A New Approach for Identifying Seasonal Autoregressive Time Series Forecasting Models

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Abstract

One of the most powerful and widely used methodology for forecasting economic time series is the class of models called seasonal autoregressive models. In this article we present a new approach not only for identifying the order of such models, but also the degree of differencing required to induce stationarity to the data. Once the order of the process is determined the identified structure is tested to see if it can be simplified. Because inflation affects many different aspects of an insurer´s financial situation, it is important to study the generating mechanisms of inflation. For this reason we also wanted to illustrate the performance of the identification method using inflation series from a number of industrialized countries.

Résumé

L´une des méthodes les plus puissantes et les plus utilisées pour prévoir les séries chronologiques de l´économie est la catégorie des modèles que l´on appelle les séries chronologiques autorégressives et saisonnières. Dans cet article nous allons présenter une nouvelle façon pas seulement d´identifier l´ordre de ces modèles, mais aussi d´identifier le degré de différentiation nécessaire pour provoquer la stationnarité des données. Une fois l´ordre du processus déterminé, le structure identifié sera testé pour voir s´il est possible de le simplifier. Etant donné que l´inflation a l´effet sur plusieurs aspects de la situation financière d´un assureur, il est important d´étudier les mécanismes qui produisent l´inflation. Pour cette raison nous avons également voulu illustrer le fonctionnement de la méthode d´identification en utilisant des séries d´inflation d´un certain nombre de pays industriels.

Keywords

Identification, order determination, residual white noise test, seasonal models, autoregressions, estimation, nonzero parameter elements, order selection.

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1. Introduction

One of the most powerful and widely used methodology for forecasting economic time series is the class of models known as seasonal autoregressive processes. In the past few years Pukkila, Koreisha, and Kallinen (1990) and Koreisha and Pukkila (1993a, 1993b, 1995) proposed new order identification methods for ARMA(p,q), ARIMA(p,d,q) and VAR(p) structures based on a new white noise test and on linear estimation methods. Their studies have shown that for samples of 100 or more observations these procedures can quite accurately identify the process governing the behavior of the data. In this article we show how their residual white noise test can be augmented and used, not only to identify seasonal autoregressive models (additive and multiplicative), but also of the degree of differencing (regular and seasonal) required to induce stationarity in the data. The 2-stage procedure proposed here requires sequential application of the white noise test to seasonal and regular autoregressive structures of residuals series of models fitted to the data. Its performance, as we shall show, is also remarkably good. Unless otherwise stated we use the notation and conventions of Box and Jenkins (1976).

The article is organized as follows: in section 2 we describe the new identification methodology; in section 3 we demonstrate the performance of the methodology by developing inflation forecasts for 10 industrialized countries; and in section 4 we offer some concluding remarks.

2. The Identification Methodology

For nonseasonal univariate processes the new identification procedure consists in systematically fitting increasing-order ARIMA(p,d,q) structures to the data using fast linear estimation procedures, and then checking if the residuals behave like white noise. Since a white noise process can be viewed as an AR(0) process, and conversely a non-white noise process can be approximated by an AR(k) process, $k>0$, then, if, on systematically fitting increasing order AR(k) models to the residuals associated with a fitted ARIMA(p,d,q) structure it is ascertained that the best fit occurs for $k>0$, then the hypothesis that the residuals behave like a white noise process is rejected. A different ARIMA(p,d,q) structure must be fitted to the data and its residuals re-tested. The order of the process is determined when the residuals associated with the ARIMA(p,d,q) process can be shown to be white noise. The residual white noise test, i.e. the determination of the order of the AR(k) process governing the residuals of the ARIMA(p,d,q) structure fitted to the observed series $z_1, z_2, \ldots, z_N$, namely $\hat{a}_t(p,d,q)$, is performed on the basis of minimization of functions of the form,

$$\delta(k) = N \log \hat{\sigma}^2_{k,p,d,q} + k\, g(N),\ k=0,1,2,\ldots k^*, \tag{1}$$

where $\hat{\sigma}^2_{k,p,d,q}$ is an estimate of the residual variance associated with the AR(k)th fit of the series $\hat{a}_t(p,d,q)$; g(N) is a prescribed nonnegative function, and k^* is an apriori determined upper bound which is typically set equal to $\sqrt{N}$. (This number has been determined empirically; see Koreisha and Pukkila, 1990, and Koreisha and Yoshimoto, 1991.) Note that it might not be necessary to evaluate (1) for all $k \le k^*$ since at the first opportunity which can be demonstrated that the residuals do not follow an AR(0) process, the order of the fitted ARIMA structure would be changed or increased. However, to show that the residuals from a fitted structure are white noise it is necessary to evaluate (1) for all values of k up to k^*. Koreisha and Pukkila have referred to this identification approach as the residual white noise autoregressive (RWNAR) order determination criterion, and when appropriate, a suffix denoting the particular penalty function g(N) that is used in (1) is appended to RWNAR, e.g. RWNAR-BIC, if g(N) = log N. The identification performance of these approaches has been shown to be remarkably good, particularly when compared to other procedures. (See Koreisha and Yoshimoto (1991) for a comparison in identification performance of ARMA(p,q) structures between the RWNAR and approaches such as the corner method and extended sample autocorrelations, and Koreisha and Pukkila (1993) for a contrast between their approach and other order determination criteria in the case of VAR(p) models.)

It should be readily apparent that by construction the RWNAR-i procedures as well as traditional order determination criteria which are based on the minimization of functions of the form,

$$\delta(p,d,q) = N \log \hat{\sigma}^2_{p,d,q} + (p+q)\, g(N),\ p=0,1,2,\ldots,p^*;\ \ q=0,1,2,\ldots,q^*, \tag{2}$$

where $\hat{\sigma}^2_{p,d,q}$ is an approximation of the residual variance associated with the fitted ARIMA(p,d,q) structure, and p^* and q^* are respectively apriori determined upper AR and MA order limits, would have great difficulty in identifying structures with gaps or seasonal or mixed seasonal models. These methods more than likely would severely underestimate the model order because as the order of the process being tested is increased the reduction in the residual variance (if any), due to the gap in the parameter structure, would be minimal in relation to the nonnegative penalty function g(N) that is increasing at a rate commensurate with the order of the fitted process. These procedures, however, can be modified to yield not only the correct order of the models with gaps, but also to identify the degree of regular and seasonal differencing necessary to induce stationarity in the data. In this study we will focus on the identification of AR processes, i.e., q=0.

Suppose that the seasonal span were known to be s. Moreover, suppose that the white noise test is first performed just on seasonal lags Ks, K =1,2,..., of residuals from ARI(p,d) x SARI(0,D)s structures fitted to the data, namely $\hat{a}_t(p,d,D)$. (Multiplicative models with P>1, as will be discussed later, are identified after the order of a fully extended model is identified.) In other words the check for "whiteness" is first conducted on seasonal lags Ks, and evaluated on the basis of minimization of functions of the form,

$$\delta(K) = N\log\hat{\sigma}^2_{Ks,p,d,D} + K\ g(N),\ K=0,1,2,...,K^*, \tag{3}$$

where $\hat{\sigma}^2_{Ks,p,d,D}$ is an estimate of the residual variance associated with the SAR(K) fit of the series $\hat{a}_t(p,d,D)$, and K^* is an apriori determined limit related to N. If the residual white noise hypothesis is rejected, then the order of process fitted to the data is increased and the test is repeated on the new set of residuals. If the residual white noise hypothesis cannot be rejected based on successive fits of SAR(K) structures to the residuals of the model fitted to the data, then the test is performed on the basis of consecutive lags, i.e. by determining the order of k which minimizes,

$$\delta(k) = N\log\hat{\sigma}^2_{k,p,d,D} + k\ g(N), k=0,1,2,...,k^*, \tag{4}$$

The procedure terminates when it can be ascertained that (3) and (4) are minimized for k=K=0. In this study like in Pukkila, Koreisha, and Kallinen (1990), and for reasons elaborated in Koreisha and Yoshimoto (1991) we set g(N) = log N.

The sequence in which parameters were included in the model used to fit the data was determined from the very simple formula, j=d+Ds+p, for j=1,2,.... If s>0, say s=4, then when j=1, two structures were tested, ARI(1,0) and ARI(0,1); when j=2, three structures were evaluated, ARI(2,0), ARI(1,1), and ARI(0,2); and when j=4, all the model structures enumerated below were examined:

p	d	Ds
0	4	0
1	3	0
2	2	0
3	1	0
4	0	0
0	0	4

Savings in the number of calculations may be possible using other rules or by arbitrarily excluding differencing orders not generally found with real data.

This 2-stage procedure (2-RWNAR) will yield the order of the AR process governing the behavior of the data, but, as can be easily surmised, the identified model structure may not be very parsimonious. In this study we used the t-ratios of the parameters of the identified structures to determine whether it is possible to delete some parameters from the fully extended form of the model without affecting the whiteness of the residuals. We used the following "build-up" strategy: First a single parameter model based on the order of the identified structure is estimated, and the 2-RWNAR test is performed on the residuals of this model. Then a new lag based on the parameter which had the largest t-value (in absolute terms) among those in the originally identified structure is added to the single parameter model and the 2-RWNAR test is performed on the residuals of the new model. Incorporation of additional lags based on t-ratios continues until all lags are reintroduced into the model structure. Among the model structures which transform the data into white noise, the simplified structure minimizing the usual (traditional) order determination criterion based on the corresponding number of estimated parameters is selected.

Model structures selected in this fashion can contain parameters whose t-ratios are less than $|2|$. Consequently, the simulation results which shall be reported in the next section will be conservative. There are many other procedures that can be used to eliminate parameters from a model. This is simply one which we found to work well.

Multiplicative model structures are also tested if the order of the identified model is greater than or equal to s, and if there is a non-zero parameter at a seasonal lag (1s, 2s, etc.) and at least one other at a non-seasonal lag. Depending on the identified order of the model selected by the 2-RWNAR method discussed above, progressively higher order non-seasonal AR structures up to order s-1 are added to the SAR(P) model to determine if the residuals from the thus constructed multiplicative model are white noise. Among the subset of multiplicative structures which yield white noise residuals we selected the one which minimized the order selection criterion based on the appropriate values for p and P. We then checked if the selected multiplicative form yielded a lower value for the minimizing criterion than the best parsimonious (gapped) additive structure to see which model form best fitted the data.

There are several methods that can be used to estimate autoregressions. These include among others: the Yule-Walker method, successive prewhitening using low order autoregressions (Koreisha and Pukkila, 1987a and 1987b), ordinary least squares, and Burg's algorithm (Burg, 1975). In this study Burg's algorithm was used to obtain estimates for $\phi_1, \phi_2, \ldots, \phi_p$ and to generate $\hat{a}_t(p,d,D)$. This is because Burg's procedure yields estimates which are consistently less biased than say Yule-Walker estimates and because it always produces stationary autoregressive estimates. For simplicity, speed and ease of calculation, however, estimates for $\sigma^2_{Ks,p,d,D}$ and $\sigma^2_{k,p,d,D}$ in (3) and (4) were obtained using Yule-Walker fits to the SAR(K) and AR(k) structures associated with the $a_t(p,d,D)$ series. (See also Pukkila and Krishnaiah, 1988; Pukkila, Koreisha, and Kallinen, 1990; and Koreisha and Pukkila, 1991a.) Even though the Yule-Walker method is known to produce biased parameter estimates when the generating process is far from white noise (Lysne and Tjostheim, 1987), for processes close to white noise the statistical properties of these estimates are very similar to those obtained from less biased approaches such as least squares and Burg's algorithm (Pukkila, 1988). Since in testing the null hypothesis of white noise there is no interest in the actual estimates of the autoregressions in (3) and (4), only in departures from white noise, it is not critical, as will be shown in the next section, to use more exact estimation procedures.

To obtain parameter estimates of more parsimonious models, i.e. models with gaps, we used a hybrid of the Burg and Yule-Walker procedures. From the fully parametrized model structure estimated via Burg's algorithm, estimates of the derived partial autocorrelations were transformed into autocorrelations. These were then used in the Yule-Walker equations associated with the corresponding "gapped" (constrained) autoregressive model to obtain estimates for ϕ_i.

Multiplicative models were estimated in three stages. First, the data were filtered with an SAR(P) model using the estimates obtained from the fully parametrized identified structure. Estimates for the non-seasonal AR(p) parameters were obtained using Burg's algorithm on the residuals of the SAR(P) fit. Finally new estimates for the seasonal AR parameters were obtained by filtering out the thus derived AR(p) component using Yule-Walker equations for a seasonally gapped structure, i.e. the hybrid method discussed above. (This estimation procedure, as it should be apparent, can be allowed to iterate to generate more refined estimates.)

3. Application to Inflation Series of Some Industrialized Countries

The small sample performance of this 2-stage identification method has been documented in Koreisha and Pukkila, 1995, using simulated data for a variety of model structures, sample sizes, and degrees of differencing. In this paper using monthly inflation series for 10 industrialized countries spanning the periods from 1978 to 1995 (see Appendix for more details on sources and time periods covered by each series), we study the forecast performance of the models identified by the 2-RWNAR approach. Each series contains 200 observations; 188 observations were used to identify and estimate the model for each of the countries. The remaining 12 observations were used to contrast the forecast performance of the identified structures. Models were constructed based on actual and logged data.

Table 1 contains the identified model structures for each of the countries based on actual data along with some remarks associated with the residuals series from each model. As can be seen the data for all countries require some degree of differencing to induce stationarity. For most series simple first order differencing is sufficient to induce stationarity; data for the US and France, however, require second order differencing. With the exception of the U.K., the residual autocorrelations for each of the models, as measured jointly by the Q statistics appear to be zero, suggesting there is no additional information that can be extracted from the data to append to the identified model. Some series had a few individual autocorrelations with t-ratios greater than 2 in absolute value, many, however, at lags that did not make much intuitive sense. The Q statistics for the U.K. inflation series, on the other hand, are significant at the 5% level, and at least three individual autocorrelations (lags 5, 20, and 24) also appear to be significant. Incorporation of autoregressive parameters at lags 5 and 20 did not noticeably affect the behavior of the residuals. Moreover, these additional parameters were not found to be statistically significant (at the 5% level). Adding an autoregressive parameter at lag 24, however, changed the residual autocorrelation function substantially: the Q(12) statistics increased to 34.1, but as anticipated the Q(24) statistics dropped to 47.1, and the t-ratio of the individual autocorrelation at lag 24 changed to 0.2. Significant (5% level) autocorrelations persisted at lags 2, 4, 5, and 6. The forecasts from this modified model, however, as shown in Table 2 are markedly inferior to the ones generated by the model identified by the 2-RWNAR procedure.

Looking more carefully at the forecast errors as measured by the percent deviations from actual (Table 2), we see that for all countries the first three step-ahead forecasts are considerably less than 1%. The deviations for 6-steps and 12-steps either stay the same or increase modestly for most countries. For Canada, Italy and the U.K. the deviations exceed 1% for the larger horizons. It should be noted, however that the data for these countries contained some of the greatest degree of variability in this study.

Figure 1 contains graphs of the forecasts for 12 periods along with their respective 95% forecast confidence limits. As can be seen the actual inflation values (post identification/estimation sample) appear to fall well within the forecasts 95% confidence interval, except for a few values for Italy and Canada which noted earlier contained data with a high degree of variability.

We also identifed forecasting models based on logarithms of the inflation series. With the exception of Finland, France, Germany, and the U.S. the identified model structures in terms of logs were the same as those based on actual data. The model structures identified for France and the U.S. in terms of logs were slightly more parsimonious than those based on actual data, ARI(4,2) and ARI(3,2) for France and the U.S. respectively. However, those identified for Finland and Germany changed considerably:

Finland:

$$\underset{(0.002)}{(1 - 1.052B} \underset{(0.007)}{- 0.325B^{12}} \underset{(0.007)}{+ 0.379B^{13})} \ln z_t = \underset{(0.000)}{0.015} + \hat{a}_t$$

Germany:

$$\underset{(0.068)}{(1- 0.296B} \underset{(0.071)}{- 0.140B^3} \underset{(0.076)}{- 0.096B^4} \underset{(0.071)}{+ 0.114B^5} \underset{(0.071)}{+ 0.071B^7}$$

$$\underset{(0.077)}{- 0.111B^8} \underset{(0.072)}{- 0.187B^9} \underset{(0.070)}{- 0.168B^{11}}) \ln z_t = \underset{(0.000)}{0.000} + \hat{a}_t$$

The logged Finnish data did not appear to require any degree of differencing to induce stationarity, but as can be seen the sum of the AR coefficients is very near one suggesting that an alternative model based on differenced series may also yield white noise residuals. The model structure for the German series contains 4 parameters that are statistically insignificant (5% level). Deletion of these parameters as we will show in Table 3 does not noticeably impact the forecasts.

It should be noted, however, that the residual autocorrelation from the models fitted to the logged data for most countries had higher Q statistics and more t-ratios of individual autocorrelations that were greater than 2 in absolute value than those from models fitted to the actual data. The Q(12) statistics for France and the US, for instance, increased to 28.6 and 28.3, respectively, suggesting that the first 12 autocorrelation values could not be jointly viewed as being zero; several of the low-lag residual autocorrelations had t-ratios that were larger than 2 in absolute value. The Q(12) statistics for the U.K. dropped to 20.4, but the Q(24) remained significant at 46.3.

The forecasts generated from these models, as shown in Table 3, are generally inferior to those obtained from the actual data particularly for longer horizons. Surprisingly the log transformation did not improve the forecasts for Canada, Italy, and the U.K. The deterioration of the Finnish forecasts must be due to the fact that the identified model appears to be almost non-stationary. Finally,it should be noted that, the 95% forecast confidence intervals, not shown here for the sake of brevity, widen as the forecast horizon increases at a much faster rate than those of the models based on actual data.

Table 1. Models Identified by the 2-Stage RWNAR Method Inflation Series for Selected Industrialized Countries. For each model residual remarks are given.

Austria

$$\underset{(0.056)}{(1-0.739B^{12})}(1-B)z_t = \underset{(0.038)}{0.105} + \hat{a}_t$$

t-ratio at lag 12 = 2.25
Q(12) = 15.9 Q(24) = 20.1 Q(36) = 31.2

Canada

$$(1-\underset{(0.067)}{0.180B^3}-\underset{(0.068)}{0.187B^4}-\underset{(0.073)}{0.391B^{12}})(1-B)z_t = \underset{(0.056)}{0.085} + \hat{a}_t$$

None of the t-ratios of individual autocorrelations on their absolute value were greater than 2.
Q(12) = 8.7 Q(24) = 19.9 Q(36) = 27.5

Denmark

$$(1-0.223B^{6}-0.393B^{12})(1-B)z_t = 0.194 + \hat{a}_t$$

(0.067) (0.067) (0.063)

t-ratio at lag 13 = 3.38
Q(12) = 13.1 Q(24) = 34.7 Q(36) = 53.6

Finland

$$(1-0.384B^{12})(1-B)z_t = 0.408 + \hat{a}_t$$

(0.074) (0.070)

t-ratio at lag 3=2.25; t-ratio at lag 13=2.63; t-ratio at lag 36=2.22
Q(12) = 15.7 Q(24) = 39.5 Q(36) = 60.9

France

$$(1+0.568B+0.596B^{2}+0.378B^{3}+0.346B^{4}+0.295B^{5}+0.058B^{6})(1-B)^{2}z_t= \hat{a}_t$$

(0.074) (0.083) (0.090) (0.090) (0.083) (0.075)

t-ratio at lag 12=2.13
Q(12) = 17.5 Q(24) = 30.4 Q(36) = 47.5

Germany

$$(1-0.286B)\,(1-0.355B^{12})(1-B)z_t = 0.115 + \hat{a}_t$$

(0.072) (0.072) (0.0259

t-ratio at lag 24=2.13; t-ratio at lag 26=2.22
Q(12) = 15.6 Q(24) = 28.8 Q(36) = 49.1

Italy

$$(1-0.171B-0.184B^{3}-0.444B^{12})(1-B)z_t=0.147+ \hat{a}_t$$

(0.064) (0.063) (0.064) (0.081)

None of the t-ratios of individual autocorrelations on their absolute value were geater than 2.
Q(12) = 9.6 Q(24) = 22.9 Q(36) = 32.8

Japan

$$(1-0.634B^{12})(1-B)z_t = \hat{a}_t$$

(0.54)

None of the t-ratios of individual autocorrelations on their absolute value were greater than 2.
Q(12) = 9.6 Q(24) = 22.9 Q(36) = 32.8

United Kingdom

$$(1\text{-}0.117B\text{-}0.621B^{12})(1\text{-}B)z_t = 0.117 + \hat{a}_t$$
$$\quad(0.057)\;(0.056)\qquad\qquad(0.057)$$

t-ratio at lag 5 =2.13; t-ratio at lag 20=2.22
t-ratio at lag 24=2.33
Q(12) = 25.0* Q(24) = 53.9* Q(36) = 94.9*

United States

$$(1+0.421B+0.436B^2+0.395B^3+0.325B^5+0.178B^5+0.343B^6)(1\text{-}B)^2z_t=0.008+\hat{a}_t$$
$$(0.070)\;(0.076)\;(0.079)\;(0.079)\;(0.076)\;(0.071)\qquad(0.019)$$

None of the t-ratios of individual autocorrelations on their absolute value were greater than 2.
Q(12) = 12.1 Q(24) = 22.5 Q(36) = 39.2

* Significant at least at the 5 % level.
Tabulated values for Q(12)=21.03, Q(24)=36.42, and Q(36)=51.00; for α=0.05.

Table 2. Percentage of Deviation from Actual of Several Step-Ahead Forecasts from Model Structures Identified by the 2-Stage RWNAR Method

	Forecast Ahead				
Country	1-Step	2-Steps	3-Steps	6-Steps	12-Steps
Austria	+0.031	-0.093	+0.196	-0.111	-0.270
Canada	+0.209	+0.342	+0.694	+1.653	+1.489
Denmark	-0.256	+0.177	-0.098	-0.192	-0.581
Finland	-0.024	+0.746	+0.703	+0.541	-0.709
France	-0.164	-0.409	-0.252	+0.020	+0.013
Germany	-0.062	-0.185	-0.238	-0.0152	-0.469
Italy	-0.076	+0.086	-0.015	+1.162	+1.700
Japan	+0.467	+0.593	+0.285	-0.589	+0.059
U.K.	+0.047	+0.338	+0.501	+1.085	+0.568
U.K.(other)*	+0.140	+0.606	+1.056	+1.588	+1.294
U.S.	-0.114	+0.073	+0.278	+0.519	+0.417

*Forecast deviations generated by the modified model:

$$(1\text{-}\ 0.124B - 0.478B^{12} - 0.284B^{24})\ (1 - B)\ z_t = 0.016 + \hat{a}_t$$
$$(0.056)\;(0.072)\quad(0.072)\qquad\qquad(0.061)$$

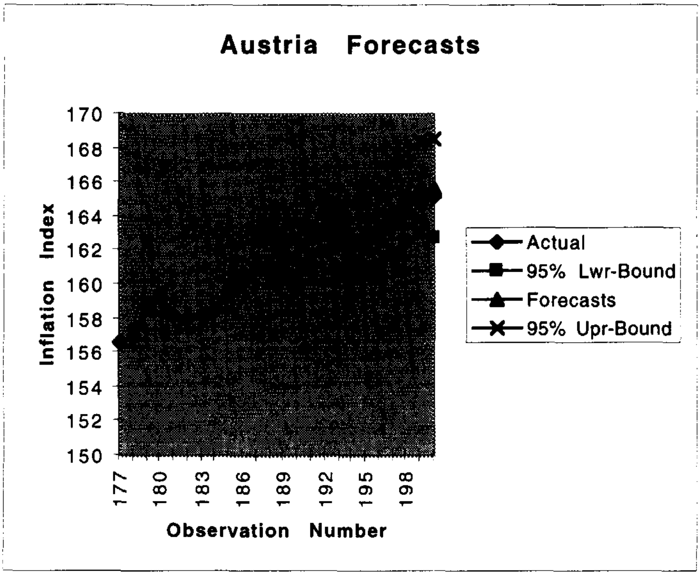
Table 3. Percentage of Deviation from Actual of Several Step-Ahead Forecasts from Model Structures Identified by the 2-Stage RWNAR Method Based on Logged Data

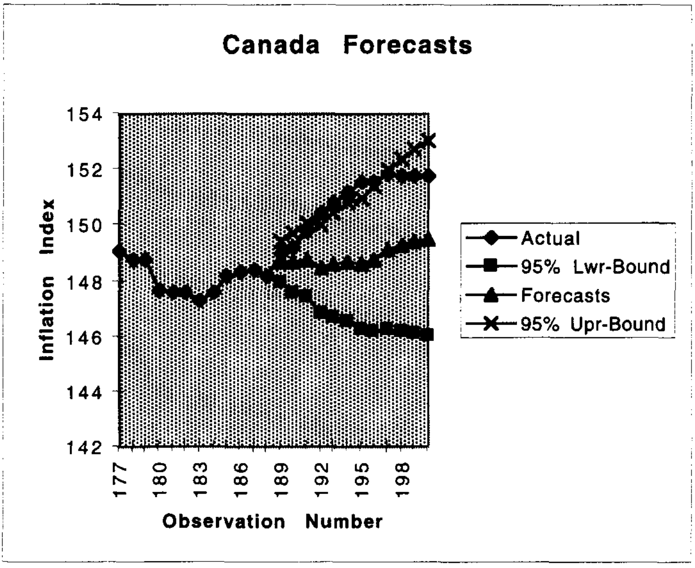
	Forecast Ahead				
Country	1-Step	2-Steps	3-Steps	6-Steps	12-Steps
Austria	-0.164	-0.380	-0.130	-0.661	-1.110
Canada	+0.204	+0.347	+0.716	+1.701	+1.700
Denmark	+0.072	+0.116	-0.164	-0.350	-0.407
Finland	+0.277	+1.333	+1.582	+2.341	+3.138
France	-0.121	-0.419	-0.307	-0.085	-0.251
Germany	-0.141	-0.405	-0.502	-0.343	-0.936
(other)*	-0.081	-0.325	-0.421	-0.203	-0.805
Italy	-0.084	+0.081	+0.014	+1.253	+2.015
Japan	+0.467	+0.575	+0.275	-0.593	+0.049
U.K.	+0.110	+0.475	+0.692	+1.545	+1.410
U.S.	-0.187	+0.012	+0.249	+0.585	+0.418

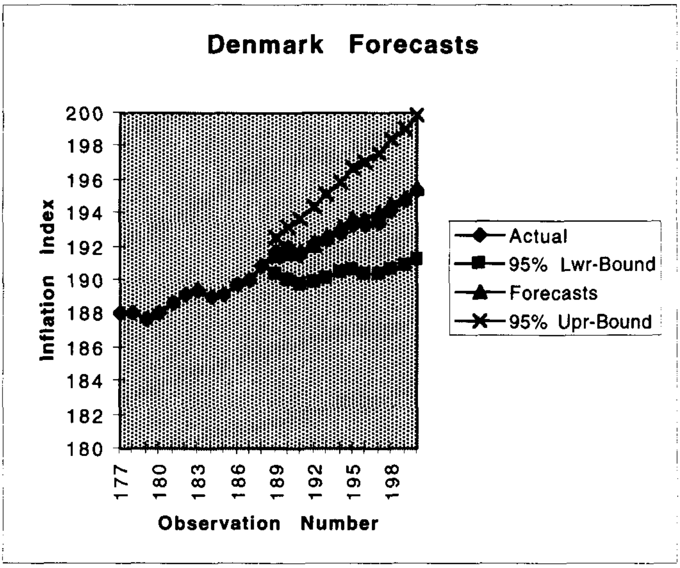
*Germany forecasts based on the model:

$$\underset{}{(1-} \underset{(0.068)}{0.299B} - \underset{(0.067)}{0.143B^3} - \underset{(0.067)}{0.201B^9} - \underset{(0.069)}{0.171B^{11}})\ \ln z_t = \underset{(0.071)}{0.001} + \hat{a}_t$$

Figure 1. Graphs of the forecasts for 12 periods along with their 95 % confidence limits.



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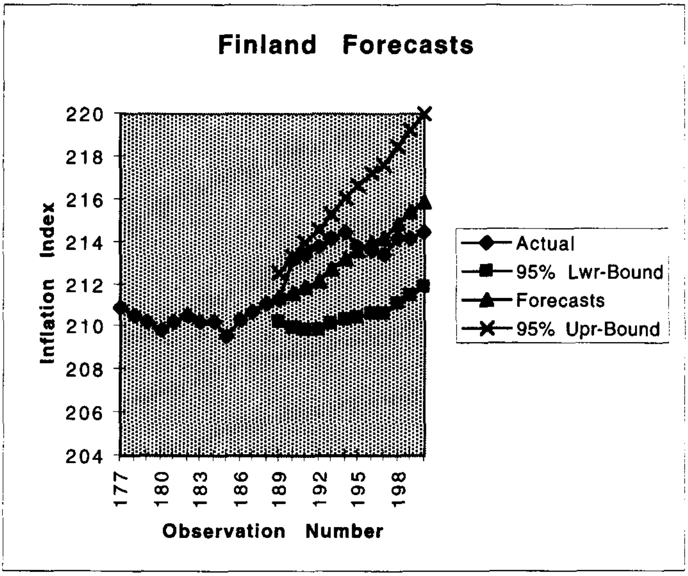
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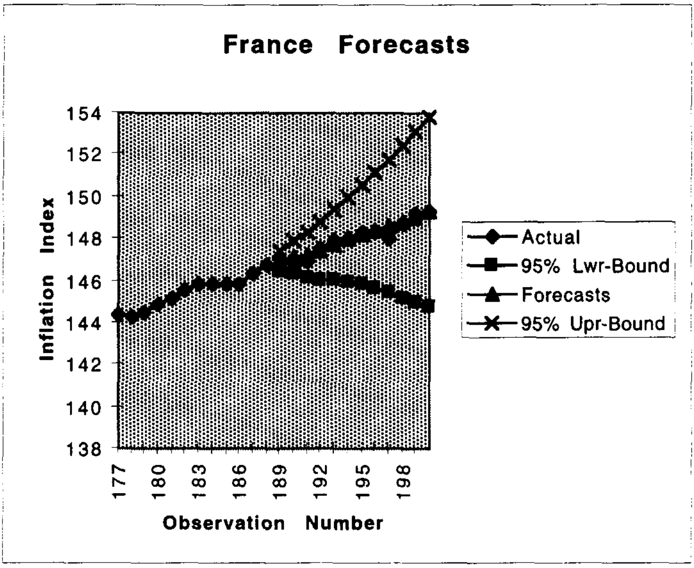
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Observation Number
Actual
95% Lwr-Bound
Forecasts
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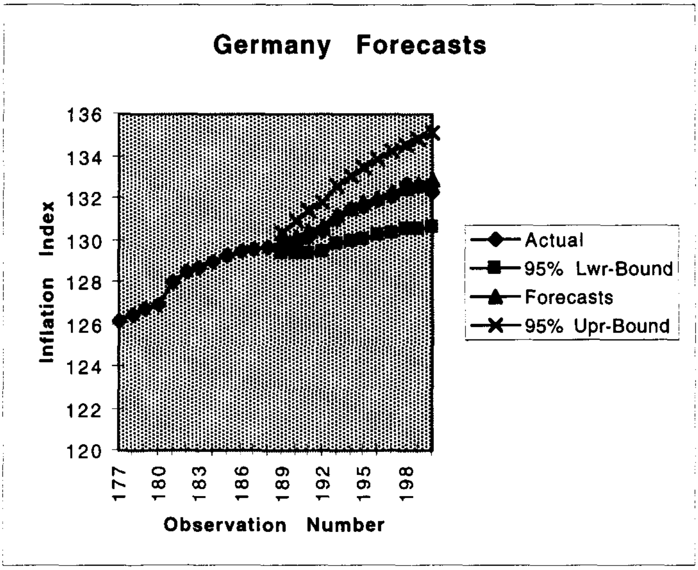
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Inflation Index
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Observation Number
Actual
95% Lwr-Bound
Forecasts
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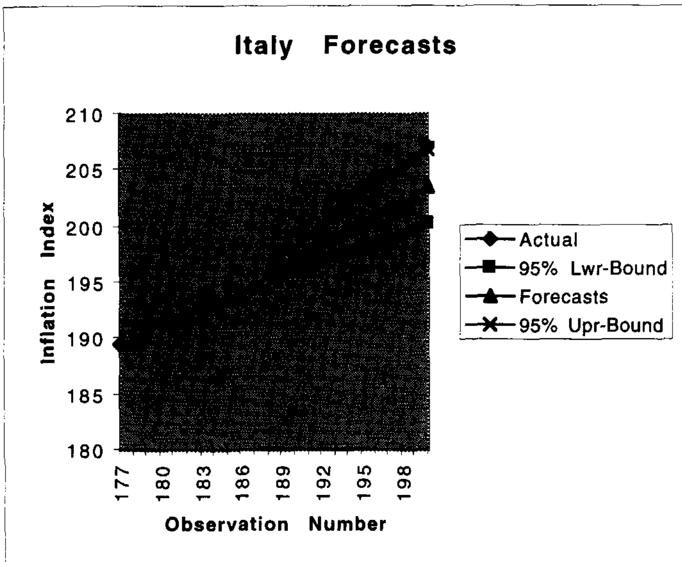
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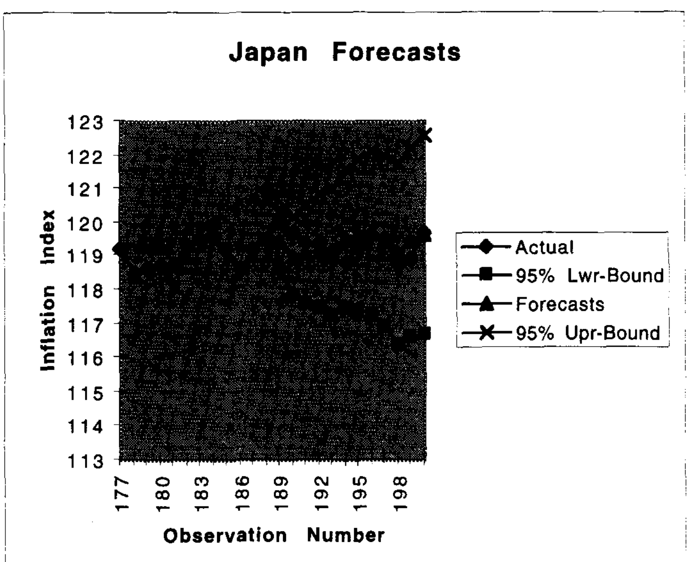
Actual
95% Lwr-Bound
Forecasts
95% Upr-Bound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Inflation Index
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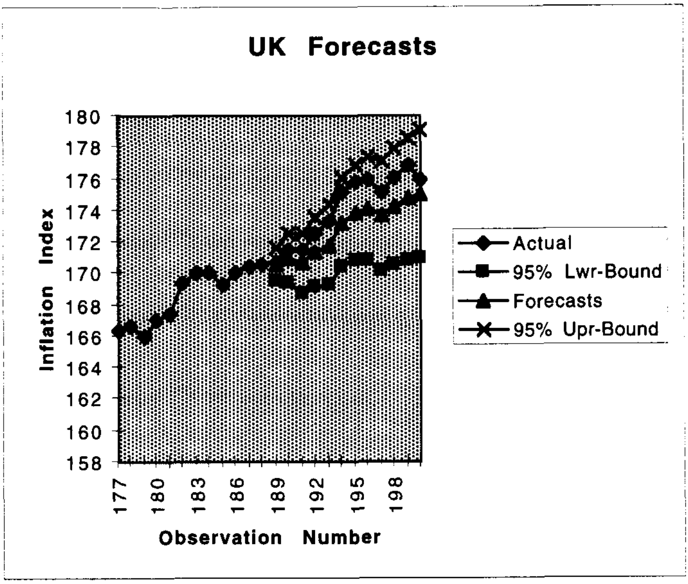
Observation Number
Actual
95% Lwr-Bound
Forecasts
95% Upr-Bound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Observation Number
Actual
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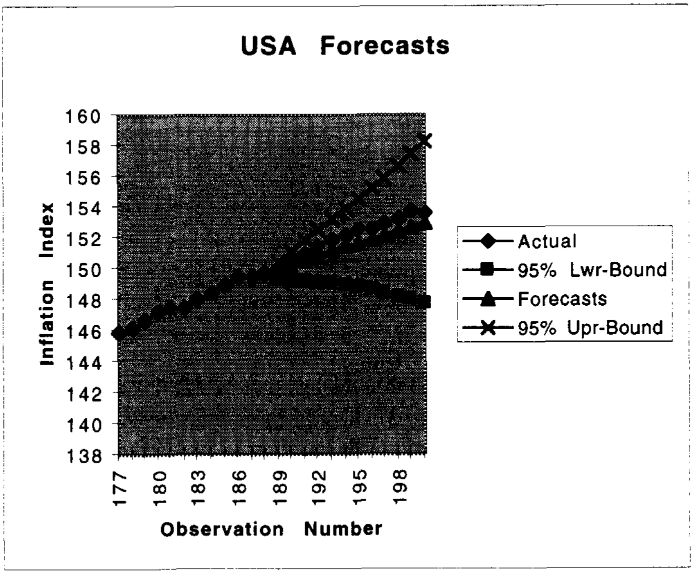
Forecasts
95% Upr-Bound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Inflation Index
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Observation Number
Actual
95% Lwr-Bound
Forecasts
95% Upr-Bound

4. Concluding Remarks

The 2-RWNAR approach appears to effectively identify structures of seasonal AR models that can be used to generate useful forecasts. The inflation forecasts, particularly those generated from models constructed from actual data for the 10 industrialized countries we studied were extremely accurate. In most cases the forecast errors as measured by the percentage deviation from actual were substantially less than 1% even for 12-steps ahead. Moreover, as noted earlier many of the search and estimation procedures that we used were based on experimentation. It is possible that more systematic approaches may yield even better results.

Appendix - Inflation Data

The data for US, UK, Canada, France, Germany, Italy, Japan were obtained from The University of Michigan clearing house of data. The World Wide Web (WWW)address for the file containing these data is:

gopher://una.hh.lib.umich.edu:70/00/ebb/indicators/BCIH/BCIH-15.dat

This file contains historical data provided by the US Department of Commerce. Data updates for some countries were found at:

gopher://una.hh.lib.umich.edu:70/00/ebb/indicators/BCIC/BCIC-15.dat.

These data have not been seasonally adjusted. The base year for the inflation index was not the same for all countries. They varied from 1982 to 1984.

The data for Austria, Denmark, and Finland were obtained from the United Nations Monthly Bulletin of Statistics, Volume 21, Part 1, April-June 1975, through Volume 49, No. 5-8, 1995 under the section "Consumer Price Index Numbers." The base year for all the indices of these countries is 1980. Indices for pre-1980 data were converted.

Two hundred observations were collected for each country. The specific time spans are listed below:

Austria	Jul 78-Dec 94
Canada	Mar 79-Oct 95
Denmark	Jul 78-Dec 94
Finland	Jul 78-Dec 94
France	Mar 79-Oct 95
Germany	Jan 79-Aug 95
Italy	Mar 79-Oct 95
Japan	Feb 79-Sep 95
UK	Mar 79-Oct 95
US	Apr 79-Nov 95

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