# Sensitivity Analysis on Inputs for a Bond Portfolio Management Model

Marida Bertocchi[1], Jitka Dupačová[2] and Vittorio Moriggia[3]

**Abstract**

Management of portfolio of fixed income securities is formulated as a multiperiod scenario based stochastic program with random recourse. Stochasticity is introduced by modelling the evolution of interest rates through scenarios based on a binomial lattice obtained by the Black-Derman-Toy approach. The main aim of the contribution is to discuss the sensitivity of the solution of the stochastic program with respect to various levels of the model input (the choice of scenarios, the fitted binomial lattice and the term structure). An application to the Italian market is given.

**Résumé**

La gestion du portefeuille des titres à revenus fixe est formulée par un programme stochastique fondé sur un scénario multipériodique avec récursion aléatoire. La stochasticité a été introduite par la simulation de l´évolution des taux d´intérêt à travers les scénarios basés sur un réseau binomial réalizé par la modèle de Black-Derman-Toy. Le but principal de cette contribution étudie la sensibilité de la solution du programme stochastique à l´égard de diverses niveaux des donnés introduites dans le modèle (le choix des scénarios, le réseau binomial adapté et la structure à terme). On présente une application dans le marché italien.

**Keywords**

Stochastic program, interest rate scenarios, sensitivity.

1) University of Bergamo, Department of Mathematics, Piazza Rosate 2, I - 24129 Bergamo (Italy); Tel: + 39-35-277517, Fax: + 39-35-249598, E-mail: marida@ibguniv.unibg.it

2) Charles University Prague, Department of Probability and Mathematical Statistics, Sokolovská 83, CZ - 18600 Prague (Czech-Republic); Tel/Fax: + 42-2-2323316, E-mail: dupacova@karlin.mff.cuni.cz

3) University of Bergamo, Department of Mathematics, Piazza Rosate 2, I - 24129 Bergamo (Italy); Tel: + 39-35-277711, Fax: + 39-35-249598, E-mail: cmsvitt@ibguniv.unibg.it

# 1 PROBLEM FORMULATION

We shall deal with a stochastic programming model designed for a bond portfolio management problem which has been introduced and described in detail in several papers, e.g. Golub et al. (1995), Dupačová and Bertocchi (1996). The main random factor is the stochastic process of the short term interest rates which determines the prices of bonds: Given a sequence of equilibrium future short term interest rates $r_t$ valid for the time interval $(t, t+1], t = 0, \ldots, T-1$ the fair price of a bond at time $t$ equals the total cashflow of $\{f_\tau\}$ generated by this bond in subsequent time instances discounted to $t$:

$$P_t(\mathbf{r}) = \sum_{\tau=t+1}^{T} f_\tau D_t^\tau \tag{1}$$

where $D_t^\tau = \prod_{h=t}^{\tau-1} (1 + r_h)^{-1}$ and $T$ is greater or equal to the time to maturity.

Various stochastic models of interest rates were developed; we shall use here the discrete binomial model of Black, Derman and Toy (1990) over a fixed horizon $T$ that provides a discrete distribution, say $P$, of $S = 2^{T-1}$ possible vectors $\mathbf{r}$ of interest rates concentrated with equal probabilities at points $\mathbf{r}^s \in R^T, s = 1, \ldots, S$ called *scenarios*; this is mostly considered as the input information which is used to build the optimization model and which influences the results. As we shall see, several steps precede construction of these scenarios; they are based on market data and the corresponding calibration of the binomial lattice is computationally demanding. The aim of the paper is to clarify how the uncertainties and numerical errors at distinct levels of processing the input influence the binomial lattice and, subsequently, the output of the optimization model.

The main purpose of the considered portfolio management model is to maximize the expected utility of the wealth at the end of a given period and, depending on the specific field of investment activities, to secure the prescribed or uncertain future payments. An *active trading strategy,* which allows for rebalancing the portfolio, is permitted under constraints on conservation of holdings for each asset at each time period and on conservation of cashflows.

We shall mostly use the notation introduced in Golub et al. (1995), see also Dupačová and Bertocchi (1996):

$j = 1, \ldots, J$ are indices of the considered bonds and $T_j$ the dates of their maturities;

$t = 0, \ldots, T_0$ is the considered discretization of the planning horizon;

$b_j$ denote the initial holdings (in face value) of bond $j$;

$b_0$ is the initial holding in the riskless asset;

$f_{jt}^s$ is the cashflow generated from bond $j$ at time $t$ under scenario $s$ expressed as a fraction of its face value;

$\xi^s_{jt}$ and $\zeta^s_{jt}$ are the selling and purchasing prices of bond $j$ at time $t$ for scenario $s$ obtained from the corresponding fair prices (1) by subtracting or adding fixed transaction costs and spread; the initial prices $\xi_{j0}$ and $\zeta_{j0}$ are known, i. e., scenario independent;

$L_t$ is liability due at time $t$;

$x_j/y_j$ are face values of bond $j$ purchased / sold at the beginning of the planning period, i.e., at $t = 0$, nonnegative *first stage decision variables*;

$z_{j0}$ is the face value of bond $j$ held in portfolio after the initial decisions $x_j, y_j$ have been made and the auxiliary nonnegative variable $y_0^+$ denotes the surplus.

The second stage decision variables on rebalancing, borrowing and reinvestment, $x^s_{jt}, y^s_{jt}, z^s_{jt}, y_t^{-s}, y_t^{+s}$ as well as the wealth $W^s_{T_0}$ at the end of the planning horizon are scenario dependent.

The model is

$$\text{maximize} \quad \sum_s p_s U(W^s_{T_0}) \tag{2}$$

subject to the first stage constraints on conservation of holdings

$$y_j + z_{j0} = b_j + x_j \quad \forall j \tag{3}$$

and on cashflow

$$y_0^+ + \sum_j \zeta_{j0} x_j = b_0 + \sum_j \xi_{j0} y_j \tag{4}$$

subject to the second stage constraints for individual interest rate scenarios on conservation of holdings

$$z^s_{jt} + y^s_{jt} = z^s_{j,t-1} + x^s_{jt} \quad \forall j, s, 1 \le t \le T_0 \tag{5}$$

and on cashflow (including rebalancing the portfolio) at each time period $1 \le t \le T_0$

$$\begin{aligned} &\sum_j \xi^s_{jt} y^s_{jt} + \sum_j f^s_{jt} z^s_{j,t-1} + (1 + r^s_{t-1}) y^{+s}_{t-1} + y_t^{-s} = \\ &L_t + \sum_j \zeta^s_{jt} x^s_{jt} + (1 + \delta + r^s_{t-1}) y^{-s}_{t-1} + y_t^{+s} \quad \forall s, t \end{aligned} \tag{6}$$

under nonnegativity of all variables and with

$$W^s_{T_0} = \sum_j \xi^s_{jT_0} z^s_{jT_0} + y^{+s}_{T_0} - \alpha y^{-s}_{T_0} \quad \forall s \tag{7}$$

The multiplier $\alpha$ in (7) should be fixed according to the problem area. For instance, a pension plan assumes repeated application of the model with rolling horizon and values $\alpha > 1$ take into account the debt service in the future.

Thanks to the assumed possibility of reinvestments and of unlimited borrowing, the problem has always a feasible solution. The existence of optimal solutions is guaranteed for a large class of utility functions that are *increasing and concave* what will be assumed henceforth. From the point of view of stochastic programming, it is a *scenario based multiperiod two-stage model with random relatively complete recourse* and with additional nonlinearities due to the choice of the utility function.

The size of the resulting deterministic program (2) – (7) as well as the numerical values of the coefficients result from the choice of the considered bonds, their characteristics (initial prices and cashflows) and initial holdings, from the scheduled stream of liabilities, from the used model of interest rates and the market data used to fit the model, and from the way how a modest number of scenarios was selected out of the whole population. This is the input. The main outcome is the optimal value of the objective function (the maximal expected utility of the final wealth) and the optimal values of the first-stage variables $x_j, y_j$ (and $z_{j0}$) for all $j$.

We shall assume that the models applied on the input side of the bond portfolio management problem have been fixed according to the past experience. In the context of Black - Derman - Toy model of interest rates, that we want to apply, it means that a specific form of a successfully tested yield curve is used to get the term structure. Even in this case there are numerous sources of errors that influence the input of the large scale mathematical program (2) – (7):

• The market data of the given day are used to fit the yield curve, i. e., to estimate the coefficients in the chosen nonlinear regression model and to estimate the yields or prices of zero coupon governmental bonds of all required maturities $t = 1, \ldots, T$. In addition, a plausible hypothesis about volatility of these yields (i.e., about variances of log yields) is needed. The estimated prices or yields of zero coupon governmental bonds of all maturities together with their volatilities are called the initial *term structure*. Evidently, both statistical and numerical errors enter the initial term structure.

• The next step is to build the full binomial lattice of the one-period short rates up to the horizon $T$. The base rates and volatilities of Black - Derman - Toy model are fitted by Newton - Raphson method; the idea is to get an agreement of prices and volatilities that correspond to the initial term structure with those obtained by the model. The procedure requires to solve a system of $2T$ nonlinear equations; this has been elaborated by Kang and Zenios (1994). The errors in the resulting base rates and volatilities stem not only from the numerical method but also from the estimated term structure. Given the initial error in the term structure, it is possible to deduce how errors propagate in the lattice and therefore how they influence coefficients of (2) – (7).

• A sampling procedure has to be used to get a manageable number of scenarios out of the fitted binomial lattice. One of possibilities is the nonrandom sampling strategy by Zenios and Shtilman (1993) which has been analysed already in Dupačová and Bertocchi (1996).

• The final step is solution of the large mathematical program (2) – (7) whose coefficients are burdened by all mentioned errors. The question is the sensitivity of the optimal first trading strategy and of the optimal value of the objective function on the above mentioned errors and what changes can be expected in connection with inclusion of additional out-of-sample scenarios.

In this paper we shall extend the results presented in Dupačová and Bertocchi (1996) related to the influence of sampling strategy on the optimal value. A brief description of the technique is given in Section 2 and an application to the Italian bond market is presented in Section 3. For the progress in the analysis of the impact of errors due to estimation of the initial term structure see the report Dupačová et al. (1996) and for influence of additional scenarios see Dupačová (1996a, b).

## 2 SAMPLING STRATEGY

The basic assumptions of the Black-Derman-Toy model can be summarized as follows:

• The short rate is the only factor that drives the bond prices, it can move up or down with equal probability over the next time period; the sequences of "up - down" and "down - up" moves from any fixed stage at a time point $t$ result into the same value of interest rate at the time point $t+2$ (the path independence property).

• The expected returns on all securities over one period are equal, short rates are lognormally distributed with the volatility of their logarithms that depends only on time.

• The input is the yield curve and yield volatilities valid for zero-coupon governmental bonds at a given date; this input should be available for all maturities.

• The securities are valued as the expected prices one period ahead discounted by the present short rate.

As a result, at each time point $l$, there are $l+1$ possible stages and for the given horizon $T$ there are $2^{T-1}$ equiprobable scenarios. Each of them can be represented by a random binary fraction with $T-1$ 0-1 digits, say

$$\omega^s = 0.\omega_1^s\omega_2^s\ldots\omega_{T-1}^s$$

with $\omega_l^s = 0$ or $1\ \forall l, s$. The digit 1 at the $l$-th position corresponds to the "up" move, the digit 0 corresponds to the "down" move of the one-period short term interest rate in the step $l$. This theoretical binomial lattice has to be calibrated by the existing term structure to get the base rates $r_{l0}$ and the volatility factors $k_l$ for all $l$. The corresponding one-period short term rates for scenario $s$ and for the time interval $(l, l+1]$ are then given as

$$r_l^s = r_{li_l(s)}$$

where

$$r_{li} = r_{l0}k_l^i,\ i_l(s) = \sum_{\tau=1}^{l} \omega_\tau^s \tag{8}$$

That is, $i_l(s)$ equals the number of the "up" moves for the given scenario $s$ which occur at time points $1, \dots, l$.

Our sensitivity analysis will be related to a simplified version of the deterministic sampling strategy by Zenios and Shtilman (1993): We fix $L, 1 < L < T$ and assign the index $s, s = 1, \dots, 2^L$ to each possible binary fraction of length $L$. The sample point $\omega^s$ from (0,1) is determined by one of these binary fractions and by an arbitrary continuation up to binary fraction of length $T$. We build then $S = 2^L$ scenarios $\mathbf{r}^s$.

The lower and upper bounds for $r_t^s$ with $t \geq L$ are evident:

$$r_l^{s-} = r_{l0} k_l^{i_L} \leq r_l^s \leq r_{l0} k_l^{l-L+i_L(s)} = r_l^{s+}, \, l = L+1, \dots, T-1 \, \forall s \tag{9}$$

and for $l \leq L$, $r_l^s$ are fully determined by the described choice of the path $\omega^s$. The input of our problem (2)-(7) consists thus of $S$ $T-$dimensional scenarios $\mathbf{r}^s$ whose first $L$ components are fixed whereas the subsequent $T - L$ components are subject to perturbations $\Delta^s$ such that

$$\begin{aligned} \Delta_l^s &= 0, \, l = 1, \dots, L \\ \Delta_{L+\tau}^s &= r_{L+\tau} - r_{L+\tau}^s, \, l = L+1, \dots, T-1 \end{aligned} \tag{10}$$

where $r_{L+\tau}$ satisfies (9).

We denote further $\varphi(\mathbf{r}^1, \dots, \mathbf{r}^S)$ the optimal value of (2)-(7) for the initial "input" $\mathbf{r}^1, \dots, \mathbf{r}^S$ and we indicate by asterisk the components of the corresponding optimal solution and of Lagrange multipliers.

It has been shown, see Dupačová and Bertocchi (1995), that the linearly perturbed problem that corresponds to the input $\mathbf{r}^s + \mu\Delta^s, s = 1, \dots, S$ has an optimal solution for $\mu$ small enough. Moreover, for arbitrary feasible perturbances $\Delta^s$, there exists the directional derivative of the optimal value function at the given input $\mathbf{r}^1, \dots, \mathbf{r}^S$ in any feasible direction $\Delta^s, s = 1, \dots, S$ and equals the derivative at $\mu = 0^+$ of the Lagrange function of the corresponding linearly perturbed problem evaluated at the initial optimal solution and multipliers

$$\varphi'(0^+) = \frac{\partial}{\partial \mu} L(\mathbf{x}^*, \mathbf{y}^*, \mathbf{z}^*, \mathbf{W}^*; \lambda^*; \mathbf{r}^s + \mu\Delta^s, s = 1, \dots, S)|_{\mu=0^+} \tag{11}$$

(cf. Gol'shtein (1970)).

It has been proved that we can obtain separability with respect to scenarios *and* time periods:

$$\varphi'(0^+) = \sum_{s=1}^{S} \sum_{l=L+1}^{T-1} \frac{\Delta_l^s}{1 + r_l^s} H_l^s \tag{12}$$

where

$$H_l^s = -\sum_{t=1}^{l} \lambda_{6t}^{s*} \sum_j \sum_{\tau=l+1}^{T} f_{j\tau}^s D_t^\tau(\mathbf{r}^s)(y_{jt}^{s*} - x_{jt}^{s*}) + \lambda_{6,l+1}^{s*}(1 + r_l^s)(y_l^{+s*} - y_l^{-s*})$$
$$L + 1 \le l \le T_0 - 1$$
$$H_l^s = - \sum_{t=1}^{T_0} \lambda_{6t}^{s*} \sum_j \sum_{\tau=l+1}^{T} f_{j\tau}^s D_t^\tau(\mathbf{r}^s)(y_{jt}^{s*} - x_{jt}^{s*}) + \lambda_7^{s*} \sum_j \sum_{\tau=l+1}^{T} f_{j\tau}^s D_{T_0}^\tau(\mathbf{r}^s) z_{jT_0}^{s*}$$
$$T_0 \le l \le T - 1$$

The desired directions of changes in $r_l^s$ for $l > L$ that result in decrease and/or in increase of the optimal value function can be thus obtained by inspection of the *signs* of $H_l^s$ only. The magnitude of these changes is limited by (9) (and also by the fact that this result is of a *local* character).

Notice that allowing cashflows dependent on scenarios we are able to extend the results od Dupačová and Bertocchi (1996) to portfolios that include bonds with call or put options. This means to mark the scenarios for which the option on a bond is likely to be exercized and to update the cashflows and prices that correspond to the bonds with options and to the corresponding marked scenarios accordingly. Clearly, both the program (2) - (7) and the subsequent sensitivity analysis have to based on the updated cashflows and prices.

## 3 APPLICATION TO ITALIAN BOND MARKET

The numerical results are related to the mentioned sampling strategy. Models with various utility functions have been developed and used to simulate the behaviour of an investment portfolio of fixed income securities on the Italian bond market within the time horizon of one year ($T_0 = 12$). The sample of bonds and all the informations come from a local bank; the spread was fixed at 2.5% and the transaction costs at 1%. Details are given in Bertocchi et al. (1996).

In this type of application, no liabilities are considered, liquidity can be obtained from the interbank market at the corresponding rate and the surplus can be always reinvested; accordingly, we put in (7) $\alpha = 1$. For the numerical illustration, scenarios based on real life data from Italian bond market were generated. We considered portfolios that include typical governmental bonds, paying semi-annual coupons and covering two year forward till 29 years maturities (the so called BTPs) as well as portfolios including puttable bonds (CTOs), paying semi-annual coupons with maturities of 8 years and possible exercise of the option in the 4th year or 6 years and exercise at the 3rd year.

TABLE 1

| Bonds | Qt | coupon | redemption | payment dates | | maturity |
|---|---|---|---|---|---|---|
| BTP36658 | 10 | 3.9375 | 100.1875 | 01/04 | 01/10 | 01/10/96 |
| BTP36631 | 20 | 5.03125 | 99.5313 | 01/03 | 01/09 | 01/03/98 |
| BTP12687 | 15 | 5.25 | 99.2312 | 01/01 | 01/07 | 01/01/2002 |
| BTP36693 | 10 | 3.71875 | 99.3875 | 01/08 | 01/02 | 01/10/2004 |
| BTP36665 | 5 | 3.9375 | 99.2188 | 01/05 | 01/11 | 01/11/2023 |

Actually, we studied several cases with different criteria. The first criterion depends on the number of periods $L$ used to build the fully covered paths. Table 2 lists the three different considered cases.

TABLE 2

| case | $L$ | no. of scenarios |
|---|---|---|
| A | 5 | $2^5 = 32$ |
| B | 3 | $2^3 = 8$ |
| C | 4 | $2^4 = 16$ |

For each of those cases, we built four different subcases depending on four types of paths, as described by digits $\omega_{L+1}^s, ..., \omega_{T_0+1}^s$, $(s = 1, ..., 2^L)$ of the binary fraction.

Combining these criteria we get 12 different cases, as summarized in Table 3. These 12 cases of simulation have been applied both with the linear and exponential utility function:

$$U(W_{T_0}^s) = \frac{1}{\gamma}\left(1 - e^{-\gamma W_{T_0}^s}\right)$$

where $\gamma$ represents the risk aversion factor. We studied three exponential subcases for $\gamma = 0.1,\ 0.01,\ 0.001$. Accordingly, the numerical experiments reported here consist of 48 different cases.

TABLE 3

| case | $l = 1, ..., L$ | $L+1$ | $l = L+2, ..., T_0$ | $T_0 + 1$ |
|---|---|---|---|---|
| A1 | $s = 1, ..., 2^5$ | $\omega_{L+1}^s = 0$ | $\omega_l^s = 0$ | $\omega_{T_0+1}^s = 1$ |
| A2 | $s = 1, ..., 2^5$ | $\omega_{L+1}^s = 0$ | $\omega_l^s = 0$ | $\omega_{T_0+1}^s = 0$ |
| A3 | $s = 1, ..., 2^5$ | $\omega_{L+1}^s = 0$ | $\omega_l^s = 1$ | $\omega_{T_0+1}^s = 1$ |
| A4 | $s = 1, ..., 2^5$ | $\omega_{L+1}^s = 1$ | $\omega_l^s = 1$ | $\omega_{T_0+1}^s = 1$ |
| B1 | $s = 1, ..., 2^3$ | $\omega_{L+1}^s = 0$ | $\omega_l^s = 0$ | $\omega_{T_0+1}^s = 1$ |
| B2 | $s = 1, ..., 2^3$ | $\omega_{L+1}^s = 0$ | $\omega_l^s = 0$ | $\omega_{T_0+1}^s = 0$ |
| B3 | $s = 1, ..., 2^3$ | $\omega_{L+1}^s = 0$ | $\omega_l^s = 1$ | $\omega_{T_0+1}^s = 1$ |
| B4 | $s = 1, ..., 2^3$ | $\omega_{L+1}^s = 1$ | $\omega_l^s = 1$ | $\omega_{T_0+1}^s = 1$ |
| C1 | $s = 1, ..., 2^4$ | $\omega_{L+1}^s = 0$ | $\omega_l^s = 0$ | $\omega_{T_0+1}^s = 1$ |
| C2 | $s = 1, ..., 2^4$ | $\omega_{L+1}^s = 0$ | $\omega_l^s = 0$ | $\omega_{T_0+1}^s = 0$ |
| C3 | $s = 1, ..., 2^4$ | $\omega_{L+1}^s = 0$ | $\omega_l^s = 1$ | $\omega_{T_0+1}^s = 1$ |
| C4 | $s = 1, ..., 2^4$ | $\omega_{L+1}^s = 1$ | $\omega_l^s = 1$ | $\omega_{T_0+1}^s = 1$ |

TABLE 4

| BB3 | | FAR=.1 | FAR=.01 | FAR=.001 | Linear |
|---|---|---|---|---|---|
| x | BTP36658 | 0 | 0 | 0 | 0 |
| | BTP36631 | 0 | 0 | 0 | 0 |
| | BTP12687 | 0 | 0 | 0 | 0 |
| | BTP36693 | 0 | 0 | 0 | 0 |
| | BTP36665 | 0 | 0 | 0 | 0 |
| y | BTP36658 | 7.720198 | 0 | 10 | 10 |
| | BTP36631 | 20 | 0 | 20 | 20 |
| | BTP12687 | 15 | 0 | 15 | 15 |
| | BTP36693 | 10 | 10 | 10 | 10 |
| | BTP36665 | 1.917787 | 0 | 5 | 5 |

| CC3 | | FAR=.1 | FAR=.01 | FAR=.001 | Linear |
|---|---|---|---|---|---|
| x | BTP36658 | 0 | 0 | 0 | 0 |
| | BTP36631 | 0 | 0 | 0 | 0 |
| | BTP12687 | 0 | 0 | 0 | 0 |
| | BTP36693 | 0 | 0 | 0 | 0 |
| | BTP36665 | 0 | 0 | 0 | 0 |
| y | BTP36658 | 7.613897 | 0 | 10 | 10 |
| | BTP36631 | 20 | 0 | 20 | 20 |
| | BTP12687 | 15 | 0 | 15 | 15 |
| | BTP36693 | 10 | 10 | 10 | 10 |
| | BTP36665 | 2.439581 | 0 | 5 | 5 |

| AA3 | | FAR=.1 | FAR=.01 | FAR=.001 | Linear |
|---|---|---|---|---|---|
| x | BTP36658 | 0 | 0 | 0 | 0 |
| | BTP36631 | 0 | 0 | 0 | 0 |
| | BTP12687 | 0 | 0 | 0 | 0 |
| | BTP36693 | 0 | 0 | 0 | 0 |
| | BTP36665 | 0 | 0 | 0 | 0 |
| y | BTP36658 | 7.846258 | 0 | 10 | 10 |
| | BTP36631 | 20 | 0 | 20 | 20 |
| | BTP12687 | 15 | 2.880924 | 15 | 15 |
| | BTP36693 | 10 | 10 | 10 | 10 |
| | BTP36665 | 4.197942 | 0 | 5 | 5 |

and for $l > T_0 + 1$ the values $\omega_l^s$ alternate up and down (1 or 0) starting with the indicated value of $\omega_{T_0+1}$.

The optimal strategy of rebalancing depends on the case considered, i. e., on the choice of the risk aversion parameter and on the number and type of scenarios. The main results, i. e., the trading strategy at the beginning, are summarized in Table 4 and for an illustration of typical subsequent scenario based decisions see the attached Figures 1–3.

The results on sensitivity analysis differ essentially for the linear utility function and for the exponential one. For the nonlinear case, the Lagrange multipliers are negligible what gives the coefficients $H_l^s$ in (12) and the derivative $\varphi'(0^+)$ near to zero. It means that the output is not sensitive on the choice of interest rate scenarios from the full binomial lattice as described in Zenios and Shtilman (1993).

For the linear case, however, the Lagrange multipliers are non zero and we have obtained negative values for $H_l^s$ for $L \leq l < T_0$ and positive values for $l \geq T_0$ in all considered experiments. Notice, however, the following differences: The cases A1-2, B1-2 and C1-2 do not allow to choose scenarios with lower interest rates than those applied for $L + 1 \leq l < T_0$. Hence, all admissible changes $\delta_l^s, L + 1 \leq l < T_0$ lead to a local decrease of the optimal value. On the other hand, for A4, B4 and C4 all admissible changes $\delta_t^s, L+1 \leq t < T_0$ provide a local increase of the optimal value. Similarly, one can discuss the cases A3, B3 and C3 and also the influence of admissible changes of scenarios for $t > T_0$.

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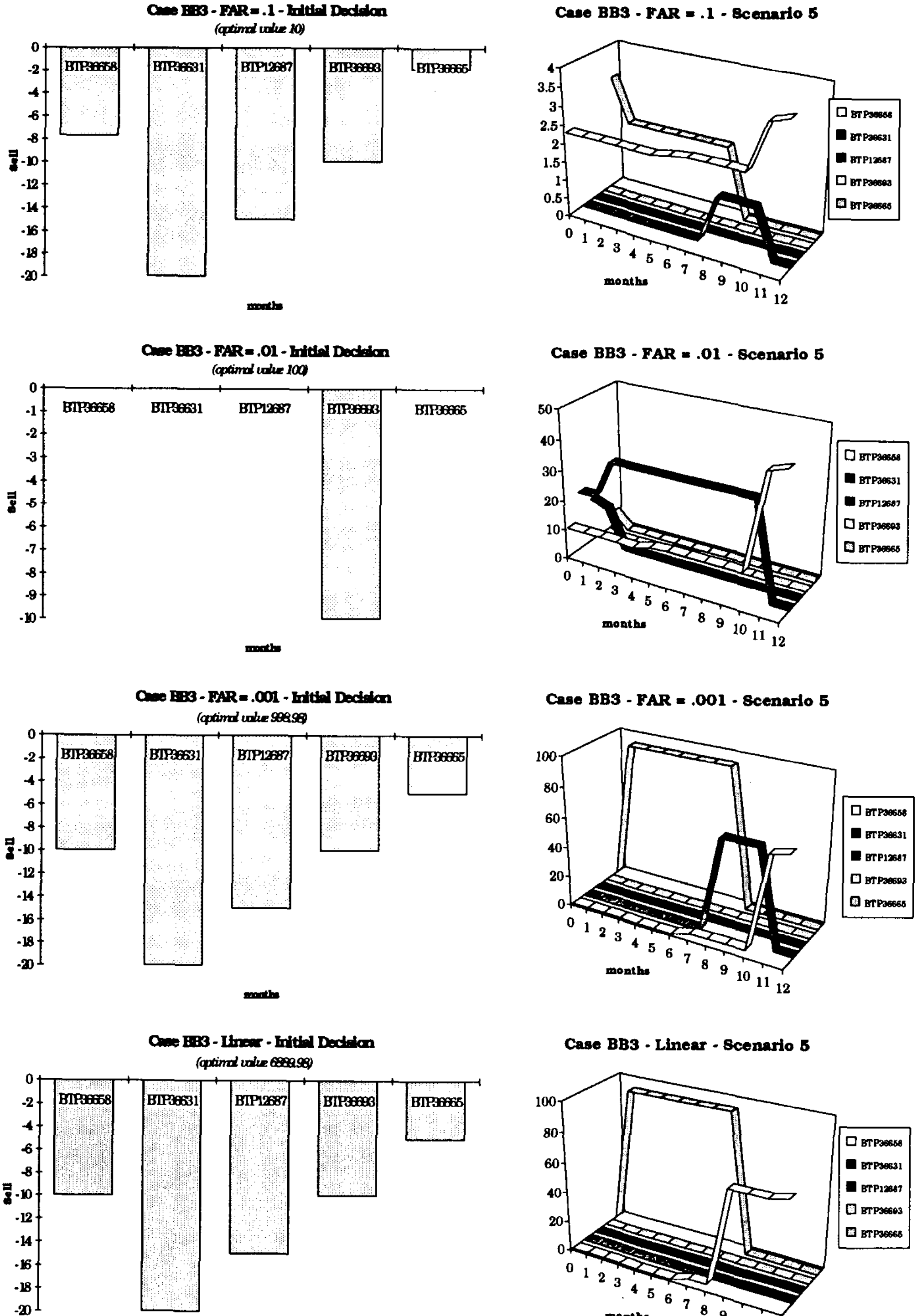
Case BB3 - FAR = .1 - Initial Decision
(optimal value 10)
BTP36658
BTP36631
BTP12687
BTP36693
BTP36665
Sell
months
Case BB3 - FAR = .1 - Scenario 5
months
Case BB3 - FAR = .01 - Initial Decision
(optimal value 100)
Sell
months
Case BB3 - FAR = .01 - Scenario 5
months
Case BB3 - FAR = .001 - Initial Decision
(optimal value 998.98)
Sell
months
Case BB3 - FAR = .001 - Scenario 5
months
Case BB3 - Linear - Initial Decision
(optimal value 6989.98)
Sell
months
Case BB3 - Linear - Scenario 5
months

Figure 1

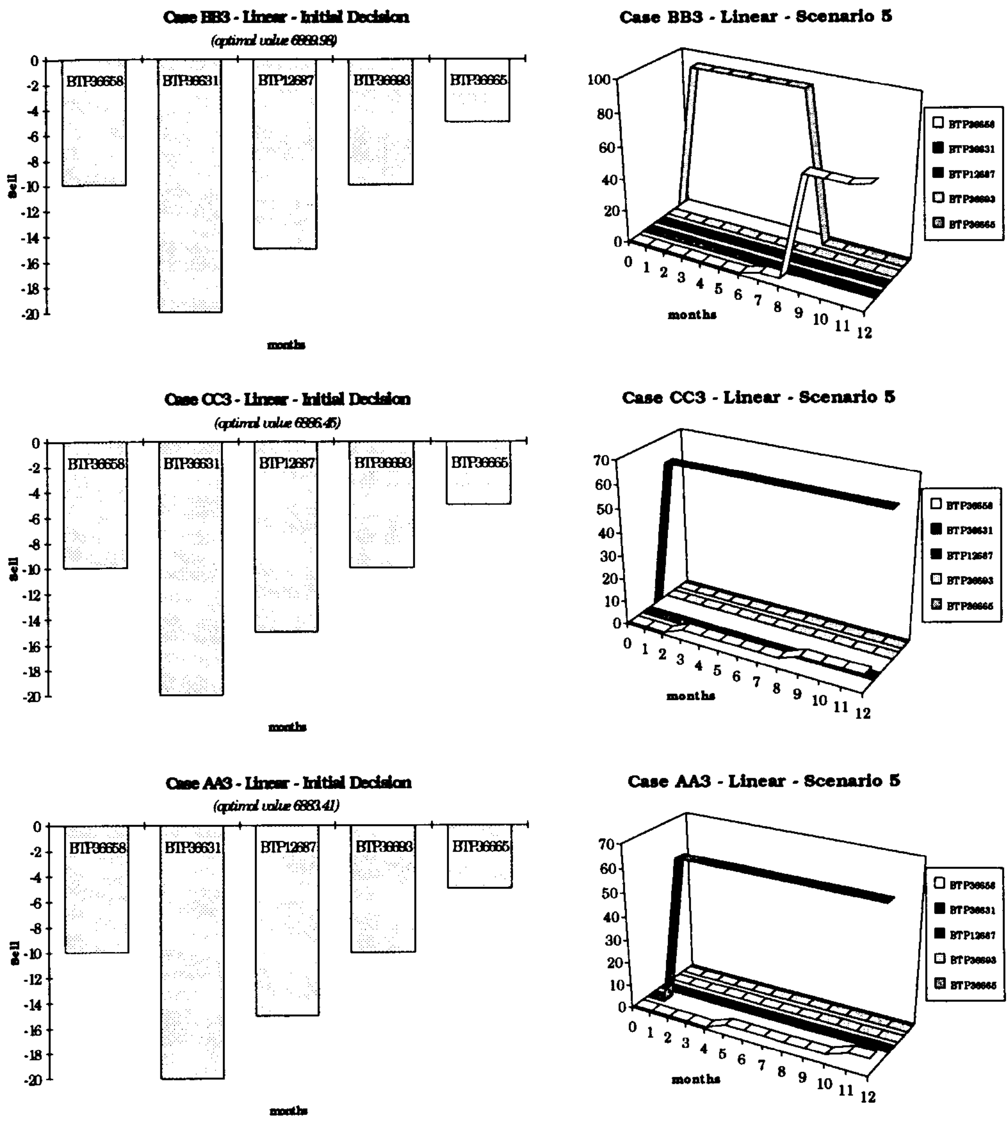
Case BB3 - Linear - Initial Decision
BTP36658
BTP36631
BTP12687
BTP36693
BTP36665
Sell
months
Case BB3 - Linear - Scenario 5
months
Case CC3 - Linear - Initial Decision
(optimal value 6886.45)
BTP36658
BTP36631
BTP12687
BTP36693
BTP36665
Sell
months
Case CC3 - Linear - Scenario 5
months
Case AA3 - Linear - Initial Decision
(optimal value 6883.41)
BTP36658
BTP36631
BTP12687
BTP36693
BTP36665
Sell
months
Case AA3 - Linear - Scenario 5
months

**Figure 2**

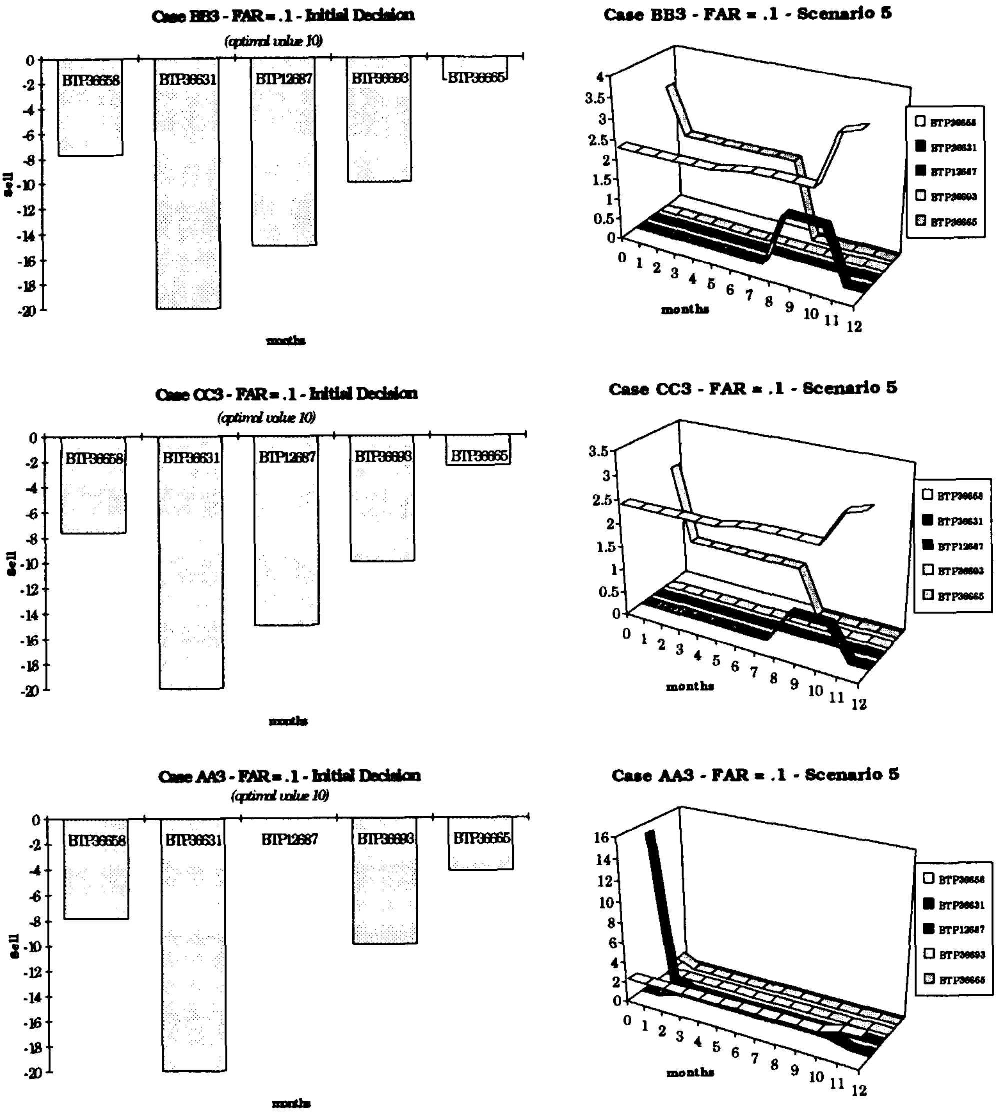
Case BB3 - FAR = .1 - Initial Decision
(optimal value 10)
BTP36658
BTP36631
BTP12687
BTP36693
BTP36665
Sell
months
Case BB3 - FAR = .1 - Scenario 5
months
Case CC3 - FAR = .1 - Initial Decision
(optimal value 10)
BTP36658
BTP36631
BTP12687
BTP36693
BTP36665
Sell
months
Case CC3 - FAR = .1 - Scenario 5
BTP36658
BTP36631
BTP12687
BTP36693
BTP36665
months
Case AA3 - FAR = .1 - Initial Decision
(optimal value 10)
BTP36658
BTP36631
BTP12687
BTP36693
BTP36665
Sell
months
Case AA3 - FAR = .1 - Scenario 5
BTP36658
BTP36631
BTP12687
BTP36693
BTP36665
months

Figure 3