A General Framework for Financial Risk

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Abstract

Financial risk is caused by many factors. Among these are inadequate asset cash flows, excessive liability cash flow and interest rate fluctuations. These risk factors are commonly referred to in North American actuarial literature as C1, C2 and C3, respectively. Most often, financial risk factors are studied independently. Coordinated study is hampered by the lack of a generalized framework. In this paper, we provide some concepts that can be used to establish a generalized framework which allows for a unified study of all financial risk factors.

Keywords

C1-, C2-, C3-risk.

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A GENERAL FRAMEWORK FOR FINANCIAL RISK

INTRODUCTION

Financial risk is caused by many factors. Among these are inadequate asset cash flows, excessive liability cash flows and interest rate fluctuations. These risk factors are commonly referred to in North American actuarial literature as C1, C2 and C3, respectively. Most often, financial risk factors are studied independently. Co-ordinated study is hampered by the lack of a generalized framework. In this paper, we provide some concepts that can be used to established a generalized framework which allows for a unified study of all financial risk factors.

Cash Flow

An important element in this paper's presentation is the relationship between an asset or liability's value and its future cash flow. As regards assets, this relationship is a fundamental element of modern finance theory. See Dothan [1] and Duffie [2] for a general discussion of this subject. Sethi [5] and Ross [4] are two sources specifically focussed on the development of an asset's value from a given stream of cash flows.

As regards liability valuation, actuarial literature provides some examples of the relationship between value and cash flows. A good basic discussion can be found in the 1979 Report of the Committee on Valuation and Related Problems presented in the Record of the Society of Actuaries [6]. Frequently, discussions of liability values related to their cash flows are in the context of immunization work, such as Kocherlakota, Rosenbloom and Shiu [3] and Wise [7].

In this paper, we take as given that assets and liabilities, or more generally, *contracts*, are defined by their potential future cash flows.

Strategies and Scenarios

Most of the terms used in this paper are in common use elsewhere in financial and actuarial literature. One exception is the concept of *strategies* and *scenarios*. Strategies and scenarios are analogous to states.

The notion of a *state* is fundamental in asset pricing theory. A state is a state of the world. Security prices are formed first in terms of "state prices", one for each state of the world, then defined as the state-price weighted sum of its payoffs in the different states (Duffie [2]). We find it useful to expand the idea of a state by giving it two dimensions. One dimension encompasses the elements that we, the contract holder, can control, called *strategies*. The other dimension encompasses the elements that we cannot control, called *scenarios*.

SINGLE CONTRACT

Single Time Dimension

We begin our development with a simple example of four contracts. The four contracts all involve only a single cash payment that will be made one year hence.

1. Contract 1 entitles us to receive the proceeds of a 10.75 unit, risk free, one year government bond. We will use the word "unit" in preference to a specific currency. The terms of the bond are that a single payment of 10.75 units will be made one year hence. We will assume that the current (time 0) value of this contract is 10, meaning that our current risk free yield curve has a one year

interest rate of 7.5%.

2. Contract 2 entitles us to receive the proceeds of an 11.825 unit risk free 2 year government bond sold one year hence. The terms of the government bond are similar to the bond described above, except payment is 2 years hence. The current price of the bond is 10 units. With the one year risk free interest rate at 7.5%, this price generates an implied one year forward rate at time 1 of 10%. The terms of contract 2 are that we must sell the bond at time 1 year.

3. This contract requires us to pay 100 units at time 1 in the event that a certain insurable event occurs during the year. We call this event the "accident". If an accident does not occur, the contract entitles us to receive 2.3 units at time 1.

4. This contract is a corporate bond. It promises to pay us 110 units at time 1 but, unlike the risk free government bonds, this loan is subject to default. We will assume that the risk of default depends on the one year interest rate in effect at time 1.

The amount of cash resulting from these contracts will depend on the outcome of certain events over which we have no control. Among these events are the one year interest rate in effect at time 1 and the outcome of an insurable event.

For contract 2, the one year interest rate in effect at the end of year 1 will govern the sale price. The estimation of future interest rates and their probability of occurrence is the subject of much research in modern finance. As this is not the direction or objective of our discussion, we assume, instead, a simple probability function for our demonstration. More complex constructions can, of course, be substituted. We assume that a one year interest rate effective at time 1 of 5% occurs with probability .5 and a one year rate of 10% occurs also with probability .5. This construction generates an average one year interest rate at time 1 of 7.5%, the same as the current one year rate, effective at time 0. Note that the current yield curve's implied one year forward rate at time 1 is 10%.

With respect to the insurable event, we assume that an accident happens with probability .01; no accident happens with probability .99.

In our example, we will assume that the risk of default depends on the one year interest rate in effect at time 1. If the rate is 5%, the probability of default is 1%. If the one year rate at time 1 is 10%, the probability of default is 10%.

Our first example has eight states or scenarios. Namely,

1. forward rates of 5% and an accident and no default;
2. forward rates of 5% and no accident and no default;
3. forward rates of 10% and an accident and no default;
4. forward rates of 10% and no accident and no default.
5. forward rates of 5% and an accident and default;
6. forward rates of 5% and no accident and default;
7. forward rates of 10% and an accident and default;
8. forward rates of 10% and no accident and default;

The information we have about the cash flows from our contracts can be summarized in Table 1 below:

TABLE 1								
Cash Flows at Time 1								
					1	**2**	**3**	**4**
Scenario	**rate**	**acc**	**def**	**probability**	**RF1**	**RF2/1**	**Insurance**	**Bond**
1	5%	yes	no	0.00495	10.75	11.26	-100.00	110.00
2	5%	no	no	0.49005	10.75	11.26	2.30	110.00
3	10%	yes	no	0.00475	10.75	10.75	-100.00	110.00
4	10%	no	no	0.47025	10.75	10.75	2.30	110.00
5	5%	yes	yes	0.00005	10.75	11.26	-100.00	0.00
6	5%	no	yes	0.00495	10.75	11.26	2.30	0.00
7	10%	yes	yes	0.00025	10.75	10.75	-100.00	0.00
8	10%	no	yes	0.02475	10.75	10.75	2.30	0.00
Expected Value					**10.75**	**11.01**	**1.28**	**106.70**

The calculated cash flows in Table 1 are easily derived. For contract 1, called RF1, the risk free government bond, we are certain to receive 10.75 units at time 1. This receipt is independent of interest rates in effect at time 1 and independent of whether or not an accident or default occurs. For contract 2 (RF2/1) the amount of our receipt depends on the one year rate in effect at time 1. We have calculated the price (our receipt) as the year 2 payment of 11.825 units, divided by 1 plus the scenario's interest rate. Contract 2's cash flow is independent of whether or not an accident or default occurs. The insurance payment (contract 3), of course, depends on the occurrence of the accident and the bond payment (contract 4) depends on the occurrence of default. Probabilities of each scenario and the expected value of each contract are easily calculated.

Table 1 allows us to compare the different contracts. It displays all of the information we have available to assess the risk inherent in each contract. It shows, as expected, that the risk free 1 year bond is, indeed, risk free. That is, regardless of what the future holds our cash flow from this contract is always the same; 10.75 units. This is, incidentally, the only contract of the four for which this is true.

Current time equivalents

Although, since we are dealing with a single point in time when all payments are made, we have no particular need to develop current time value equivalents, we display Table 2 in preparation for a discussion of multi-time dimension contracts in the next section. Table 2 states each term in Table 1 in units calibrated against the current value, at time 0, of a risk free investment. In our example the relationship between a risk free value at time 1 and its value at time 0 is 1.075 to 1. Table 2 cash flow values are Table 1 values divided by 1.075.

TABLE 2								
Cash Flow Values at Time 0								
					1	**2**	**3**	**4**
Scenario	**rate**	**acc**	**def**	**probability**	**RF1**	**RF2/1**	**Insurance**	**Bond**
1	5%	yes	no	0.00495	10.00	10.48	-93.02	102.33
2	5%	no	no	0.49005	10.00	10.48	2.14	102.33
3	10%	yes	no	0.00475	10.00	10.00	-93.02	102.33
4	10%	no	no	0.47025	10.00	10.00	2.14	102.33
5	5%	yes	yes	0.00005	10.00	10.48	-93.02	0.00
6	5%	no	yes	0.00495	10.00	10.48	2.14	0.00
7	10%	yes	yes	0.00025	10.00	10.00	-93.02	0.00
8	10%	no	yes	0.02475	10.00	10.00	2.14	0.00
Expected Value					**10.00**	**10.24**	**1.19**	**99.26**

The assumption we make with this equivalence is that we are ambivalent as to whether we own 10 currency units or contract 1. They are equivalent. This equivalence makes sense in the context of each scenario for the other contracts as well. What we mean by this is the following: assuming that a given scenario is a true representation of the state of the world, the cash flows in that scenario of contracts 2, 3 and 4 are as certain as the cash flow of contract 1. Using contract 3 as an example, if scenario 1 occurs, then we know for certain that we will need to make a cash payment of 100 units at time 1. If 10.75 units at time 1 has a time 0 value of 10 then 100 unit payment has a time zero value of 93.02.

Before proceeding to the multi-time situation, we note some interesting aspects of the values in Table 2. If the price of the corporate bond were set at its expected value, i.e. 99.26, this would imply a risk weighted interest rate of 10.8% , compared to the risk free rate of 7.5%. Typically, bonds would be priced by selecting such a risk weighted rate and discounting the cash flows using the risk weighted rate. The process used to generate the Table 2 expected values and the process of setting a price using the risk weighted rate produce the same result with consistent assumptions. Note however, that for contract 2 and 3, meaningful risk weighted discount rates cannot be developed.

Multiple Time Dimensions

Table 3 adds a second time period during which our sample contracts can generate cash flows. It also allows us to add a strategy.

First, we extend our earlier example by allowing the insurance contract to continue for another year. In our extended example, the insurance contract (contract 3) continues for a second time period, with the cash flows and their probabilities the same as in the first period. The only event at time 2 that we need to monitor is

TABLE 3

Cash Flows at Time [1, 2]

						1	2	3	4	5
Scenario	**rate**	**acc**	**def**	**a2**	**probability**	**RF1**	**RF2/1**	**Insurance**	**Bond**	**RF2**
1	5%	yes	no	yes	0.0000495	[10.75, 0]	[11.26, 0]	[-100.00, -100.00]	[110.00, 0]	[0.00, 11.825]
2	5%	no	no	yes	0.0049005	[10.75, 0]	[11.26, 0]	[2.30, -100.00]	[110.00, 0]	[0.00, 11.825]
3	10%	yes	no	yes	0.0000475	[10.75, 0]	[10.75, 0]	[-100.00, -100.00]	[110.00, 0]	[0.00, 11.825]
4	10%	no	no	yes	0.0047025	[10.75, 0]	[10.75, 0]	[2.30, -100.00]	[110.00, 0]	[0.00, 11.825]
5	5%	yes	yes	yes	0.0000005	[10.75, 0]	[11.26, 0]	[-100.00, -100.00]	[0.00, 0]	[0.00, 11.825]
6	5%	no	yes	yes	0.0000495	[10.75, 0]	[11.26, 0]	[2.30, -100.00]	[0.00, 0]	[0.00, 11.825]
7	10%	yes	yes	yes	0.0000025	[10.75, 0]	[10.75, 0]	[-100.00, -100.00]	[0.00, 0]	[0.00, 11.825]
8	10%	no	yes	yes	0.0002475	[10.75, 0]	[10.75, 0]	[2.30, -100.00]	[0.00, 0]	[0.00, 11.825]
9	5%	yes	no	no	0.0049005	[10.75, 0]	[11.26, 0]	[-100.00, 2.30]	[110.00, 0]	[0.00, 11.825]
10	5%	no	no	no	0.4851495	[10.75, 0]	[11.26, 0]	[2.30, 2.30]	[110.00, 0]	[0.00, 11.825]
11	10%	yes	no	no	0.0047025	[10.75, 0]	[10.75, 0]	[-100.00, 2.30]	[110.00, 0]	[0.00, 11.825]
12	10%	no	no	no	0.4655475	[10.75, 0]	[10.75, 0]	[2.30, 2.30]	[110.00, 0]	[0.00, 11.825]
13	5%	yes	yes	no	0.0000495	[10.75, 0]	[11.26, 0]	[-100.00, 2.30]	[0.00, 0]	[0.00, 11.825]
14	5%	no	yes	no	0.0049005	[10.75, 0]	[11.26, 0]	[2.30, 2.30]	[0.00, 0]	[0.00, 11.825]
15	10%	yes	yes	no	0.0002475	[10.75, 0]	[10.75, 0]	[-100.00, 2.30]	[0.00, 0]	[0.00, 11.825]
16	10%	no	yes	no	0.0245025	[10.75, 0]	[10.75, 0]	[2.30, 2.30]	[0.00, 0]	[0.00, 11.825]
Expected Value						**[10.75, 0]**	**[11.01, 0]**	**[1.28, 1.28]**	**[106.70, 0]**	**[0.00, 11.825]**

whether or not the accident occurs. Scenarios 1 through 8 are the same as in the previous section and assume the accident occurs at time 2. Scenarios 9 through 16 are the same as 1 through 8 except that the accident doesn't occur.

The second extension in our example is the introduction of the notion of a strategy. Contract 2 is a risk free 2 year bond that we sell at time 1. What if we chose to not sell this bond at time 1, instead holding it to maturity at time 2? By making this choice, we have altered the cash flow from the bond in a manner that differs from the alteration caused by the occurrence of different scenarios. Different interest rate scenarios in effect at time 1 will cause the contract 2 cash flows to differ. The different interest rates at time 1 are not controllable by us, the contract holder. The occurrence of a specific interest rate is a random event and we can assign a probability of occurrence to each event. We have displayed these different scenarios as different rows in Tables 1, 2 and 3 with a probability of occurrence assigned to each row. On the other hand, the alteration in the cash flows that occurs because we choose to hold the bond to its maturity at time 2 is controllable by us, the contract holder. The exact cash flow outcome of our decision to hold the bond until maturity could depend on the outcome of random scenarios (in our example it doesn't). But, in the context of a given random scenario, the difference between the cash flows from contract 2 and holding the bond until maturity are known with complete certainty, in the same way that the differences in cash flow between two contracts (contract 1 compared to contract 2, for example) are known with complete certainty (in the context of a given strategy). We will define the strategy of holding the two year bond to maturity as contract 5, using the designation RF2 to refer to it. We use a new column in Table 3 to display the new strategy.

Figures in Table 3 are presented in pairs. The first element is the cash flow at time 1, the second element is the cash flow at time 2. All of the information we know about the cash flows of our 5 contracts and the 16 scenarios that represent all of the possible futures is displayed in Table 3. Unfortunately, the number pairs are more difficult to interpret that single values. This difficulty, of course, would be magnified considerably by the addition of more time dimensions. Therefore, we produce Table 4 using the same device that we used to produce Table 2.

Table 4 shows the time 0 cash equivalents of the number pairs in Table 3. The risk free interest rate between time 0 and time 1 is 7.5%. We know this for a fact. We also know, at time 0, with complete certainty, that the risk free 2 year interest rate between time 0 and time 2 is 18.25%. We know that for each scenario, the cash flow pairs in Table 3 will occur with complete certainty (if that scenario occurs). Then, assuming the risk free rate is unique, Table 4 is a unique time 0 cash equivalent of the number pairs in Table 3.

Reviewing Table 4, we note that contract 1 is risk free. So is the 2 year government bond if we hold it until maturity (contract 5). Note that the expected value of selling the 2 year government bond is higher than the expected value of holding it until maturity. This is a consequence of the difference between the current yield curve's implied forward rate at time 1 (10%) and the one year interest rates at time 1 in the scenarios (5% and 10% with an average of 7.5%).

An alternative to using the current yield curve to establish the time 0 value is to establish the present cash equivalent based on each scenario's specified interest rate. For example, for scenarios 1 and 2, we would use 5% rather than 10% to develop the time 1 equivalent of the time 2 cash flows. For scenario's 3 and 4, we would use 10%, etc. This approach would not alter the time 0 value for RF2/1 (since all of the cash flows for this contract occur at time 1). But, it would alter the current values of RF2. In fact, using scenario specific rates would make the RF2 values exactly equal to the RF2/1 values displayed in Table 4. As a consequence, RF2 ceases to appear to be risk free. Since this outcome is contrary to expectations, we will use the current yield

TABLE 4										
Cash Flow Values at Time 0										
						1	**2**	**3**	**4**	**5**
Scenario	**rate**	**acc**	**def**	**a2**	**probability**	**RF1**	**RF2/1**	**Insurance**	**Bond**	**RF2**
1	5%	yes	no	yes	0.0000495	10	10.48	-177.59	102.33	10
2	5%	no	no	yes	0.0049005	10	10.48	-82.43	102.33	10
3	10%	yes	no	yes	0.0000475	10	10.00	-177.59	102.33	10
4	10%	no	no	yes	0.0047025	10	10.00	-82.43	102.33	10
5	5%	yes	yes	yes	0.0000005	10	10.48	-177.59	0.00	10
6	5%	no	yes	yes	0.0000495	10	10.48	-82.43	0.00	10
7	10%	yes	yes	yes	0.0000025	10	10.00	-177.59	0.00	10
8	10%	no	yes	yes	0.0002475	10	10.00	-82.43	0.00	10
9	5%	yes	no	no	0.0049005	10	10.48	-91.08	102.33	10
10	5%	no	no	no	0.4851495	10	10.48	4.08	102.33	10
11	10%	yes	no	no	0.0047025	10	10.00	-91.08	102.33	10
12	10%	no	no	no	0.4655475	10	10.00	4.08	102.33	10
13	5%	yes	yes	no	0.0000495	10	10.48	-91.08	0.00	10
14	5%	no	yes	no	0.0049005	10	10.48	4.08	0.00	10
15	10%	yes	yes	no	0.0002475	10	10.00	-91.08	0.00	10
16	10%	no	yes	no	0.0245025	10	10.00	4.08	0.00	10
Expected Value						**10**	**10.24**	**2.27**	**99.26**	**10**

curve to transform the cash flow vectors. However, the use of a scenario specific transformation provides some interesting insights and its merits should not be entirely ignored.

It should be evident that the time dimension can be extended beyond two (actually, our examples have 3 time points, including time 0). A more general algebraic form will be presented below. First, we will extend the discussion, using the same 5 contracts, to a multiple contract situation.

MULTIPLE CONTRACTS

Up to this point, we have considered each of the contracts as independent items in our analysis. With the development of contract 5, we have introduced the notion of strategic variations. That is, we have allowed for an analysis of how we, the contract holder, would fare if we chose to sell the 2 year bond (contract 2) or hold it to maturity (contract 5). We now extend this idea to accommodate analysis of how we would fare if we held more than one contract (using the term *portfolio* to refer to our holdings) and how we would fare if we altered the contracts that compose our portfolio.

We now proceed to establish a portfolio of contracts. The portfolios we will develop will use variations of the contracts from the previous section. We describe below the nature of the variations we will consider.

Constant multiple

We can hold 10 of the contract described as RF1, the one year government bond. We can accurately describe this as [107.5, 0] using the form of cash flow description shown in Table 3. The cash flow equivalent, at time 0, of these 10 contracts is 100 units.

In general terms, if we use the term **C** to denote a vector of cash flows $[c_t] = [c_0, c_1, \ldots c_n]$ then we recognize that k**C** = $[kc_t]$, where k is a constant. Note that in defining **C** we have included, for generality, a first term at time 0 to allow for cash held at that date. The vectors in Table 3 have excluded this term as it is 0 for all of the examples and its inclusion would add unnecessary clutter.

If we use L to denote the transformation of a cash flow vector into a scalar by the application of unique risk free interest rates in effect at time 0, then, $L(k\mathbf{C}) = kL(\mathbf{C})$.

Negative constant multiple

We can hold the "negative" of the contract described as RF1. Using the form of Table 3, we would describe this as [-10.75, 0]. The general form is already described above, recognizing that the constant k can take on negative values.

In financial literature the terms "long" and "short" are frequently used to described opposite sides of the same contract. We are long in a contract if we receive cash, like RF1. We are short if we pay cash, like -RF1. This also points out the fact that each contract has, at least, two contract holders. If we own contract RF1, someone else must own -RF1.

Sum of contracts

If we hold two or more contracts, our combined cash flow from these is simply the sum of cash flows at each point in time. Referring again to Table 3, if we own both RF1 and Bond, our total cash flow is [120.75, 0] for scenarios 1 through 4 and 9 through 12 and [10.75, 0] for the other scenarios.

Where specific reference to a strategy and scenario are important, we will define $\mathbf{C}(a,b) = [c_t (a,b)]$ as the vector of cash flows of a contract for strategy a and scenario b. We will use $\mathbf{C}_i$ to denote the cash flow vector for contract i, when we are considering more than one contract. Then:

$$\mathbf{C}_1 (a,b) + \mathbf{C}_2 (a,b) = [c_{1,t} (a,b) + c_{2,t} (a,b)]$$

and

$$L(\mathbf{C}_1 (a,b) + \mathbf{C}_2 (a,b)) = L(\mathbf{C}_1 (a,b)) + L(\mathbf{C}_2 (a,b))$$

This notation forces us to clarify what we mean precisely when we consider a situation like contract 2 and its variant, contract 5. If we mean that these two options are different strategies for the same, single, contract, then they are strategic variations of the same contract. That is, using our general notation, we can describe them as $\mathbf{C}(1, b)$ and $\mathbf{C}(2, b)$. Strategy 1 here means the sale of the government bond at time 1 (i.e. contract 2) and strategy 2 means holding the same government bond to maturity. If we mean that they are truly different contracts and, thus, we can own both at the same time, then we would describe them using our notation as $\mathbf{C}_2(a,b)$ and $\mathbf{C}_5(a,b)$.

Multiple Strategies

Before constructing a number of portfolios for our study, we define one additional simple contract. This contract is the "uninsured" version of the insurance contract. Its cash flows are minus 100 in the event that an accident occurs (this accident occurs with probability .01 at time 1 and probability .01 at time 2). Its cash flows are 0 in the event an accident does not occur. With reference to Table 3, the vectors by scenario are the same as for the insurance contract except that the value 2.3, wherever it occurs is replaced by 0. We will use the word "uninsured" for this contract.

We now construct four strategies.

In strategy 1 we hold a portfolio of contracts consisting of -10 units of RFI, one Uninsured and 1.05 of Bond . The details by contract type and their sum for strategy 1 is shown in Table 5.

For strategy 2 we hold the same contracts as strategy 1 but also hold a minus Insurance. Minus insurance is the negative of Insurance. In the insurance contract as previously described we are the "insurer" in the contract. Minus Insurance is the contract from the "insured's" perspective. That is, we are the insured.

Strategy 3 is the same as strategy 2 except that we hold 10.4223 units of RF2 instead of 1.05 units of Bond. The multiple 10.4223 was selected so that the time 0 value of the two investments is the same. While the move from strategy 1 to strategy 2 requires us to incur a cost, strategy 3 has been constructed so that it represents an equal cost substitute for strategy 2.

Strategy 4 is like strategy 3 except that we replace RF2/1 with RF2. That is, we assume we sell our 2 year government bond at time 1.

Table 6 shows the cash flow vectors by strategy and scenario. Table 7 shows the cash equivalent of Table 6 values.

TABLE 5

Cash Flows at Time [1, 2]

Scenario	rate	acc	def	a2	probability	-10RF1	UnInsured	1.05Bond	S1
1	5%	yes	no	yes	0.0000495	[-107.50, 0]	[-100.00, -100.00]	[115.50, 0]	[-92.00, -100.00]
2	5%	no	no	yes	0.0049005	[-107.50, 0]	[0.00, -100.00]	[115.50, 0]	[8.00, -100.00]
3	10%	yes	no	yes	0.0000475	[-107.50, 0]	[-100.00, -100.00]	[115.50, 0]	[-92.00, -100.00]
4	10%	no	no	yes	0.0047025	[-107.50, 0]	[0.00, -100.00]	[115.50, 0]	[8.00, -100.00]
5	5%	yes	yes	yes	0.0000005	[-107.50, 0]	[-100.00, -100.00]	[0.00, 0]	[-207.50, -100.00]
6	5%	no	yes	yes	0.0000495	[-107.50, 0]	[0.00, -100.00]	[0.00, 0]	[-107.50, -100.00]
7	10%	yes	yes	yes	0.0000025	[-107.50, 0]	[-100.00, -100.00]	[0.00, 0]	[-207.50, -100.00]
8	10%	no	yes	yes	0.0002475	[-107.50, 0]	[0.00, -100.00]	[0.00, 0]	[-107.50, -100.00]
9	5%	yes	no	no	0.0049005	[-107.50, 0]	[-100.00, 0]	[115.50, 0]	[-92.00, 0]
10	5%	no	no	no	0.4851495	[-107.50, 0]	[0.00, 0]	[115.50, 0]	[8.00, 0]
11	10%	yes	no	no	0.0047025	[-107.50, 0]	[-100.00, 0]	[115.50, 0]	[-92.00, 0]
12	10%	no	no	no	0.4655475	[-107.50, 0]	[0.00, 0]	[115.50, 0]	[8.00, 0]
13	5%	yes	yes	no	0.0000495	[-107.50, 0]	[-100.00, 0]	[0.00, 0]	[-207.50, 0]
14	5%	no	yes	no	0.0049005	[-107.50, 0]	[0.00, 0]	[0.00, 0]	[-107.50, 0]
15	10%	yes	yes	no	0.0002475	[-107.50, 0]	[-100.00, 0]	[0.00, 0]	[-207.50, 0]
16	10%	no	yes	no	0.0245025	[-107.50, 0]	[0.00, 0]	[0.00, 0]	[-107.50, 0]
Expected Value						**[-107.50, 0]**	**[-1.00, -1.00]**	**[112.04, -1.00]**	**[3.54, -1.00]**

TABLE 6

Cash Flows at Time [1, 2]

Scenario	rate	acc	def	a2	probability	S1	S2	S3	S4
1	5%	yes	no	yes	0.0000495	[-92.00, -100.00]	[8.00, 0]	[-107.50, 123.24]	[9.87, 0]
2	5%	no	no	yes	0.0049005	[8.00, -100.00]	[5.70, 0]	[-109.80, 123.24]	[7.57, 0]
3	10%	yes	no	yes	0.0000475	[-92.00, -100.00]	[8.00, 0]	[-107.50, 123.24]	[4.54, 0]
4	10%	no	no	yes	0.0047025	[8.00, -100.00]	[5.70, 0]	[-109.80, 123.24]	[2.24, 0]
5	5%	yes	yes	yes	0.0000005	[-207.50, -100.00]	[-107.50, 0]	[-107.50, 123.24]	[9.87, 0]
6	5%	no	yes	yes	0.0000495	[-107.50, -100.00]	[-109.80, 0]	[-109.80, 123.24]	[7.57, 0]
7	10%	yes	yes	yes	0.0000025	[-207.50, -100.00]	[-107.50, 0]	[-107.50, 123.24]	[4.54, 0]
8	10%	no	yes	yes	0.0002475	[-107.50, -100.00]	[-109.80, 0]	[-109.80, 123.24]	[2.24, 0]
9	5%	yes	no	no	0.0049005	[-92.00, 0]	[8.00, -2.30]	[-107.50, 120.94]	[9.87, -2.30]
10	5%	no	no	no	0.4851495	[8.00, 0]	[5.70, -2.30]	[-109.80, 120.94]	[7.57, -2.30]
11	10%	yes	no	no	0.0047025	[-92.00, 0]	[8.00, -2.30]	[-107.50, 120.94]	[4.54, -2.30]
12	10%	no	no	no	0.4655475	[8.00, 0]	[5.70, -2.30]	[-109.80, 120.94]	[2.24, -2.30]
13	5%	yes	yes	no	0.0000495	[-207.50, 0]	[-107.50, -230]	[-107.50, 120.94]	[9.87, -2.30]
14	5%	no	yes	no	0.0049005	[-107.50, 0]	[-109.80, -2.30]	[-109.80, 120.94]	[7.57, -2.30]
15	10%	yes	yes	no	0.0002475	[-207.50, 0]	[-107.50, -2.30]	[-107.50, 120.94]	[4.54, -2.30]
16	10%	no	yes	no	0.0245025	[-107.50, 0]	-109.80, -2.30]	[-109.80, 120.94]	[2.24, -2.30]
Expected Value						**[3.54, -1.00]**	**[2.26, -2.28]**	**[-109.78, 120.97]**	**[4.93, -2.28]**

TABLE 7

Cash Flow Values at Time 0

Scenario	rate	acc	def	a2	probability	S1	S2	S3	S4
1	5%	yes	no	yes	0.0000495	-170.15	7.44	4.22	9.19
2	5%	no	no	yes	0.0049005	-77.12	5.30	2.08	7.05
3	10%	yes	no	yes	0.0000475	-170.15	7.44	4.22	4.22
4	10%	no	no	yes	0.0047025	-77.12	5.30	2.08	2.08
5	5%	yes	yes	yes	0.0000005	-277.59	-100.00	4.22	9.19
6	5%	no	yes	yes	0.0000495	-184.57	-102.14	2.08	7.05
7	10%	yes	yes	yes	0.0000025	-277.59	-100.00	4.22	4.22
8	10%	no	yes	yes	0.0002475	-184.57	-102.14	2.08	2.08
9	5%	yes	no	no	0.0049005	-85.58	5.50	2.28	7.24
10	5%	no	no	no	0.4851495	7.44	3.36	0.14	5.10
11	10%	yes	no	no	0.0047025	-85.58	5.50	2.28	2.28
12	10%	no	no	no	0.4655475	7.44	3.36	0.14	0.14
13	5%	yes	yes	no	0.0000495	-193.02	-101.95	2.28	7.24
14	5%	no	yes	no	0.0049005	-100.00	-104.08	0.14	5.10
15	10%	yes	yes	no	0.0002475	-193.02	-101.95	2.28	2.28
16	10%	no	yes	no	0.0245025	-100.00	-104.08	0.14	0.14
Expected Value						**2.44**	**0.17**	**0.18**	**2.66**

As we can see from Tables 6 and 7, different strategies have different outcomes depending on the scenario that occurs. Although S1 has a higher time 0 expected value than S2, this higher value is at the expense of considerably greater volatility in the range of possible outcomes. In effect, in strategy 2, we incur an expected cost to transfer the accident risk.

Strategy 3 removes the default risk from the portfolio and has done so without reducing expected return, relative to strategy 2. In this case, the risk has been reduced by giving up the potential for higher returns under some scenarios.

Strategy 3 and 4 also provide an interesting comparison. Strategy 4 is the same as strategy 3 except that we sell the 2 year bond at time 1. The higher expected value for S4 is generated by the fact that, in our example, the year 1 interest rates with their associated probabilities of occurrence differ from the current yield curve's implied forward rate. Note also that, if we use a scenario specific transformation (i.e. a year 1 to 2 rate of 5% for scenarios 1 and 2, for example) S4 would have the same time zero values as S3.

Before proceeding to a discussion of the application of our general risk framework to real portfolios and business problems, we summarize the essential features of our framework.

1. cash flow

A vector of future cash flows is the fundamental building block in our analysis. All contracts, meaning assets, liabilities and, in general, financial events, need to be decomposed into their future cash flows.

2. scenarios and strategies

A contract's cash flows are subject to two forces, which are mutually exclusive and exhaustive. Uncontrollable forces generate scenarios. Uncontrollable forces are those that we, the contract holder cannot control. Controllable forces generate strategies. Strategies typically consist of buying, selling or altering contracts. It is helpful to consider scenarios and strategies as two dimensions in a table of outcome vectors. We have adopted the convention of showing scenarios as rows and strategies as columns in a two dimensional matrix.

3. linear transformation of cash flow vectors

The analysis date (time 0) yield curve of unique risk free rates provides a meaningful, and unique, transformation of the cash flow vectors. If we recognize that the cash flow vectors in a portfolio of contracts form a subset of a real vector space (a real n-space if we allow for n time elements in our vector), then the construction of the time 0 risk free equivalent is a linear transformation mapping the real n-space to the real number line.

Transformations other than a constant current yield curve may be useful. For example, the yield curve anticipated in each scenario could be used. We note however, by example, that if the anticipated yield curves by scenario and their associated probability are not consistent with the current curve, analytical results will differ from those developed using the constant current yield curve transformation.

REAL PORTFOLIOS

In order to demonstrate the elements of our general risk analysis framework, we have constructed contracts, scenarios and strategies by a process of synthesis. That is, we started with simple contracts, adding complexity and additional elements, then combining these contracts to develop a portfolio for study. In the practical application of these ideas to real portfolios of assets and liabilities, the process is more a process of analysis than of synthesis. In real portfolios, the contracts and the information that we have about them exist as data elements in a record keeping system. In order to apply these ideas it is necessary for us to find the essential data elements and organize them in a manner consistent with our framework and its decision making processes.

A portfolio's balance sheet is the best starting point. Typically, this provides an inventory of contracts with enough information about each contract to begin the development of cash flow vectors.

Homogeneity

If a portfolio contains a number of identical contracts (like 100 units of the same corporate bond issue) there is generally no need to consider each of these individually, as separate contracts. Instead we can normally deal with the total as a single contract.

Where a number of contracts are similar but not identical, there may also be opportunities for combination. For example, consider a portfolio that includes a large number of individual life insurance policies. Assuming each insurance policy is written on a different life, its future cash flow depends on the very unique circumstances that determine the survival of each life. Clearly, these insurance policies are not identical in the same way that units of a corporate bond issue are identical. However, using common actuarial techniques, we can construct a survivorship model that generates cash flows for these insurance policies in total. Total results would be much more credible than the specific results of each individual policy. Furthermore, dealing with cash flows in total would make studying scenario and strategy variations easier than studying these variations at the individual policy level.

A case for combining similar but not identical contracts can also be made for retail mortgages. Again, common patterns of behaviour allow for a more meaningful total result.

Scenario selection

In our example, we have developed contracts, portfolios of contracts and the scenarios that impact cash flows in a controlled manner. We have been able to develop scenarios so that we have a complete and exhaustive picture of the future. The scenarios in our sample are the only scenarios that can occur. We are able to assign a probability to each scenario and the sum of the probabilities of all the scenarios is one. This will not normally be so in real portfolios.

In real applications we are not able to construct all possible scenarios nor are we able to enumerate all of the individual events that collectively comprise each scenario. Nonetheless, by concentrating our efforts on the events that impact the cash flows of the contracts in our portfolio we can construct useful and meaningful scenarios. Such constructions should attempt to maintain a consistency between the elements that comprise a scenario, for example the relationship between interest rates and the change in price indices should reasonable. The past may be a good guide, but not the sole measure of what is reasonable.

In practice, except for a very simple portfolio of contracts, we are not able to construct all of the possible scenarios that could occur nor are we likely to be able to assign accurate probabilities to those scenarios that we are able to construct. However, by constructing a reasonable number of diverse scenarios much useful information can be developed.

Definition of contracts

Our examples and the discussion above assume that the contracts in our portfolio are those typically recorded on a balance sheet as either assets or liabilities. But this framework does not require us to restrict our definition in this manner. Any vector of cash flows could be included. For example, a capital expenditure could be analyzed in this fashion. Likewise, a business enterprise's balance sheet could be enhanced by including the impact of cash flow from new sales.

APPLICATIONS

The framework presented here is sufficiently general to accommodate all manner of financial analysis. By way of example, two applications are discussed briefly.

Dynamic Solvency Testing

Many regulatory jurisdictions are studying the merits of implementing a dynamic test of the solvency of financial institutions, particularly life insurance companies. A dynamic solvency test is a projection of a company's balance sheet surplus for some years into the future, assuming that a variety of different futures might occur. The outcome under each selected future is projected and in this way information is developed as to the company's ability to withstand the impact of potential future financial problems. Currently, dynamic solvency testing is done by developing future income statements and balance sheets and selecting some future date, for example, five years hence, at which the outcome of the different scenarios is viewed. While this approach is meaningful, it unfortunately conveys the impression that a problem is some years off and can be dealt with at a leisurely pace. Furthermore, this approach may fail to include the impact of events that occur after the study horizon.

The general framework proposed here addresses all financial consequences of the test scenarios in terms of current, time 0, values. Furthermore, it allows us to develop and analyze the impact of adopting different strategies in terms of the current financial impact of their outcomes.

Derivatives

Current financial reporting methods do not effectively incorporate derivatives. The principal reason for their shortfall in this regard is that, in large part, the essential elements of a derivative are contained in the variability of cash flows rather than their expected value. Another major shortfall of traditional methods is their inability to incorporate the potential cash flows of derivatives with the cash flows of other more traditional financial contracts. By allowing such combination this generalized framework permits a quantitative analysis of the risk and rewards of including derivatives in a portfolio of contracts.

CONCLUSION

This paper presents, by means of numerical examples and some simple algebraic equations, a new framework for the analysis of financial contracts and portfolios of financial contracts. The objective of this new framework is to allow for an analysis of the combined impact of all of the risks that can effect a portfolio.

The essential elements of this framework are threefold: cash flow, strategies and scenarios, and the linear transformation of cash flow vectors to establish a current value.

The author hopes that this paper will encourage others to explore the application of these ideas to current financial problems and to develop further the theoretical foundation presented here.

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