Indirect Cost, Reinsurance Pricing, and the Availability of Disaster Insurance

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Abstract

The insurer's indirect financial market costs are added to the direct costs associated with the insured event to illustrate the necessary conditions for the existence of disaster insurance und reinsurance. Investors' negative response to an insurer's huge, disaster-related liability exposures may lead to availability problem unless the insurer's value losses in the financial market because of these exposures can be minimized. Three different pricing schemes for disaster reinsurance contracts are investigated. The one which is based on the Option Pricing Theory is rejected because it leads to market failure.

Résumé

Les coûts indirects financiers du marché de l'assureur sont ajoutés aux coûts directs associés à l'événement assuré pour répondre aux conditions nécessaires applicables à l'assurance-désastre et à la réassurance. La réponse négative des investisseurs aux vastes risques de catastrophe d'un assureur peut amener au problème de disponibilité en cas de pertes de valeur du marché financier de la part de l'assureur parce que ces cas peuvent être minimisés. Trois différents schémas de prix pour contrats de réassurance-désastre sont etudiés. Celui qui se base sur la théorie d'Option de Prix est rejeté parce qu'il amène à une perte de marché.

Keywords

Indirect cost, options, disaster insurance, asset value losses, fair price.

Mots clefs

Coût indirect, options, assurance de désastre, pertes des valeur de biens, prix équitable.

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INDIRECT COSTS, PRICING OF REINSURANCE, AND THE AVAILABILITY OF DISASTER INSURANCE

I. Introduction

Pricing of reinsurance has been widely studied. Borch (1962) uses equilibrium theory to derive fair pricing of a reinsurance contract. Doherty and Garven (1986) give the formula of reinsurance premiums by applying the Option Pricing Theory. Venter (1993) values the reinsurance contract by using the Arbitrage Pricing Theory. In this paper, a reinsurance contract is considered as a put option to the primary insurer with the price of the reinsurance contract being subsequently determined under three different conditions of the reinsurance market -- a market monopolized either by the primary insurer or by the reinsurer and a market where the competitive conditions prevail. Consequently, three different pricing schemes for the reinsurance contract are given--the minimum one, the maximum one, and the fair one.

The approach followed in this paper to the problem of pricing of reinsurance differs from all of those suggested by previous studies, including that of Doherty and Garven(1986). First of all, indirect losses as well as direct losses to insurance firms resulting from the occurrence of the insured event are considered in price determination. Indirect losses to insurance firms(both primary insurer and reinsurer) associated with the insured event include losses resulting from declines of the firm's stock and bonds values in the financial market. Investors in the financial market always respond negatively to large liability exposures by a firm. Such indirect losses to a primary insurer or a reinsurer tend to be quite significant when the insurer suffers a huge liability exposure due to the occurrance of the insured event, like an earthquake or a hurricane.

Indirect costs affect the decision of insurers as to whether or not to be engaged in the market competition. Aware of potential indirect costs, some insurance firms may not want to do business in the area of disaster insurance. This may well explain why many primary insurers and reinsurers withdrew from the disaster insurance market.

Availability of insurance and reinsurance has been a serious problem in disaster insurance. For instance, following the big earthquake in the Bay area in San Francisco, California, many residents in that area found that they can no longer buy coverage of earthquake insurance from their original insurers. Similarly, following Hurricane Andrew, many residents in the eastern coast of the United States were declined disaster insurance coverage by most insurance companies.

Several previous studies indicate that the unavailability of reinsurance is one of the major reasons which caused the primary insurers' unwillingness to continue their business. For instance, Berger, Cummins and Tennyson (1989) show that the unavailability of reinsurance cover contributed to the crisis of liability insurance. Hershbarger (1994) demonstrates that inaccessibility of disaster reinsurance leads to the unavailability of disaster insurance.

Conventional explanations for the unavailability of insurance or reinsurance include non-diversification of risk, adverse selection, and moral hazard (Borch, 1990). Given the nature of disaster insurance, these three factors may account, to some extent, for the non-existence of insurance or reinsurance, but they cannot fully explain the whole picture. The huge indirect costs associated with disaster insurance significantly contributed to the failure of the disaster insurance market.

Regarding the pricing of reinsurance, prior studies assume that the reinsurance market is perfectly competitive. In this paper, based on different conditions of insurance markets, three different pricing schemes for reinsurance are introduced. In addition, the necessary conditions for the existence of the reinsurance market and, therefore, the existence of the primary insurance market are given. Especially, the current research shows that both the primary and the reinsurance markets may fail under a price derived from the Option Pricing Theory while these markets can exist otherwise.

The rest of the paper is organized as follows: the next section explores different pricing schemes of the reinsurance contract and gives conditions for the existence of reinsurance. Section 3 gives a numerical example, which shows how both primary insurers and reinsurers may benefit by cooperating with each other. Section 4 demonstrates the effects of sizes of firms, indirect costs and the reinsurance premiums on the availability of insurance. The last section provides some concluding remarks.

II. Options and the Value of Reinsurance

2.1. The Choice of Strategies and the Value of Reinsurance

Suppose there are two strategies for the primary insurer. Under strategy one the insurer's net asset takes a random value X_1 with a distribution F_1 while under strategy two, the firm's net asset has a random value X_2 with the distribution F_2, i. e. $X_1 \sim F_1(\alpha_1, \sigma_1)$ and $X_2 \sim F_2(\alpha_2, \sigma_2)$; where α_1 and σ_1 are the mean and standard deviation of X_1, respectively; α_2 and σ_2 are the mean and standard deviation of X_2. In addition, $\alpha_1 < \alpha_2$ and $\sigma_1 < \sigma_2$. Min. (X2) < 0 < Min(X1). In other words, the primary insurer will have a higher value on average under strategy two, but its variance will be higher also. Moreover, the insurer may face the risk of bankruptcy under strategy two.

Assume that the primary insurer will not take strategy two without reinsurance either because it does not want to bear the risk [1], associated with this strategy, or because regulation prohibits the insurer from doing so [2].

Now, suppose that a reinsurance firm offers an option to the primary insurer. Let Ex be the exercise price and O_p be the price of the option, Ex and Op ≥ 0. Then, one can illustrate what is the price of the option and under what conditions both the primary insurer and the reinsurer are willing to share risks and benefits.

The expected net benefit for the primary insurer from taking strategy two instead of one is $\Delta = E(X_2 \mid X_2 \geq Ex) + Ex * P(X_2 < Ex) - E(X_1) - O_p$.

On the other hand, the expected payment the reinsurer encounters is

$P_m = E[(Ex - X_2) \mid X_2 < Ex] = Ex * P(X_2 < Ex) - E(X_2 \mid X_2 < Ex)$.

Then, the necessary condition for the existence of reinsurance is $O_p \geq P_m$ and $\Delta \geq 0$. Consequently, one has

(1) $E(X_2 \mid X_2 \geq Ex) + Ex * P(X_2 < Ex) - E(X_1) - O_p \geq 0$;

(2) $O_p \geq Ex * P(X_2 < Ex) - E(X_2 \mid X_2 < Ex)$.

Denote $\Delta_1 = E(X_2) - E(X_1)$ and $\Delta_2 = O_p - Ex * P(X_2 < Ex) + E(X_2 \mid X_2 < Ex)$.

Thus,

$\Delta = \Delta_1 - \Delta_2$. From (1) and (2), one has $\Delta_1 = E(X_2) - E(X_1) \geq 0$

In a reinsurance market monopolized by the primary insurer, the reinsurer earns zero profit, i. e. $\Delta_2 = 0$ or O_p = Ex * P($X_2 <$ Ex) - E(X_2 | $X_2 <$ Ex). On the other hand, being a monopoly in the market, the reinsurer can seize all profits as much as it can, so $\Delta = 0$, i. e. O_p = Ex * P($X_2 <$ Ex) + E(X_2 | $X_2 \geq$ Ex) - E(X_1). In other words, O_p = Ex * P($X_2 <$ Ex) - E(X_2 | $X_2 <$ Ex), denoted by P_{min}, is a minimum premium that the reinsurer requires; and O_p = E(X_2 | $X_2 \geq$ Ex) + Ex * P($X_2 <$ Ex) - E(X_1), denoted by P_{max}, is the maximum premium that primary insurer is willing to pay for reinsurance. Let O^*_p be the actual price of reinsurance, then the necessary conditions for the existence of reinsurance are $P_{max} \geq O^*_p \geq P_{min}$.

In addition, one can use the option pricing theory to derive a fair price for the option, where the exercise price is Ex and the present value of the asset is E(X_1). Let P_{fair} be the fair pricing from OPT; one has P_{fair} = E {Max. (Ex -X_2, 0)}, where E is the expectation operator. Thus, one has three different price schemes, P_{min}, P_{max} and P_{fair}. The necessary condition for the existence of reinsurance and therefore insurance is that $P_{min} \leq P_{max}$. When the reinsurance is priced according to P_{fair}, the necessary conditions for existence of both primary and reinsurance markets are $P_{min} \leq P_{fair} \leq P_{max}$.

Furthermore, one may consider the reinsurer's asset losses from the adverse event of the disaster. Let L_R be the asset value losses to the reinsurer when the reinsurer encounters the liability exposure, then the expected cost to the reinsurer is

$$P_m = Ex \cdot P(X_2 < Ex) - E(X_2 \mid X_2 < Ex) + L_R \cdot P(X_2 < Ex)$$
$$= [Ex + L_R] \cdot P(X_2 < Ex) - E(X_2 \mid X_2 < Ex)$$

Now, the necessary conditions for the existence of reinsurance become

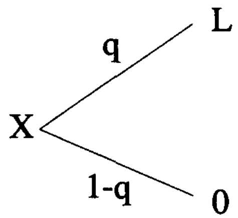
(1) $\Delta = E(X_2 \mid X_2 \geq Ex) + Ex * P(X_2 < Ex) - E(X_1) - O_p \geq 0$;

(2) $\Delta_2 = Op - [Ex + L_R] P(X_2 < Ex) - E(X_2 \mid X_2 < Ex) \geq 0$;

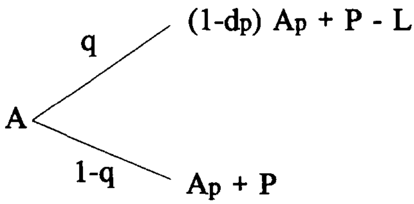
Satisfying condition 1 will motivate the primary insurer to underwrite policies while having condition 2 will make the reinsurer have an incentive to participate in the disaster insurance.

2.2. Two - State Options and Pricing of Reinsurance

Suppose that the primary insurer has net assets A_p without underwriting disaster insurance. Assume further that, disaster insurance has a loss distribution that has total claim indemnities of L with a probability of q.



Assume that the default rate of primary insurer's asset is d_p when the insured event occurs and that the total premiums earned by the insurer are P. Then, the primary insurer's contingent assets have the following distribution after participating in the disaster insurance scheme:



The expected net assets to the primary insurer after underwriting the disaster insurer are $(1 - q\, d_p)A_p + (P - qL)$. The primary insurer may participate in issuing disaster insurance policies when $(1 - q\, d_p)A_p + (P - q\, L) - A_p > 0$ or

$$- q\, d_p Ap + (P - qL) > 0 \tag{2.1}$$

From (2.1), one has $P > q\, L + q\, d_p Ap$. In other words, the premium that the insurer charges is at least as large as the expected payments, q L, plus the expected indirect losses, $q\, d_p A_p$.

The insurer's probability of insolvency after taking the disaster insurance is q which, if it is too big to satisfy certain regulatory requirements, will force the primary insurer to cede some premiums in order to satisfy solvency regulation.

Assume that the reinsurer has net assets A_R . The form of reinsurance takes excess of loss. Let Ex and O_p be respectively, the exercise price and option price (called the premium of reinsurance)[3]. In other words, if $(1- d_p) A_p + (P - L) - O_p < Ex$, the reinsurer will compensate the primary insurer. Also, assume that the default rate of the reinsurer's asset is d_R when the reinsurer needs to pay claims to the primary insurer. Thus, the expected costs to the reinsurer are $P_m = (Ex + d_R A_R) q - E(X_2 \mid X_2 < Ex) = q \{Ex + d_R A_R - [(1- d_p) A_p + (P - L) - O_p]\}$. The reinsurer has an incentive to do business only when $O_p \geq P_m$.

Now, with reinsurance, the primary insurer's expected net assets are $(1-q)\{(1-d_p) A_p + (P - L) - O_p\} + q\, Ex$. Thus, the net benefit for the insurer by participating in the disaster insurance business is

$$(1-q)\{(1- d_p) A_p + (P - L) - O_p\} + q\, Ex- A_p \qquad (2.2)$$

Thus, the necessary conditions for the existence of disaster insurance are

(1) $\Delta = (1-q)(A_p + P - O_p) + q\, Ex- A_p \geq 0$;

(2) $\Delta_2 = Op - q \{Ex + d_R A_R - (1- d_p) A_p - (P - L) + O_p \} \geq 0$;

When the primary insurer has the monopoly power over the reinsurer, one has $\Delta_2 = 0$. Hence, $P_{min} = O_p = [q/1-q] \; [Ex + d_R A_R - (1- d_p) A_p - P + L]$. (2.3)

On the other hand, when the reinsurance market is monopolized by the reinsurer, one has $\Delta = 0$, i. e.

$$P_{max} = O_p = P - [q/1-q] \; Ap + [q/1-q] \; Ex \qquad (2.4)$$

Moreover, one can derive the price of the reinsurance using the option pricing theory. Once again, let Ex be the exercise price of the option and let p be the hedging probability. Notice that A_p is the present value of the primary insurer without option. Then, one has

$$p \, [(1 - d_p)A_p + P - L] + (1-p)(A_p + P) = A_p \qquad (2.5)$$

One can solve for p from the above equation and has

$$p = [P/L+d_p A_p]. \qquad (2.6)$$

Then, the expected value of the option is $p \, [Ex - (1 - d_p)A_p - P + L]$. In other words, the fair price of the option is

$$P_{fair} = O_p = [P/L+d_p A_p] \, [Ex - (1-d_p)A_p - P + L] + q \, d_R A_R \qquad (2.7)$$

In (2.7), the second term q $d_R A_R$ is the indirect costs to the reinsurer. One may consider it as the transaction costs to the firm. Thus, one may view the first term in (2.7) as the fair price without transaction costs.

Now, one has three different price schemes for the reinsurance contract. The minimum price P_{min}, the maximum one P_{max}, and the fair one P_{fair}. The necessary condition for the existence of the disaster reinsurance and insurance, as stated before, is that $P_{min} \leq P_{max}$. In addition, when the reinsurance contract is priced with P_{fair}, the necessary conditions will be $P_{min} \leq P_{fair} \leq P_{max}$.

III. Numerical Example

Assume that primary insurer has a net asset value of $100 million. If the insurer underwrites policies in a line like disaster insurance, it will, say, encounter potential direct losses of $300 million with a probability 0.20. In addition, when the insured disaster occurs, assume that the insurer's assets are defaulted in the financial market and the default rate is 10%. To avoid its bankruptcy, without reinsurance the primary insurer must charge premiums totaling at least $210 million. Notice that the expected claim costs are only $ 60 million (.20 * $ 300 million) and the expected indirect costs associated with the disaster are $ 2 million (.20 * 10% * $100 million). Therefore, the insurance premiums could be as low as $ 62 million. Suppose that consumers are willing to be insured when the premiums they pay don't exceed $ 80 million. However, the primary insurer can not take advantage of this business due to solvency regulation which considers the insurer's probability of insolvency of 20% when engaged in disaster insurance to be too high.

Suppose that a reinsurance firm has net assets of $300 million. The default rate of the reinsurer's assets is 5% when the reinsurer is engaged in disaster reinsurance. Let Ex be the exercise price the reinsurer offers to the primary insurer and O_p be the price of the option or the premium of reinsurance. From the discussion in section 2.2, one knows that the minimum premium of reinsurance to be:

$$P_{min} = .2/.8 \; [\; Ex + 5\% * 300 - .9*100 - P + 300 \;] = .25 \; [\; Ex - P \;] + 56.25$$

On the other hand, the maximum premium is

$$P_{max} = P - .25 * 100 + .25 \; Ex = P + .25 \; Ex - 25$$

And, the fair premium derived from the OPT is

$P_{fair} = [P/L+d_pA_p] [Ex - (1-d_p)A_p - (P - L)] + q\ d_RA_R$

$= P/310 [Ex - 90 + 300 - P] + 15 q = 3 + P/310 (Ex + 210 -P)$

To have $P_{max} \geq P_{min}$, one needs $P + .25 Ex - 25 \geq .25 [Ex - P] + 56.25$; or $P \geq 65$. In other words, the premiums the policyholders pay must be at least \$ 65 million dollars, which are the total expected costs consisting of the expected direct losses of \$60 million, the expected indirect costs for the primary insurer of \$2 million, and the expected indirect costs for the reinsurer of \$3 million.

In case Ex = 0, one has

$P_{min} = 56.25 - .25 P$; $P_{max} = P -25$; and $P_{fair} = 3 + P(210 - P)/310$.

When Ex = 100, one has

$P_{min} = 81.25 -.25 P$; $P_{max} = P$; and $P_{fair} = 3 + P(310 - P)/310$.

Furthermore, one can explore the pricing of reinsurance given the insurance premiums paid by the policyholders. Suppose that P = 65, then one has

$P_{min} = P_{max} = .25 Ex + 40$; and

$P_{fair} = 3 + 65/310 (Ex + 145) = 33.40 + .21Ex$

So, $P_{fair} < P_{min}$. In other words, if the reinsurance premium is set to equal the P_{fair} determined from the OPT, the reinsurer will suffer a net loss and, therefore, it will have no incentive to do business.

In fact, it can be verified that P must equal to 79.28 to guarantee that $P_{fair} \geq P_{min}$ for Ex = 0.

If P = 79.28 and Ex = 0, then $P_{min} = P_{fair} = 36.43$, and $P_{max} = 54.28$.

It is interesting to notice that a fair reinsurance premium derived using the OPT lead to insurance market failure.

IV. Indirect Costs, Pricing of Reinsurance and Availability of Insurance

The previous section indicates that the reinsurance market and thus insurance market may fail because of the indirect costs and pricing of reinsurance.

First, indirect costs may lead the failure of the insurance markets because of the following reasons. The lowest insurance premiums the policyholders need to pay are the expected losses plus the expected total indirect losses. Therefore, when there are huge indirect losses from the financial market associated with the insured events, the loading factor in the premiums charged to policyholders will be so big that no consumers would want to purchase insurance. On the other hand, when insurance firms can not, because of rate regulation, adjust their premiums to cover the indirect costs, no firms will have the incentive to underwrite disaster insurance policies.

Secondly, the pricing of reinsurance using the OPT may cause the failure of the reinsurance market. Notice that the OPT is based on the assumption that the reinsurance market is perfectly competitive, or there is no arbitrage. If so, the price of the option is whatever its value will be to the primary insurer. As a result, the price derived from the OPT does not guarantee the existence of the reinsurance market because it ignores the profit incentives of the reinsurance firms.

The example demonstrated in section 3 shows that one may have $P_{fair} < P_{min}$; in other words, a reinsurer may not be compensated enough when the price is set up according to P_{fair}. On the other hand, one can find a case with $P_{fair} > P_{max}$. This can happen when the reinsurer suffers huge indirect losses denoted by $D_R A_R$[4].

In addition, the size of the firm affects the availability of insurance. In order to underwrite policies like the ones in disaster insurance, a firm must have enough assets. However, a big firm may not have any competitive advantages due to the indirect costs. To see this, let two firms have assets A_1 and A_2, respectively, with $A_1 > A_2$. And let d_1 and d_2 be the default rate of their assets when the insured event occurs. Thus, the total defaulted assets to the big firm are $d_1 A_1$ and the total defaulted assets to the small firm are $d_2 A_2$. The big firm will have its advantage or low indirect costs only when $d_1 A_1 < d_2 A_2$, or $d_1 < d_2 A_2/A_1$. Assume that the big firm has twice the assets of the small firm. Then, the big firm has lower indirect costs only when its default rate is less than half of that of the small firm.

V. Conclusions

By making insurers' cost include indirect costs from the financial market as well as direct costs associated with the insured event, this paper illustrates conditions for the existence of disaster insurance and reinsurance. The study demonstrates that investors' negative response to an insurer's huge liability exposures related to the disaster could prohibit that insurer from continuing its coverage to the policyholders. A large insurer is not necessarily more willing to engage in underwriting disaster insurance unless it can minimize its value losses in the financial market caused by the liability exposures.

Moreover, the paper gives three different pricing schemes for reinsurance -- a maximum price, a minimum price, and a fair price, based on certain market conditions. The study demonstrates that a fair reinsurance premium derived from the OPT may lead the failure of all insurance markets.

Endnotes

1. An insurer will be able to bear such a risk when it can raise extra money from the financial market. Here, the assumption is that raising needed money is too costly for the insurer in the short time.

2. Each insurer is required to have a minimum surplus ratio by the state's solvency regulation.

3. The relation between the stop loss and the put option is straight forward. Let $\bar{L}$ be the stop loss point, i. e. the reinsurer will compensate the primary insurer for all extra losses when the primary insurer's loss exceeds $\bar{L}$; then, $Ex = (1 - d_p) A_p + P - O_p - \bar{L}$.

4. Notice that $D_R A_R$ occurs in the P_{fair} but it does not occur in the P_{max}.

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