Market-Consistent Valuation of Long-Term Insurance Contracts

Valuation Framework and Application to German Private Health Insurance

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Motivation

- What is the shareholders value from long-term insurance contracts?
- How do the special characteristics of German health insurance affect the shareholders value and risk associated with the value?
- How big is the financial risk of companies private health insurance portfolios?

Literature review

- Projection of insurance contracts in balance sheet setup:
  - Kling et al. (2007)
  - Gerstner et al. (2008)
- Market-consistent valuation of single insurance contracts:
  - Bacinello (2003)
- Market-consistent valuation of portfolios of insurance contracts:
  - Sheldon and Smith (2004)
  - Castellani et al. (2005)
  - Wüthrich et al. (2010)
- Principles of market-consistent embedded value:
  - CFO Forum (2009)

Agenda

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Framework

Stochastic Environment: External factors

$(\Omega, \mathcal{F}, P)$ prob. space
$X_1, \ldots, X_n$ r.v.
with $X_i : \Omega \to \mathbb{R}^T$
$1 \leq i \leq n$

↓

Insurance Company: Cash flows

$Y = f(X_1, \ldots, X_n)$

↓

Valuation: Value

e.g. $\mathbb{E}(g(Y))$

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JY-Model (Heath-Jarrow-Morton framework, under P)

$$df_n(t,T) = \alpha_n(t,T)dt + \varsigma_n(t,T)dW_n^P(t)$$
$$df_r(t,T) = \alpha_r(t,T)dt + \varsigma_r(t,T)dW_r^P(t)$$
$$dI(t) = I(t)\mu(t)dt + I(t)\sigma_I dW_I^P(t)$$

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$t \in [0,T]$, $I(0) = I_0 > 0$, $f_n(0,T) = f_n^M(0,T)$ and $f_r(0,T) = f_r^M(0,T)$.
(W_n^P, W_r^P, W_I^P) Brownian motion with correlations $\rho_{n,r}$, $\rho_{n,I}$ and $\rho_{r,I}$.

Stochastic Environment

- Assume that for $t \in [0, T]$:

$$\varsigma_n(t,T) = \sigma_n e^{-a_n(T-t)} \quad \text{and} \quad \varsigma_r(t,T) = \sigma_r e^{-a_r(T-t)}.$$

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JY-Model (Short rate framework, under Q^n)

$$dn(t) = (\vartheta_n(t) - a_n n(t))dt + \sigma_n dW_n(t)$$
$$dr(t) = (\vartheta_r(t) - \rho_{r,I}\sigma_r\sigma_I - a_r r(t))dt + \sigma_r dW_r(t)$$
$$dI(t) = I(t)(n(t) - r(t))dt + I(t)\sigma_I dW_I(t)$$

(W_n, W_r, W_I) Brownian motion with correlations $\rho_{n,r}$, $\rho_{n,I}$ and $\rho_{r,I}$ and

$$\vartheta_n(t) = \frac{\partial f_n(0,t)}{\partial T} + a_n f_n(0,t) + \frac{\sigma_n^2}{2a_n}\left(1 - e^{-2a_n t}\right)$$
$$\vartheta_r(t) = \frac{\partial f_r(0,t)}{\partial T} + a_r f_r(0,t) + \frac{\sigma_r^2}{2a_r}\left(1 - e^{-2a_r t}\right).$$

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Private health insurance policies in Germany

- Life-long guarantee of coverage for predefined medical reimbursement.
- Insurer renounces the right to cancel the contract.
- Annual premium calculated by principle of equivalence (similar to life-techniques) assuming future claim development based on current situation.
- No benefits in case of death or surrender.
- Policyholder pays additional premium until age 60 (10 percent of actuarial premium).
- Additional reserve serves to limit excessive premium increase.

Special contract characteristics

- Increase of average claim size (linked to inflation) leads to check of basis for actuarial calculation.
- If increase of average claim size is significant then insurance company adjusts expected claim size and interest rate.
- Premium payment includes surcharge (surcharge factor).

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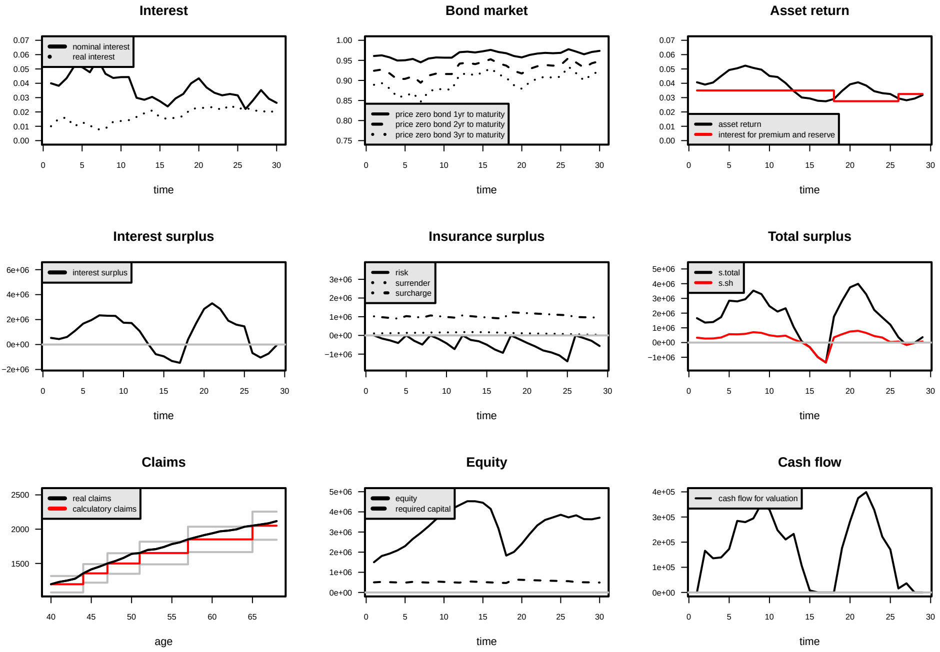
Options and Guarantees

- Short-term interest rate guarantee.
- Medical reimbursement.
- Surrender.

Surplus distribution

- Surplus resulting from asset development, risk and cost surplus is aggregated at the end of the year.
- Positive surplus: Shareholders receive at most 20 % (surplus factor).
- Negative surplus: Up to 100 % financed by shareholders.
- Asymmetric impact on shareholders value.


Interest
nominal interest
real interest
time
Bond market
price zero bond 1yr to maturity
price zero bond 2yr to maturity
price zero bond 3yr to maturity
time
Asset return
asset return
interest for premium and reserve
time
Interest surplus
interest surplus
time
Insurance surplus
risk
surrender
surcharge
time
Total surplus
s.total
s.sh
time
Claims
real claims
calculatory claims
age
Equity
equity
required capital
time
Cash flow
cash flow for valuation
time

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Valuation

Definition: Present Value of Future Profits

$$PVFP := \mathbb{E}^{Q^n}\left(\sum_{t=0}^{T} \exp\left(-\int_0^t n_u du\right) Y_t\right)$$

Y_t cash flow between shareholder and company

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Y_t cash flow between shareholder and company
Generate K scenarios of external factors $X_1, \ldots, X_T$ under risk-neutral measure Q^n.

Present Value of Future Profits Estimation

$$\widehat{PVFP} := \frac{1}{K}\sum_{k=1}^{K}\left(\sum_{t=0}^{T} B_{k,t} Y_t^k\right)$$

with $B_{k,t}$ discount factor for scenario k at time t.

Valuation

Generate sample of K scenarios of external factors $X_1, \ldots, X_n$ under risk-neutral measure Q^n.

Definition: Average Scenario

$$X^*_{i,t} := \frac{1}{K} \sum_{k=1}^{K} X^k_{i,t}$$

This scenario yields

PVFP of Average Scenario

$$PVFP^{AS} := \sum_{t=0}^{T} B^*_t Y^*_t$$

with B^*_t discount factor at time t.

Quantification of Financial Risk

Def.: Time Value of Financial Options and Guarantees

$$TVFOG := PVFP^{AS} - \widehat{PVFP}$$

Time value is the additional value ascribable to the potential for benefits under the option to increase in value prior to expiry (CFO Forum, 2009).

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Example: one model point

- Contracts specifics
  - 5,000 contracts from male insureds
  - age 40, contracts exist since 10 years
  - initial annual premium 1,764 € and average claim size 1,197 €

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- Capital market (values from Jarrow and Yildirim (2003))
  - Nominal: $a_n = 0.03398$, $\sigma_n = 0.00566$, $n_0 = 0.04$
  - Real: $a_r = 0.04339$, $\sigma_r = 0.00299$, $r_0 = 0.01$
  - Inflation: $\sigma_i = 0.00874$, $I_0 = 100$
  - Dependency: $\rho_{n,r} = 0.01482$, $\rho_{r,i} = -0.32127$, $\rho_{n,i} = 0.06084$

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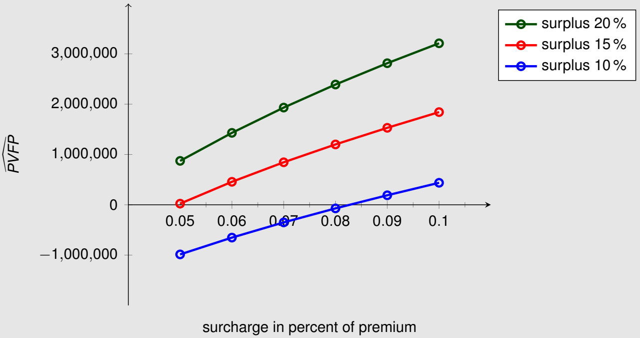
- Liablities at $t = 0$:
  - Actuarial Reserve: $60,964,712$ €
  - Bonus Account: $5,000,000$ €
  - Equity: $1,500,000$ €

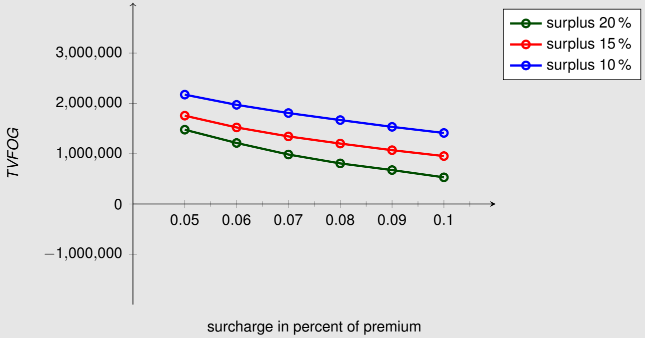
Results

Present Value of Future Profits



Results

Time Value of Options and Guarantees



Conclusion

- Surcharge factor has a big impact on present value of future profits.
- We observe an exposure to financial risk in terms of the time value.
- High variance in present value of future profits.

Thank you very much for your attention.

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