200,00	--	200,00	--	200,00	--	82,05	--	--	--
36603	CCT	--	--	200,00	--	--	--	--	--	51,78	--	--
36604	CCT	--	--	--	200,00	--	--	--	--	--	--	--
36607	BTP	7,44	200,00	--	55,76	--	--	--	--	200,00	--	--
36608	CTO	--	0,92	--	--	--	--	--	--	--	--	--
36621	BTP	--	--	--	--	--	--	--	--	--	75,71	--
36629	CCT	--	--	--	--	--	--	--	200,00	--	--	--
36632	BTP	--	--	--	--	--	--	--	--	--	--	42,07
36634	BTP	--	--	--	--	--	--	--	--	--	--	61,46

References

[1] ABI, *Guida all'Assset-Liability Management. L'approccio ABIALM*, Bancaria Editrice, 1995

[2] Banca d'Italia, *Requisiti patrimoniali individuali sui rischi di mercato*, Circolare n. 114, 4 settembre 1994

[3] Castellani, G., De Felice, M., Moriconi, F., Mottura, C., *Un corso sul controllo del rischio di tasso di interesse*, Il Mulino, Bologna, 1993

[4] Castellani, G., De Angelis, P., L., *Minimum Risk Portfolio Selection in Stochastic Immunization Framework*, Research Group on Models for Mathematical Finance, working paper, n.22, 1996

[5] Ciampolini, M., Giraldi, C., Govino, C., *L'immunizzazione di portafogli obbligazionari: un confronto empirico tra strategie alternative*, Finanza Imprese e Mercati, n. 3, 1993

[6] Comitato di Basilea, *Il trattamento prudenziale dei rischi di mercato*, Basilea, Aprile 1993

[7] Comitato di Basilea, *Emendamento dell'accordo sui requisiti patrimoniali per incorporarvi i rischi di mercato*, Basilea, Gennaio 1996

[8] Comitato di Basilea, IOSCO, *Framework for Superivisory information about the derivatives activities of banks and securities firms*, Maggio 1995

[9] Groupe of Thirty, *Derivatives: Practices and Pinciples s*, Washington, DC, Dicembre 1994

[10] De Felice, M., *Immunization Theory: an Actuarial Perspective on asset-Liability Management*, in "Financial Risk in Insurance, G. Ottaviani (ed.), Springer Verlag, 1995

[11] Drudi, F, Panetta, F., *Rischio di tasso di interesse e coefficienti patrimoniali: un'analisi dei regolamenti SIM*, Banca d'Italia, Temi di discussione, N. 180, 1992

[12] Drudi, F. Panetta, F., *Alcune considerazioni sulla volatilità dei corsi dei CCT e i coefficienti patrimoniali del Comitato di Basilea*, Banca d'Italia, 1994

[13] Esposito, M., Giraldi, C., *Some puzzling B.I.S. Arithmetics*, Banca Commerciale Italiana, Ufficio Studi, R94-13, 1994

[14] European Comunity, *Capital Adequacy Directive*, EEC 93/6, 1993.

[15] Mottura, C., *Uno schema di asset-liability management per contratti finanziari con flusso di pagamenti aleatorio*, Atti del XVI Convegno AMASES, Treviso, 1992

[16] JPMorgan, *RiskMetrics - Technical Document*, Morgan Guaranty Trust Company, New York, 1995