AFIR 2011

To hedge or not to hedge

that is the problem

Dr. Ricardo Tagliafichi

The crisis of confidence

In 2008 the crisis of confidence begins when the financial institutions, like banks, pensions funds and insurance companies can't comply with the depositors demand or catastrophes liquidations

The lack of solvency appeared due the lack of balance between assets and liabilities

A. L. M.

Asset Liability Management

The ALCO committee of each company not only must established conditions on the balance between Assets and Liabilities, also must determine a good polite on reserves hedging portfolio risks

The actuaries must estimate the risks that directly affect the investment portfolio as a result of:

Deposits

Mathematical and technical reserves

Investment Portfolio and Risks

Investment Portfolio	
Fixed income investment	Variable income investment
• Bonds • Notes • Mortgages • Loans with guarantee • Long term debt • Other loans	• Equities • Foreign exchange rate • Indices • Commodities
Risk involved	
• Market Risk • Credit Risk	• Market Risk

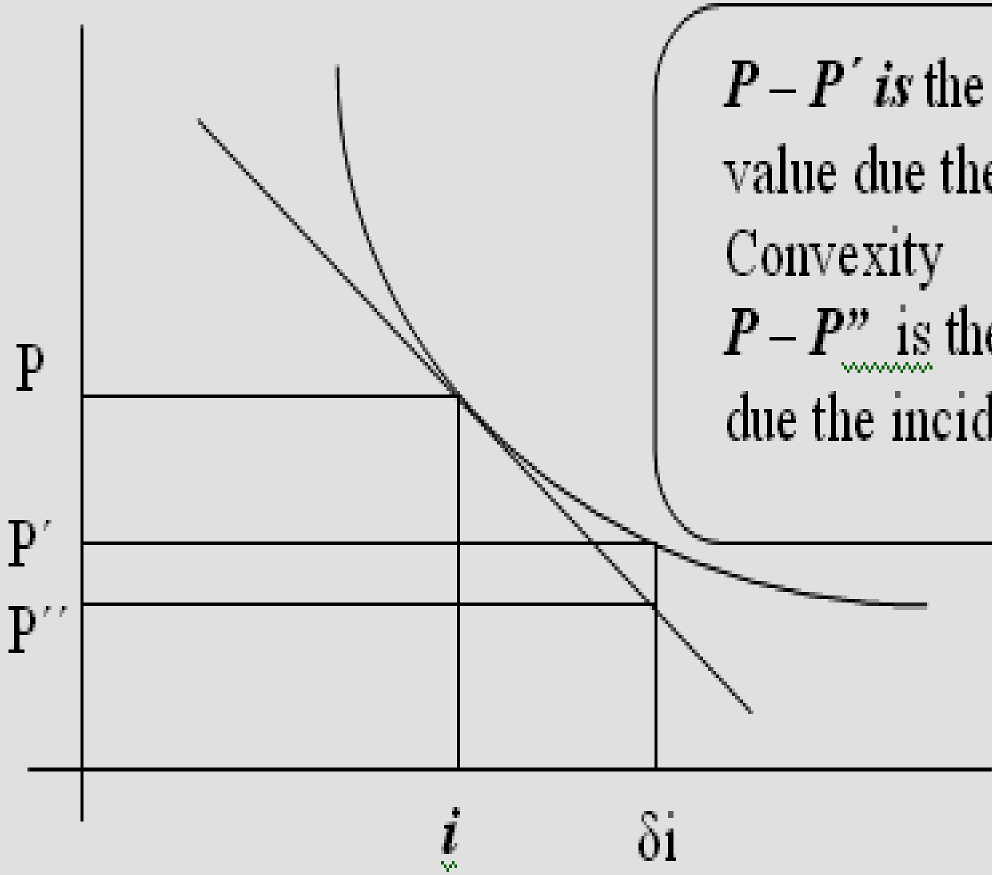
To estimate the Risks we use VaR models

There are two problems with the use of VaR

1) The model may pass the back testing and reflects the reality market movements

2) Once we estimate VaR please make reserves reducing results and try not to distributes earnings not realized

Market VaR for Fixed Rent Portfolio



P – P′ is the net variation of portfolio value due the incidence of Duration and Convexity

P – P” is the variation of portfolio value due the incidence of only Duration

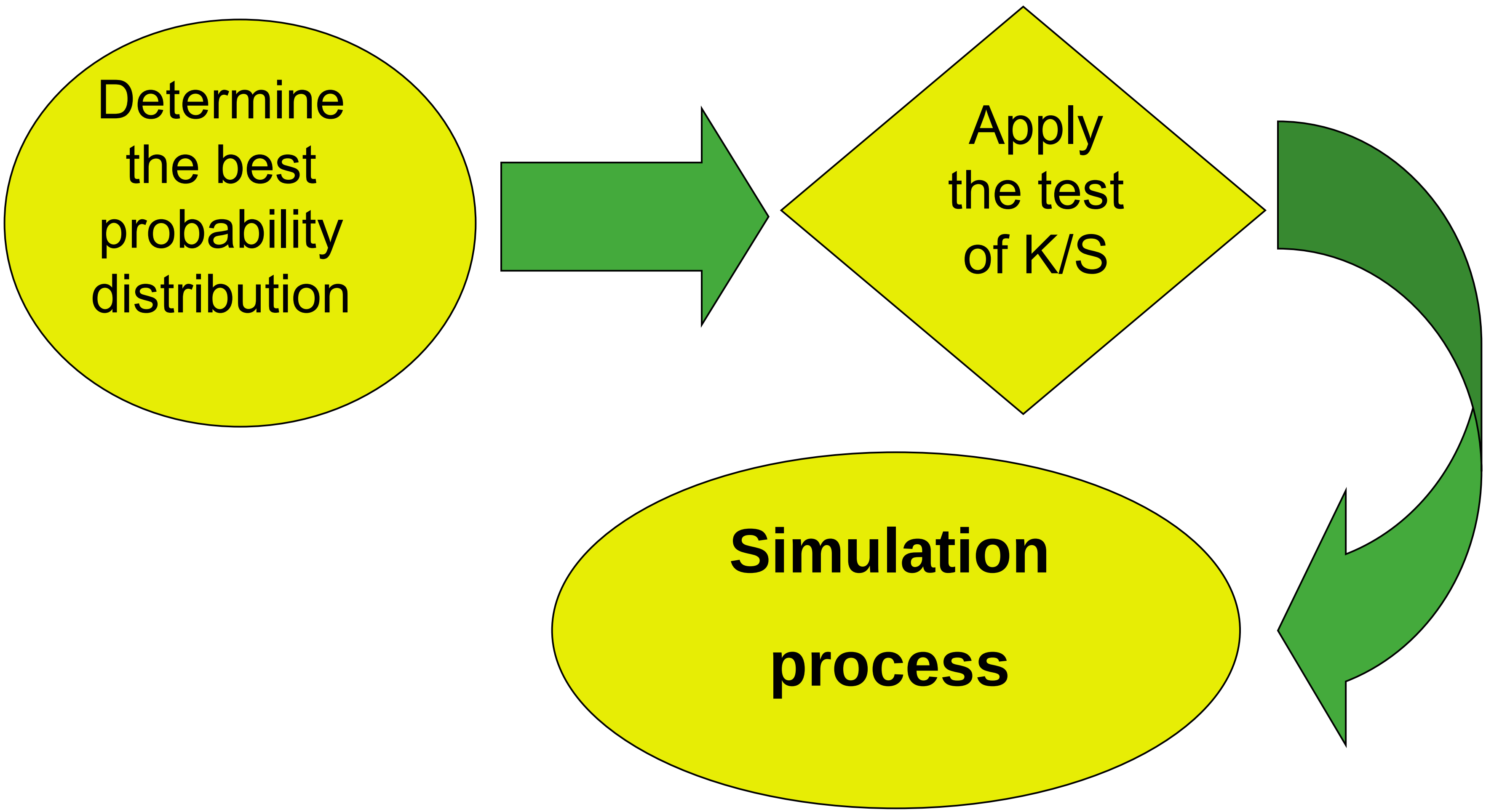
The problem is to estimate δi

$$-MD = -\frac{1}{(1+i)} \frac{\sum_{t=1}^{n} \frac{tCF_t}{(1+i)^t}}{P}$$

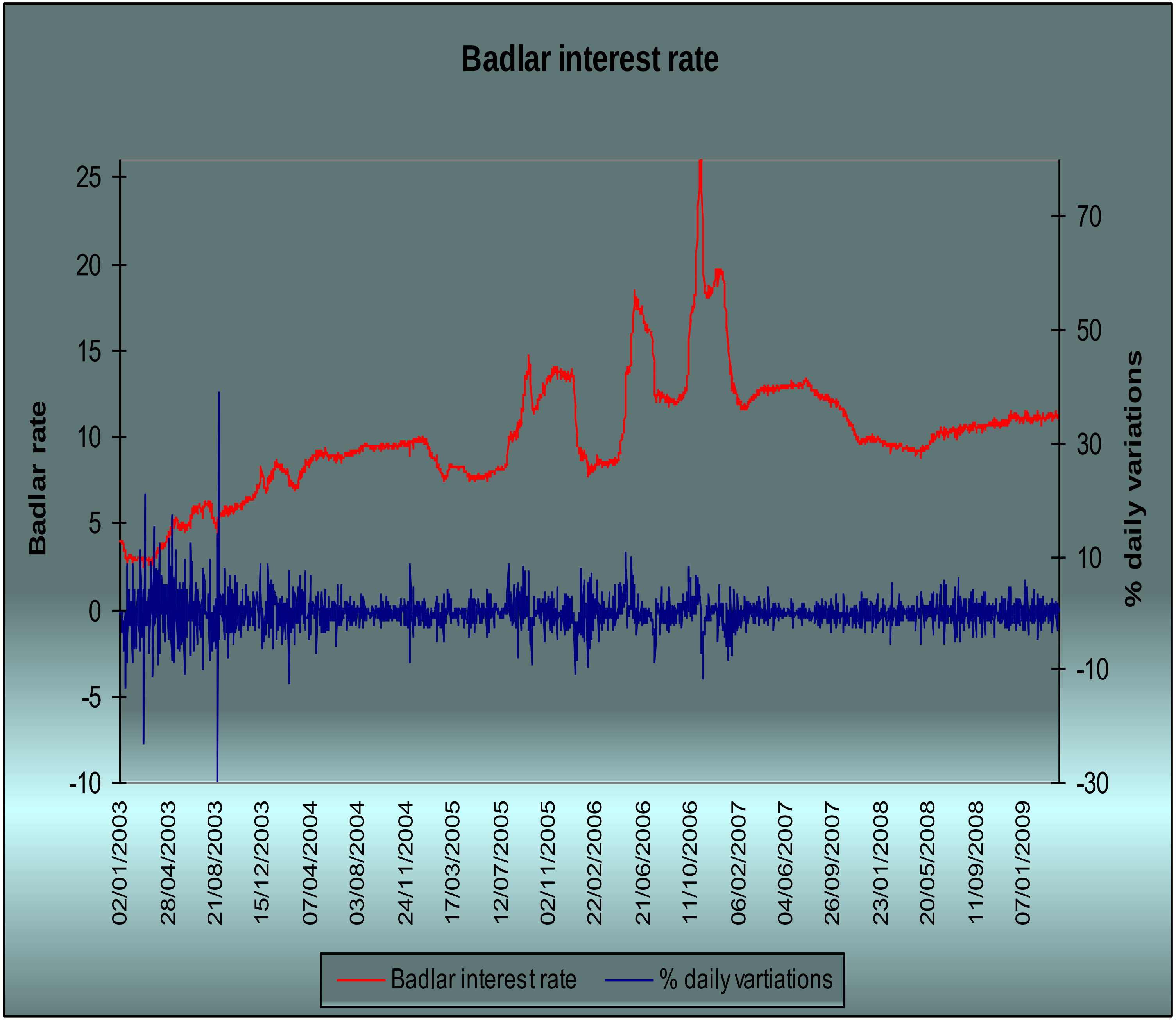
$$CV = \frac{1}{P} \sum_{t=1}^{n} \frac{t(t+1)CF_t}{(1+i)^{t+2}}$$

$$\Delta\%P = -MD\, di + 0.5\, CV\ di^2$$

Static VaR Model for interest rate to estimate δi



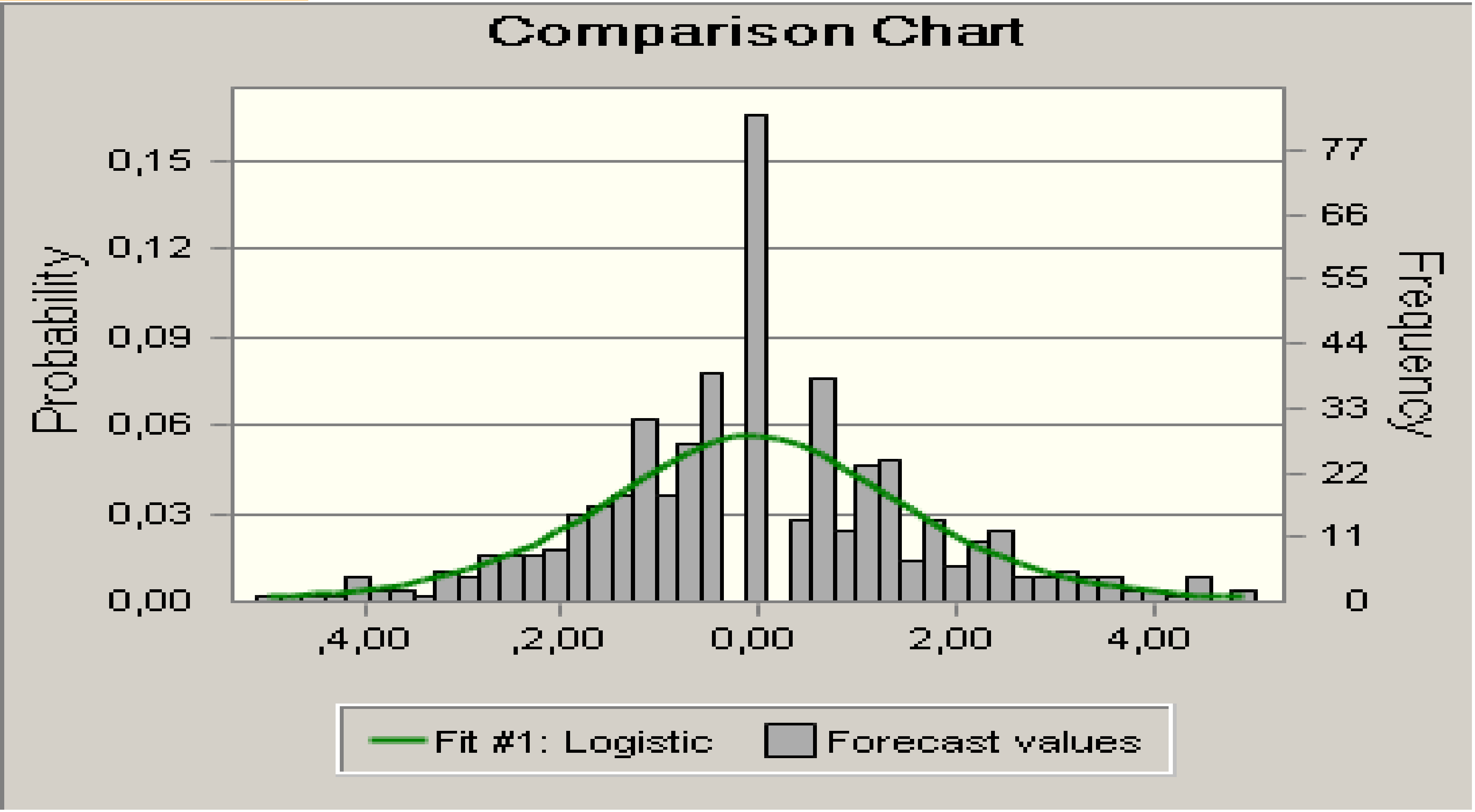
AFIR 2011



Badlar interest rate
Badlar rate
% daily variations
25
20
15
10
5
0
-5
-10
70
50
30
10
-10
-30
02/01/2003
28/04/2003
21/08/2003
15/12/2003
07/04/2004
03/08/2004
24/11/2004
17/03/2005
12/07/2005
02/11/2005
22/02/2006
21/06/2006
11/10/2006
06/02/2007
04/06/2007
26/09/2007
23/01/2008
20/05/2008
11/09/2008
07/01/2009
Badlar interest rate
% daily vartiations

AFIR 2011

The best distribution fit



The test of goodness of fit

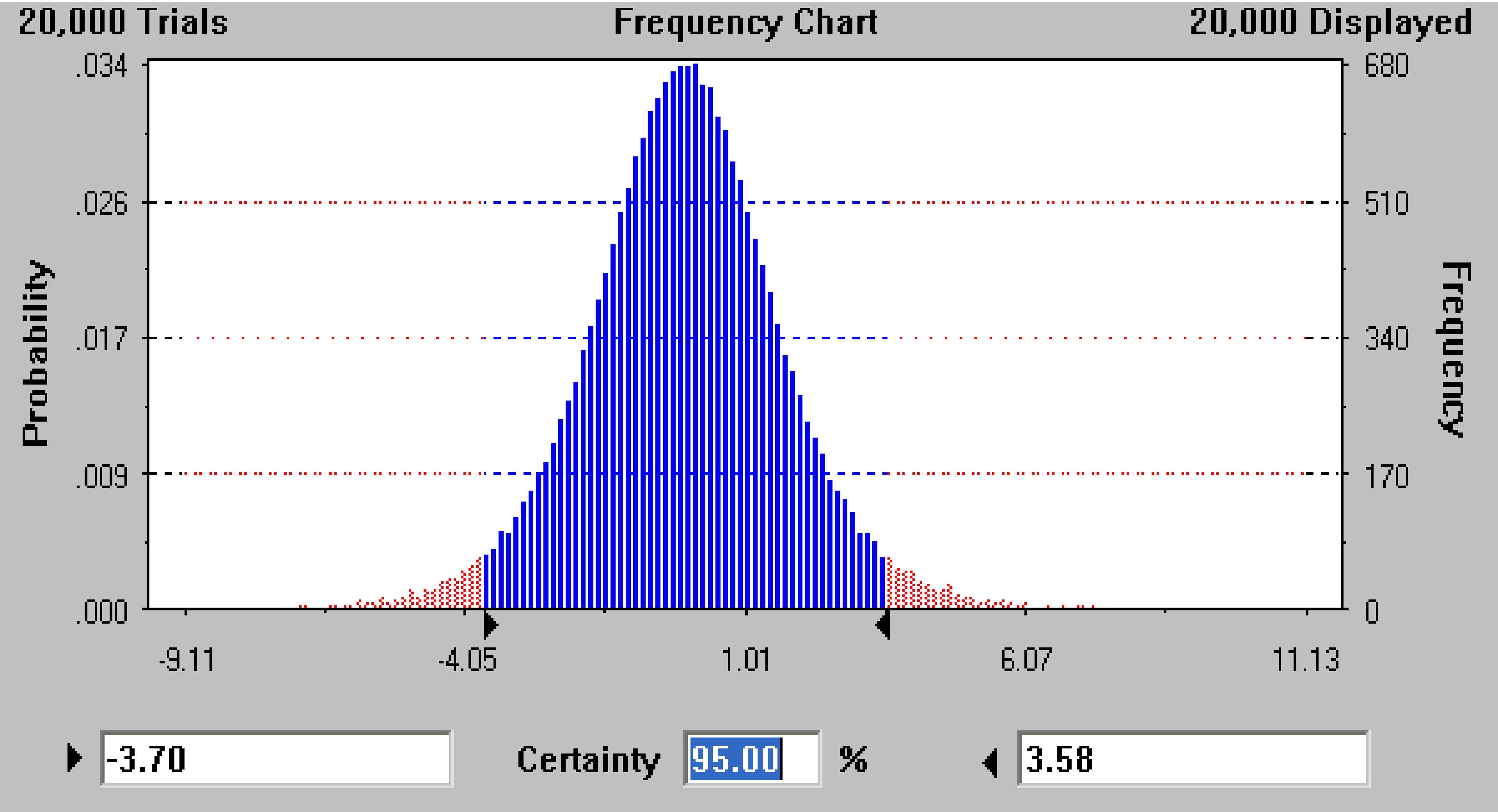
Analyzed by the K/S model

Ranked by: Kolmogorov-Smirnov

Distribution	A-D	Chi-Square	K-S	Parameters
Logistic	1,3710	326,6128	,0942	Mean=,0,06,Scale=0,99
Gamma	4,1333	303,8663	,0982	Location=,7,05,Scale=0,52,Shape=13,56702
Student's t	1,9466	278,0459	,1029	Midpoint=,0,02,Scale=1,63,Deg. Freedom=6,65
Normal	2,5494	322,3094	,1033	Mean=,0,02,Std. Dev.=1,81

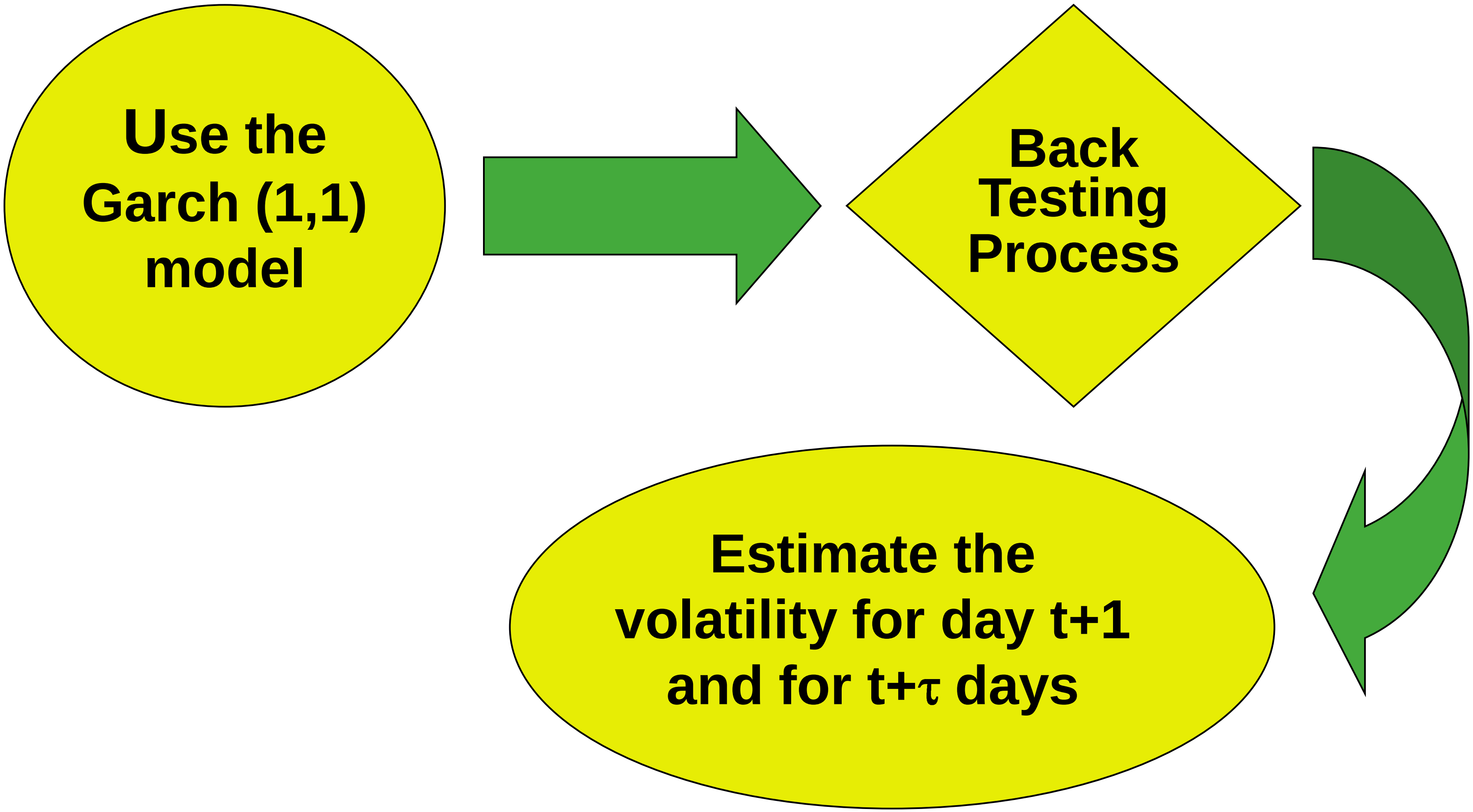
AFIR 2011

Simulation using logistic distribution



AFIR 2011

The dynamic estimation of δi



The conditions to use Garch (1,1)

1) The presence of black noise in the autocorrelation and partial autocorrelation of the residuals produced by the regression on a constant of the daily interest rate variations

2) The presence of heteroskedasticity in the model

The Garch (1,1) model

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

The properties of Garch (1,1)

If $(\alpha + \beta) < 1$, we can estimate de volatility for a period beginning at time *t* for the next τ days

$$\sigma_{t+\tau}^{2} = \frac{\omega}{1-(\alpha+\beta)} \left\{ (\tau - 1) - \left[(\alpha+\beta) \frac{1-(\alpha+\beta)^{\tau-1}}{1-(\alpha+\beta)} \right] \right\} + \frac{1-(\alpha+\beta)^{\tau}}{1-(\alpha+\beta)} \sigma_{t}^{2}$$

Also:

$$\frac{1}{1-(\alpha+\beta)}$$

Estimate days of persistence

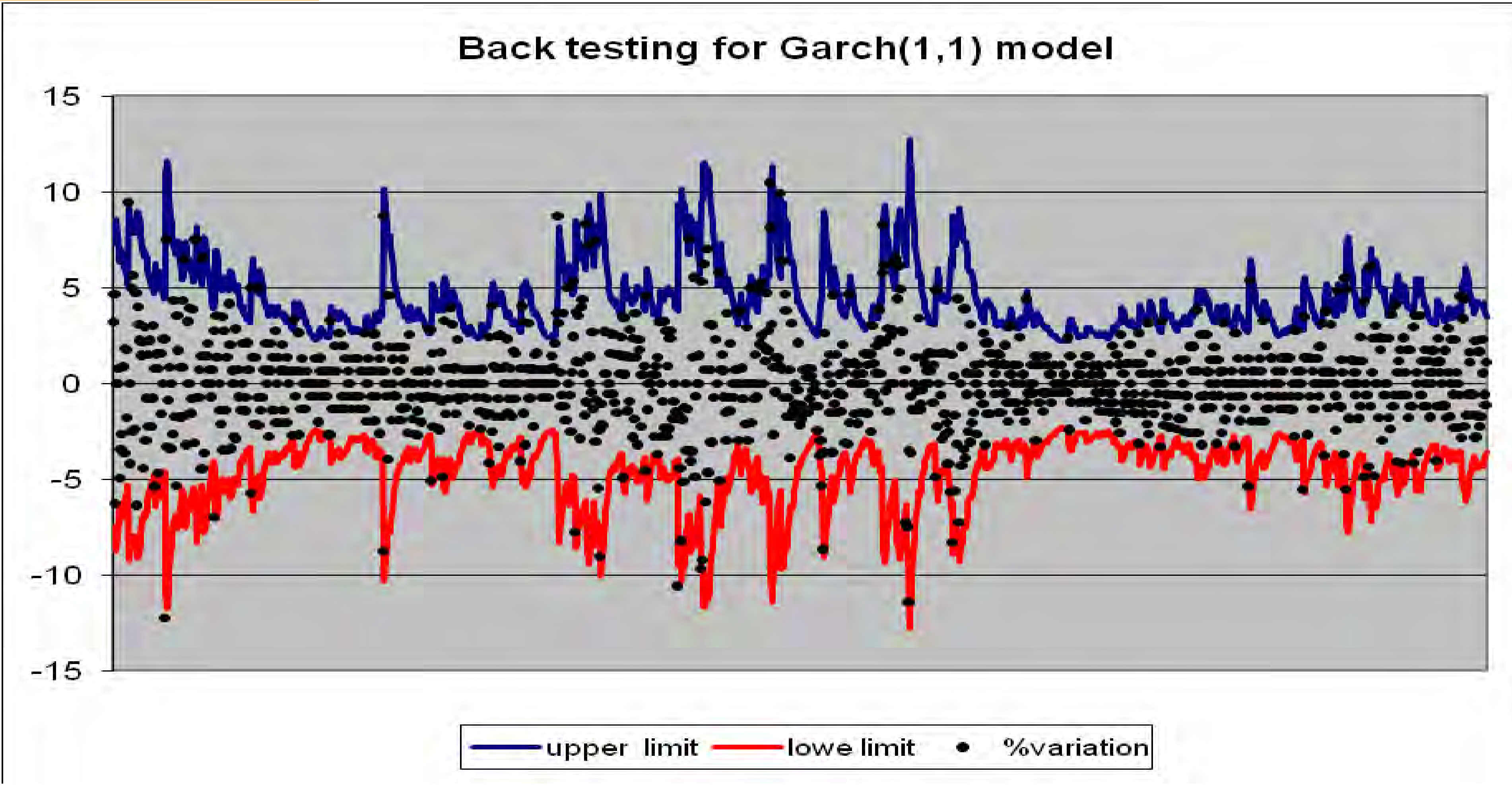
$$\frac{\omega}{1-(\alpha+\beta)}$$

Estimate traditional volatility

AFIR 2011

$$\sigma_t^2 = 0.222 + 0.179\,\varepsilon_{t-1}^2 + 0.798\,\sigma_{t-1}^2$$

$$(0.04) \quad (0.019) \qquad (0.016)$$



The volatility and δi The Normal and logistic distribution

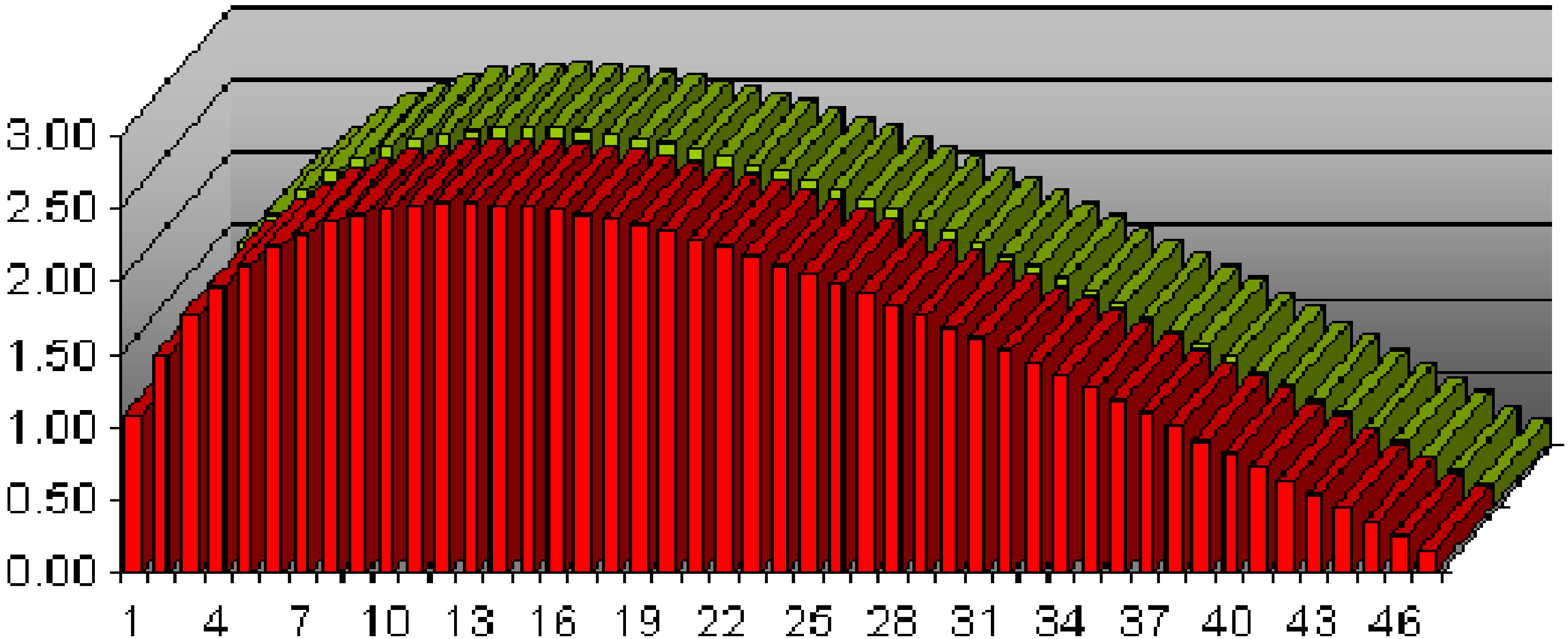
To estimate the maximum variation we use the positive branch of an interval of confidence:

$\delta i = r + z\sigma$

- Normal distribution and rule of $t^{1/2}$
- Logistic distribution and rule of $t^{1/2}$
- Normal distribution and Garch (1,1)
- Logistic distribution and Garch (1,1)

AFIR 2011

Excess of δi using rule of $t^{1/2}$ vs. Garch (1,1) model



Maximum daily loss with a 2.5% of probability for the following series

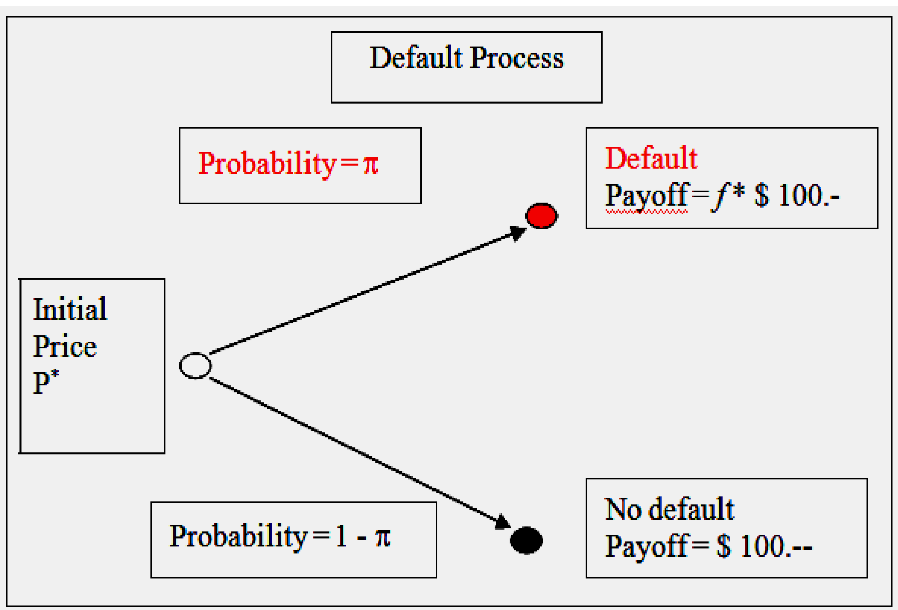
Model	Gold	Euro/USD	D. Jones
$t^{1/2}$	-2.219%	-1.326%	-2.40%
Garch(1,1)	-1.910%	-1.279%	-1.470%

AFIR 2011

The Credit Risk for fixed income

Transition matrix Vs. excess of interest rate

Initial Qualification	Qualification to the year end (% of change probability)							
	AAA	AA	A	BBB	BB	B	CCC	Def.
BBB	**0.02**	**0.33**	**5.95**	**86.93**	**5.30**	**1.17**	**0.12**	**0.18**
BB	0.03	0.14	0.67	7.73	80.53	8.84	1.00	1.06
B	0	0.11	0.24	0.43	6.48	83.46	4.07	5.20
CCC	0	0	0.22	1.30	2.38	11.24	64.86	19.79



$$P^* = \frac{100}{(1+i^*)} = \left[\frac{100}{(1+i)}\right](1-\pi) + \left[\frac{f\,100}{(1+i)}\right]\pi$$

$$\frac{(1+i)}{(1+i^*)} = 1 - \pi + f\pi$$

$$1 - \frac{(1+i)}{(1+i^*)} = \pi - f\pi$$

$$\pi = \frac{(i^* - i)}{(1+i^*)(1-f)}$$

AFIR 2011

How to cover the risks

The derivative market with different products is used to cover the different risks

CDS → Credit default Swaps to cover the Credit Risk in Fixed income investment

Options → On equities, interest rate and commodities, to reduce the market risk or to assure a maximum price to buy

Future Market → On equities, interest rate and commodities to fix a price

AFIR 2011

The mathematics of derivatives

The formulas of Black and Sholes and Black

$$C = SoN(d1) - \frac{E}{e^{i\frac{t}{n}}} N(d2)$$

$$d1 = \frac{\ln\left(\frac{So}{E}\right) + \left(i + \frac{s^2}{2}\right)\frac{t}{n}}{s\sqrt{\frac{t}{n}}}$$

$$d2 = d1 - s\sqrt{\frac{t}{n}}$$

This formula must be correct by the following reasons:

✓ The returns don't follow the rule of t ½

✓ The returns aren't *nid*

✓ The returns are correlated

✓4) There are a strong presence of heteroskedasticity in the returns

The changes proposed in the Black and Scholes formula

$$d1=\frac{\ln\left(\frac{So}{E}\right)+\left(i+\frac{s^2}{2}\right)\frac{t}{n}}{s\sqrt{\frac{t}{n}}}$$

$$d1=\frac{\ln\left(\frac{So}{E}\right)+i\frac{t}{n}+\frac{1}{2}s^2\frac{t}{n}}{s\sqrt{\frac{t}{n}}}$$

Replacing the volatility for *t* days estimated with the rule of t½ by the estimated with Garch for the next τ days

$$d1=\frac{\ln\left(\frac{So}{E}\right)+i\frac{t}{n}+\frac{s^{2\,Garch}}{2}}{s^{Garch}}$$

AFIR 2011

The variation of prime options due the volatility variation

The first derived from the price relates to the volatility

$$\Lambda = \frac{\partial C}{\partial s} = So \frac{1}{\sqrt{2\pi}} e\left(-\frac{d_1^2}{2}\right)\sqrt{t/365}$$

In the Black and Scholes model the volatility is constant during the option life. In consequence we can modify the Vega value using:

$$\Lambda^{Garch} = \frac{\partial C}{\partial s} = So \frac{1}{\sqrt{2\pi}} e\left(-\frac{d_1^{2\,GARCH}}{2}\right)\sqrt{t/365}$$

The formula of Black applied to options of interest rate and foreign exchange

$$F = (So + s)\, e^{(r-c)t}$$

$$C = e^{-rt} \left[F\, N(d1) - E\, N(d2) \right]$$

$$d1 = \frac{\ln\left(\frac{F}{E}\right) + \frac{s^2}{2} t}{s\, t}$$

$$d2 = d1 - s\, t$$

Also change the volatility using Garch (1,1) value

$$d1 = \frac{\ln\left(\frac{F}{E}\right) + \frac{s^{2Garch}}{2}}{s^{Garch}\, t}$$

$$d2 = d1 - s^{Garch}\, t$$

AFIR 2011

A trivial example

Assets	Quantity (units)	Price per unit	Portfolio value	% VaR for next 30 days	VaR
Euros	100000	1.4271	142.710	7.38	10532
Gold	100	1505	150.500	13.3	20016
Total			293.210		30548
-VaR			-30.548		
Net			262.662		

Strategic result

Concept	cash	Observations
Prime collection	5.847	Prime for sell an option on 100000 € @ 0.0584
Option Exercise	137.000	Collect the option 100000 € @ 1.37
Gold sell	150.680	Collect to sell of 100 oz of gold @ 1506.80
Portfolio cash	293.527	Compare with the exposure of 262.662 net of VaR

Conclusions

To solve the problem of to hedge or not to hedge

- A good forecast of Market VaR
- A good forecast of Credit VaR
- A good estimation of prime options
- A good strategic design

Questions