Development of an Inflation Model

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Abstract

Inflation rates have a number of interesting properties which are not reflected in traditional actuarial models, for example uncertainty as to any trend, an increasing funnel of doubt and occasional sharp jumps in the level. A simple intuitive model is developed to allow for these features. The model is parameterised, with parameters chosen by the user, for example having regard to historic data and available information embedded in the securities markets. In particular, parameters can be chosen such that the model does not make the implicit assumption that the model user is able to make better predictions of future inflation rates than the collective views of all investors as evidenced by the structure of prices in the conventional and index-linked gilt markets.

The author regards the model as being useful and reaslistic, rather than definitive, and the model is put forward with a view to stimulating debate. A number of its limitations are set out in later sections of this paper.

Résumé

Les taux d´inflation ont plusieurs propriétés intéressantes qui ne se reflètent pas dans les models "actuarial" traditionnels comme par example l´incertitude d´une tendance, la croissance de doutes ou d´occasionels violents sauts au niveau des taux. Un simple model est développé de façon à prendre en considération ces caractéristiques.

Le model est mis en paramètre avec des données choisies par l´utilisateur, par example en regard avec des données historiques et d´information disponible fixées dans les marchés. Un paramètre particulié peut être choisi de façon à ce que le model ne suppose pas que l´utilisateur soit capable de faire de meilleures prédictions sur les prochains nivaux des taux d´inflation que la plupart des investisseurs au vu de la structure des prix dans des marchés conventionnels et "index-linked gilt".

L'auteur considère le modèle étant utile réalistique plutôt que définitif et qu'il est lancé pour stimuler le débat. Un certains nombre de limitations sont exposées dans les prochaines sections de ce document.

Keywords

Inflation rates, gilt markets.

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Characteristics of Inflation Rate Series

The first step in developing the model was to examine data for inflation experience in the UK during the last approximately 70 years, and also for the period from 1661. Features of the series worthy of note for the purposes of this paper are as follows:

Mean rate of inflation

The data does not support the hypothesis that the underlying rate of inflation is constant over time or that there is some long run mean towards which current inflation rates will regress over time. For example, although the average rate of inflation during the 18th and 19th centuries was approximately 0% pa, there were extended periods over which inflation was negative, and lengthy periods during which it was positive. Similarly, during the 20th century there were long periods during which inflation was low, other periods of high inflation and yet other periods of moderate inflation. The yields on Consols bonds appear to confirm that long term expectations change over time, since yields tended to be high when recent and historic inflation rates had been high, and vice versa. This does not of course rule out the possibility of long-run reversion towards a (possibly changing) mean, as discussed below.

Inflation rate variability

The variability of inflation rates, as measured by year on year changes, varied widely over time. Variability was generally much higher during the period prior to around 1920 than post 1920, and there was some evidence that high variability was associated with periods of high inflation.

The distribution of inflation rates was not accurately represented by a normal or log normal distribution. A particular feature noted was occasional sharp jumps in the level of inflation rates, with the size of these occasional jumps being outside the range that might be expected based on a normal distribution. Generally, in the 20th century, the jumps were positive rather than negative.

Mean reversion

The results of regression analysis suggested that the rate of inflation tends to regress towards current long run expectations. The coefficient of reversion was of the order of 0.85 up to 1920 and approximately 0.5 post 1920.

For example, if current bond yields are implying a long run inflation rate of 5% pa and current inflation is 15% pa, a reversion rate of 70% results in an estimate for next year's inflation of 8% pa.

A regression analysis suggested that the reversion parameter of 0.5 was largely unaffected by the exclusion of inflation rate jump outliers.

Inflation rate predictability

There is an increasing 'funnel' of doubt with regard to future inflation rates. An analysis of economic forecasts suggested that one year ahead forecasts were twice as accurate as forecasts for inflation rates four years into the future.
Analyses of interest rate variability can also be used to gauge inflation rate predictability, since the level of interest rates provides an indicator of inflation expectations. For example, the variance of the series $\ln(i_{t+1}/i_t)$ can be compared with

that for the series ln (i_{t+n}/i_t), where i is the yield on government bonds of any specified term. Generally, it appears that variance increases with the time difference. This characteristic is presumably a function of inflation rate regime, so that it may not hold during the currency of any specific regime.

Description of the Inflation Model

The developed model consisted of two components, one reflecting inflation expectations as derived from an analysis of the yield curves of index-linked and conventional gilts, and the second to represent future inflation uncertainty.

Expected inflation

The index-linked and conventional gilt yield curves were analysed in order to provide estimates of future rates of inflation, by comparing the level of implied forward real and conventional yield curves. The difference does not, however, represent unbiased estimates of future inflation since:

- some investors in the gilt market, for example life offices, are taxed on part of their income receipts. Index-linked bonds may provide a lower pre-tax real yield than conventional bonds because the inflation component of the total return on index-linked bonds is typically not taxed.

- dated conventional bonds are subject to risk in that they could decline in value if there is an unexpected increase in inflation. Cash instruments are broadly unaffected by unexpected inflation because yields adjust on a continuous basis. The prices of index-linked gilts are also largely unaffected by changes in inflation expectations. Most studies show the yield curve to be upward sloping in a typical year and the average excess yield on long dated bonds compared with short term bonds may provide an indication that an excess return is required to compensate investors for bearing inflation risk.

- forward yield curves provide a biased estimate of expected interest rates as a result of stochastic variation. A numerical example may be useful. Consider three equally likely scenarios with interest rates at 7% pa, 10% pa and 13% pa. The expected or mean rate of interest is 10% pa. The value of a 30 year zero coupon bond is:

$$100 \times 1/3 \times (.1314 + .0573 + .0256) = 7.14$$

  The spot rate is 9.2% pa, 0.8% pa lower than the expected interest rate. When using a stochastic model one needs to adjust spot rates upward to account for this volatility effect. The effect increases over time so that the impact on forward rates is higher than on spot rates.

For the first two reasons, the real yield on conventional bonds may exceed the yield on index-linked bonds. In the analysis that follows, these effects have been modelled by assuming that the term premium increases from 0.8% pa in the first year to a constant level of 2.0% pa from year seven. The term premium has been taken to represent the excess return on long dated gilts compared with the returns on index-linked gilts.

The difference between implied inflation and expected inflation, the volatility adjustment, was estimated as the addition to nominal forward rates such that the mean present value based on stochastic projections of these rates replicates the current price structure of conventional bonds.

Variability of inflation

The model allows for variations in inflation around the expected levels.

The model is fully described in Appendix 1. In summary, projected inflation for year T in the future, before addition of the volatility adjustment, is set equal to:

- implied inflation as assessed from the yield curve analysis, plus
- an uncertainty term which increases with time T (up to a maximum set at 15 years) and depends on which of the K scenarios is being followed. This term allows for the fact that expectations can change. Effectively, each scenario is characterised with a different underlying level of inflation, plus
- an uncertainty term which reflects variability of inflation along a scenario. This uncertainty is also assumed to increase with time T (up to a maximum set at 7 years), plus
- shock terms which allow for the possibility of substantial positive or negative jumps in the inflation rate, less
- a reversion term which brings inflation back towards the trend rate for the scenario.

An issue which arose during the development of the inflation model related to the frequency with which negative inflation arises in the model. Given an increasing 'funnel of doubt' and an average rate of inflation of approximately 4% pa, negative inflation is expected to arise with increasing frequency over time, ultimately arising in some 20% of scenarios. This frequency can be reduced in the model by reducing some or all of the variability parameters or increasing the length of the low uncertainty periods. However, there are arguments to support a proportion of negative inflation periods and in this context it is perhaps worth mentioning recent press comment exploring the implications this has for the swaps market. Documentation is currently being examined to ensure that the fixed payer will also pay the (negative) floating rate coupons during a negative interest rate period. This feature is important for the model since interest rates are typically higher than inflation rates. Two particular arguments which should be considered are:

- There is an underlying economic argument for a rational economic policy to keep the money supply fixed at a constant level, so as to minimise nominal shocks to the economy and to minimise uncertainty. With a constant money supply, inflation would have a negative mean. A key benefit would be that the information flow arising from fluctuations in supply and demand would transmit much more quickly and transparently to economic agents, thereby enhancing welfare. However, the vested interests of the central banking authorities and central governments run counter to welfare enhancement in this regard, unless competitive pressures for a sound money policy take hold. In the model some 15% of scenarios have a negative overall inflation and this is implicitly the probability assigned to the circumstance that governments will adopt an optimal monetary policy. The remaining approximately 5% of negative inflation occurrences arise as a result of random fluctuations from time to time.

- The frequency of negative inflation rates also needs to be viewed in a historic setting and having regard to the reduction in inflationary expectations that has occurred in the last decade. Prior to abandonment of the gold standard, inflation had a mean of approximately 0% pa and was frequently negative. Naturally, during periods of higher inflation, the likelihood of negative inflation reduces. Since we cannot be sure what regime the government will introduce it is appropriate to incorporate both normal random fluctuations and regime shifts into the model. The overall level of standard deviation arising from the model is in fact lower than the standard deviation of the outturn of inflation over the last 80 years.

Besides consistency with historic data, the frequency of negative inflation events is not out of line with that which arises in other models, for example, the Dyson/Exley (1995) and Smith (1996) models.

For no particularly good reason, other than pragmatism, the possibility of hyper inflation was not built into the model. Appendix 2 provides summary results from a stochastic projection of 200 scenarios.

Fitting the Model

The form of the model was chosen having regard to observed and intuitive characteristics of the historic inflation rate series, and parameters constrained to fit data indicative of market expectations.

With regard to historic fit, the parameters were chosen primarily on the basis of reasonableness, and by inspecting the data, rather than using detailed statistical techniques.

The result of using statistical techniques could be better central estimates of the parameters of the underlying distribution and of the range of likely values for these parameters. However, it should be borne in mind that inflation rates are ultimately under Government control and there is therefore a limit to the amount of information that can be gleaned from analysis of historic data. Statistical techniques may be inappropriate if it is believed that the choice of regime to be applied in the future differs from that which has applied in the past. Whilst not wishing to minimise the importance of using statistical analysis, the use of ad hoc and judgemental techniques is also necessary.

Another issue requiring consideration is trending in the expected rate of inflation. For some typical parameter values, expected inflation is calculated to be under 4% pa up to projection year 30, but trends up to approximately 5% or 6% pa between years 60 and 80. Trending is common in many interest rate models, but suggests that some underlying characteristic has not been appropriately factored in. This was not a problem for the central assumption set at 31 December 1995, but needs to be watched out for in sensitivity testing.

Using the Model

The model provides a 'central' set of future stochastic interest rate scenarios. However, one can envisage a wide range of alternative plausible scenarios. One approach to allowing for this uncertainty is to present results based on the central assumption set and also on a number of sensitivities, each sensitivity corresponding to alternative choices for the model parameters. For example, in relation to the term premium, one sensitivity could assume a term premium of 1% pa and a second could assume 2½ % pa, given a central assumption of 2% pa. Appendix 3 lists the central set of parameters and also provides this author's view of a plausible range for each.

Experience with using the model will enable the number of sensitivities to be presented to be reduced.

Comparison with Alternative Models

A number of criticisms could be levelled at the model:

- Statistical models with few parameters are, other things being equal, generally to be preferred to models with many parameters. The developed model has a number of parameters and it could be argued that a simpler model would be better. The author's view is that there is no strong underlying logic for inflation to follow one process rather than another and notes that there have been a number of 'regimes' in the past. The user can input a range of parameter settings to correspond to his views as to subjective possibilities for the future. For example, the paucity of pertinent historic data makes it difficult to assess the size and frequency of positive inflation shocks, even if one believed that the past was a guide to the future. Parameters have been included to facilitate this process. To take another example, many existing models including the Exley/Dyson and Smith models, do not include a reversion parameter. The results allowing for alternative reversion parameters could be used to gauge the validity of conclusions drawn from using these other models.

- Another issue relates to statistical testing against historic data. The author questions the relevance of historic means, variances and other distributions parameters as predictors for the future. For example, why should variance over the next fifty years be related to variance over the last (say) hundred years, as might be expected from a standard approach to model fitting? For this reason, although parameter values have been selected having regard to historic data, it is suggested that an alternative approach to the fitting of parameters could be adopted. That is, parameters could be selected to track the volatility and other aspects of the distribution of market prices of conventional and index-linked gilts and derivative securities.

The question arises as to why this model should be used in preference to existing models. The following points provide examples of model features which differ from one commonly used model, the Wilkie (1995) model:

- the expected inflation rate is chosen to be consistent with gilt yield data

- forecasting ability is recognised by introducing a low uncertainty period: the standard deviation in year one is 1.3% compared with 4.3% for the Wilkie model

- uncertainty resulting from sudden jumps has been introduced

- the Wilkie model parameters have been fitted to one observation period, 1923 to 1994. The parameter k_2 introduces an uncertainty element to reflect the fact that the period 1923 to 1994 is just one outcome of many possible scenarios. In other words, there is no fixed long run rate of inflation towards which inflation in all scenarios trends. The trend rate of inflation is different for each scenario. For this reason, the developed model projects higher long-term uncertainty. For example, the standard deviation of inflation estimates after 30 years is approximately 5% pa compared with 4% pa for the Wilkie model and the standard deviation of the scenario means over 80 years is approximately 4% pa compared with of the order of 1%.

The variance characteristics of the model are reasonably close to those incorporated in the Dyson/Exley (1995) model, which, however, is of a random walk form and does not include sudden jumps. The variance characteristics also bears some resemblance to those used in the Smith (1996) equilibrium model, which incorporates a random walk and includes jump characteristics. These two models do not allow for reversion effects.

Model Applications

The model has been and could be used for a number of different applications, for example:

- to project and illustrate likely ranges of inflation rates in the future
- to provide summary statistics of the distribution of possible future inflation rates, for example, as to probabilities of very high or very low inflation
- to use as a base for a stochastic investment model of nominal equity, cash or fixed interest returns
- to assist with the pricing of inflation linked securities and derivatives.

Since the underlying distribution of future inflation rates is in fact unknown, the author would suggest presenting results on both the "central" parameter set and using the sensitivities.

Refining the Model

The author believes that there is scope for refining the model, for example:

- building-in interactions with other variables, such as interest rates, other economic policy indicators and commodity prices, using modern time series analysis tools
- better reflecting the tendency of inflation rate variance to change over time, and the characteristics of inflation rate jumps

- incorporating varying real rates of interest

- facilitating the input of market derived data on interest rate volatility

- modelling log (1+inflation), rather than inflation itself, to facilitate algebraic treatment within an economic context

- examining whether the model can be respecified in algebraic form.

Conclusions

Inflation rates are difficult to model because of the radical shifts in regime which occur from time to time. However, analysis of the historic data suggests a number of desirable properties of a good model. Since each feature will have a different economic impact according to the application being considered, the inflation model described in this paper is somewhat complex (there are many parameters). At the same time, each parameter has a clear interpretation so that the model can be readily used to accommodate alternative views.

The mean future rates of inflation are set based on the yield structure of index-linked and conventional gilts. Parameters for the model need to be selected from time to time having regard to volatility of primary securities and implied volatility underlying options on fixed interest and floating rate securities, for example in the swaps markets (since interest rate levels provide some indication of future inflation rates). The model is therefore broadly consistent with an arbitrage-free and rational expectations model of the economy. It is not fully arbitrage free because of possible internal inconsistencies between calibration of the model at successive dates. That is, in common with many other models, the probability distribution of inflation at some point in the future calculated now may differ from the implied probability distribution at the same date calculated based on rolling the model forward over the period.

Alternative approaches to modelling inflation, for example using a standard distribution function (Normal, Poisson, etc) or mathematically tractable equations, have a number of advantages to the methodology adopted here. In particular, closed form algebraic solutions facilitate the consistent pricing of assets into the future, not just at the current appraisal date. For other applications, algebraic solutions are not necessary, or cannot be found for models of appropriate complexity. The parameter oriented model described here may provide more realistic sets of results capable of ready interpretation.

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Appendix 1

Description of the Inflation Model

The inflation model is described by the following equations for inflation in year T for scenario K:

Implied inflation: $j_{TK} = e_T + C\,(\,s_1(\,T)\,Z_{TK} + s_2(T)\,Z_K + s_3\,(T)\,Z_1 + s_4\,(T)Z_2)$
$- K_5\,(\,j_{(T-1)K} - e_{T-1} - s_2(T-1)\,CZ_K)$, where

Expected inflation: $(1+j_{TK})(1+V_T)-1$, where V_T is the volatility adjustment required to adjust for the difference between expected and forward rates in a stochastic projection.

e_T is the implied rate of inflation in year T, derived from an analysis of forward rates in the index-linked and conventional gilt markets.

C is the yield on undated bonds (3½% War Loan) at the date of the appraisal. This is applied to all the variability components since many studies suggest that high bond yields are associated with periods of high inflation, and since there appears to be some association between interest rate levels and interest rate volatility.

$s_1(T)Z_{TK}$ is a term to reflect increasing variability of the inflation rate series along each scenario. $s_1(T)$ is assumed to be of the form K_1ÖT/ÖK_{10} where K_1 and K_{10} are constants and, in the expression, T is limited to a maximum value equal to K_{10}. Thus, the standard deviation increases up to K_{10}, which was set equal to 7 years, and is then constant. Z_{TK} is a normal variate with mean zero and standard deviation equal to one, and is chosen separately for each scenario and year of projection.

With C = 8.5%, setting K_1 = 0.20 gives rise to total uncertainty in inflation (excluding the effect of shock terms discussed below) of approximately 0.6% pa for the first projection year increasing to approximately 1.7% pa for year seven.

$s_2(T)Z_K$ is a term which reflects the possibility that inflation will not follow the expected trend and could, for example, decline to zero or increase. Assuming randomly distributed changes, uncertainty increases with ÖT and, for this reason, $s_2(T)$ is assumed to equal K_2ÖT/ÖK_{11}. Z_K is a normal variate with mean zero and standard deviation equal to 1 and is chosen separately for each scenario. With current expectations of inflation of say 4% pa, setting K_2 equal to 0.22 gives a one standard deviation range for expected inflation of 2½% pa to 5% pa in 7 years' time and 2% pa to 6% pa in 15 years' time. After 15 years, uncertainty in trend inflation has been assumed to remain constant.

$s_3(T)Z_1$ is a term allowing for exceptional one-off increases to the inflation rate. The frequency and severity of positive inflation shocks can be varied. Based on an examination of inflation rates in the UK since the early 1920's, the frequency was set as one year in ten and the severity was assumed to equal an increase in inflation of 10% in the year. An adjustment was made to ensure that the average contribution to mean inflation was zero and to reduce the severity of shocks in the first K_{10} projection years.

$s_4(T)Z_2$ allows for negative shocks. In practice, an examination of the inflation data series since the early 1920's showed a lower frequency and severity of negative shock terms. The shock severity was set at -5% with frequency one in forty years.

$K_5(j_{(T-1)K} - e_{T-1} - s_2(T-1)CZ_K)$ reflects the results of regression analyses which suggest that short term fluctuations in the inflation rate dampen over time. The parameter K_5 was estimated as 0.5 in the regression.