The Capital-Asset Pricing Model: Tests in real terms on a South African market portfolio comprising equities and bonds

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"The wrong sort of bees would make the wrong sort of honey"

-Winnie-the-Pooh

Presented by Taryn Leigh Reddy
AFIR Colloquium 2011
Madrid, Spain
22nd June 2011

AGENDA

- Background and further research
- Aim
- Data and periods considered
- Method
- Empirical Tests
- Conclusion

AGENDA

- Background and further research
  - The Capital-Asset Pricing Model: The case of South Africa
  - Why was this paper written?
- Aim
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- Conclusion

This paper extends on the previous work of the authors entitled "The CAPM: The case of South Africa"

Aim

- Is the CAPM valid in the South African market? In particular, does the CAPM explain excess return?
- Is the relation between return and beta linear?
- Individual sectoral indices versus portfolios of sectoral indices were considered
- Are those sectors with higher systematic risk associated with higher expected return?

Data

- Quarterly total return indices from 30 June 1995 to 30 June 2009 for ten sectoral indices
- The FTSE/JSE ALSI was selected as a proxy for the market portfolio
- The STEFI Composite Index and the Ginsberg, Malan & Carson Money -market Index

Based on the tests of this paper the CAPM could generally not be rejected

Tests

- Similar to the tests this paper except for the construction of portfolios from the ten sectoral indices
- The ten-year period (30 June 2000 to 30 June 2009) was sub-divided into ten one-year sub-periods
- Prior betas and in-period betas were determined

Conclusions

- The CAPM could be rejected for some periods, but not for all
- For the purpose of testing whether the CAPM can be used in long-term models, a test based on in-period sample betas is relevant, since such a model can generate unbiased ex-ante betas
- On the basis of the tests using in-period betas, the CAPM cannot be rejected
- The use of the model for long-term actuarial modelling in the South African market can be reasonably justified

This paper was written due to the further research which arose from the previous paper

Market Portfolio

- Expected returns are linear in beta if and only if the market portfolio is mean-variance efficient (Ross, 1978)
- The market portfolio, which is central to testing the CAPM, is unobservable in practice, therefore a market proxy is chosen
- Various authors have warned that the choice of a wrong proxy would reduce the predictive ability of the CAPM. Therefore it is possible that the results obtained in the previous study were misstated
- In this study, in order to obtain a more realistic market portfolio, conventional and index-linked bonds were included both in the composition of the market portfolio and in tests of the securities market line

Real terms

- This paper extends to reconsider the CAPM in real terms
- Although most tests of the CAPM are applied in nominal terms, it is preferable to measure returns in real terms since investor's preferences must ultimately be reflected in terms of consumption of goods and services, not merely in terms of currency
- In short-term applications the difference may be immaterial but in longer-term applications such as those typically used in actuarial modelling the difference may be material and index-linked bonds exist

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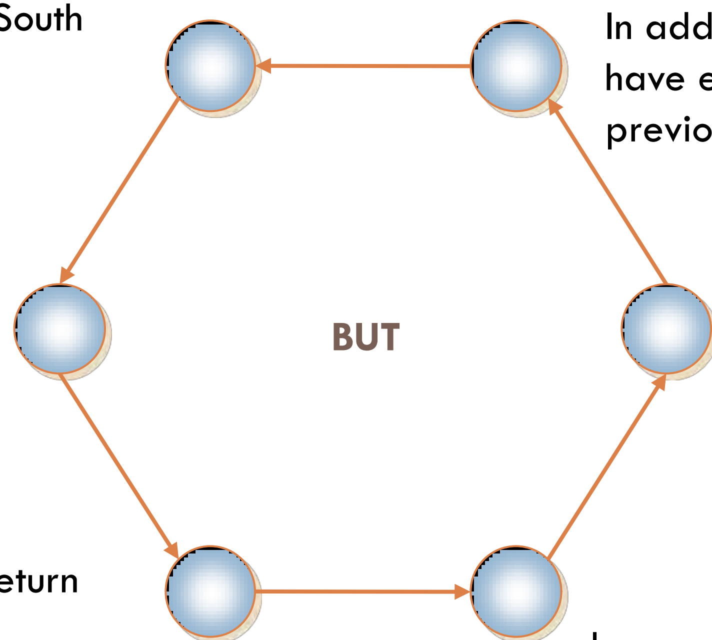
This study aimed to answer a number of questions related to the CAPM

Is the CAPM valid in the South African market?

In particular, does the CAPM explain excess return?

Is the relation between return and beta linear?



In addressing these questions we have extended the analysis in the previous study...

by including bonds in the market portfolio and...

by expressing returns in real terms

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Comprehensive data over a sufficiently long time-period was used

Data

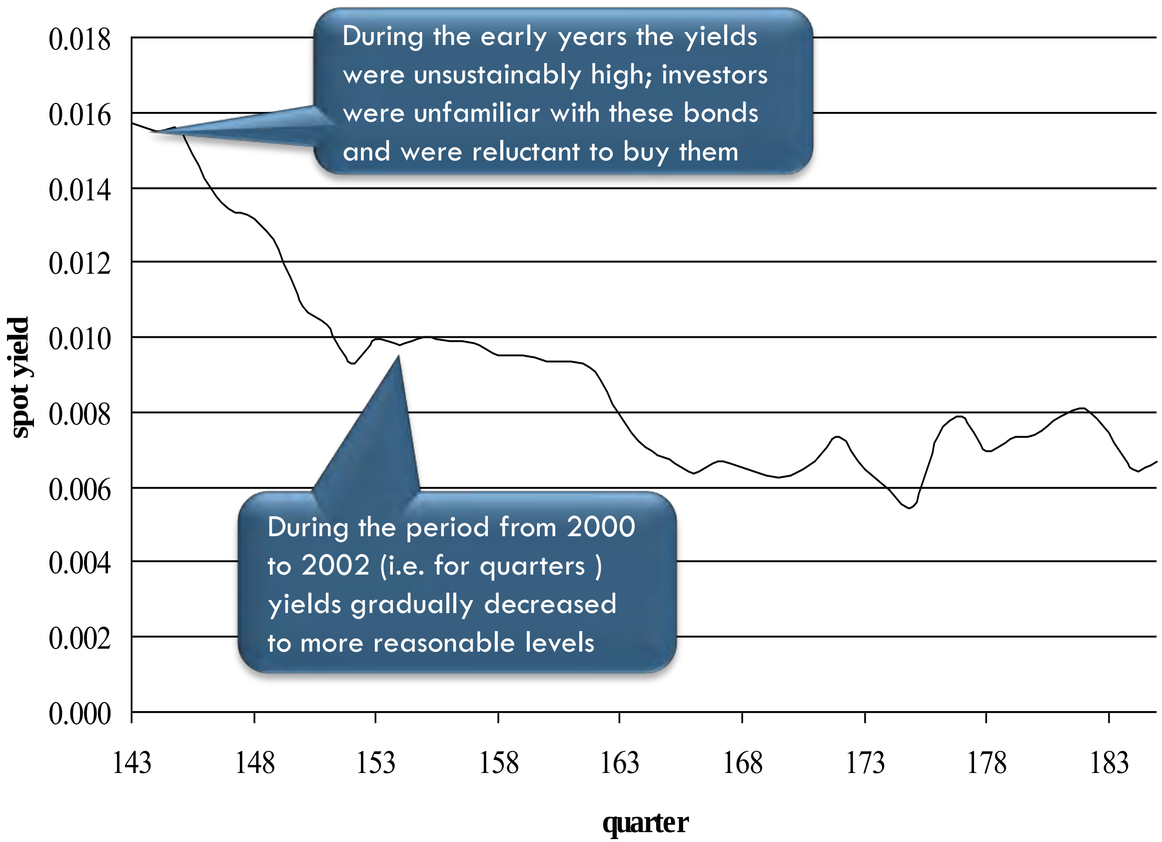
- For the investigation, quarterly total returns from the FTSE/JSE All-Share Index listed on the JSE Securities Exchange from 30 September 1964 to 31 December 2010 (i.e. for quarters $t \in [1, \ldots, 185]$). were used, together with yields on government bonds and consumer price indices over the same period
- However, this period has seen a number of changes, which may have affected the underlying assumptions of the CAPM or it's testability
- Therefore different sub-periods were considered

Sub-periods

- Due to the onset of high inflation in the 1970s the first sub-period considered was therefore from 30/9/1964 to 31/12/1972 (i.e. for quarters $t \in [1, \ldots, 33]$)
- Until 31/12/1985 the yield curve comprised only three points, which effectively represented yields on the primary market. The secondary market was not well developed. Tests of the CAPM including bonds as well as equities prior to that date may be affected by the inefficiency of the bond market. The second sub-period considered was therefore from 31/12/1972 to 31/12/1985 (i.e. for quarters $t \in [34, \ldots, 85]$)

The remaining sub periods were determined based on the data for index-linked bonds

Value of the spot yield of a government index-linked bond (with 40 quarters to redemption) for all times *t* at which there were inflation-linked bonds in issue



- Under these circumstances it is not reasonable to suppose that the inflation-linked bond market was in equilibrium with the rest of the market
- Furthermore, until 2002 there were less than four bonds in issue
- It was therefore decided to ignore inflation-linked bonds before 31/12/2002 (i.e. before $t = 153$)
- Such bonds were not included in prior periods
- The fourth and fifth periods were therefore from 30/9/1989 to 31/12/2002 (i.e. for quarters $t \in [101,\ldots,152]$) and from 31/12/2002 to 31/12/2011 (i.e. for quarters $t \in [153,\ldots,185]$) respectively

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An introduction to some methodology would be helpful before diving into the tests…

Risk-free rate

- Because this study considered the CAPM in real terms, the real risk-free return was used (the spot rate for an inflation-linked bond maturing one quarter hence)
- However inflation-linked bonds before 31/12/2002 were ignored
- Therefore for earlier periods real risk-free returns were not available , and it was not possible to apply tests of the CAPM based on the risk-free return
- However tests based on the zero-beta version of the CAPM was applied for those sub-periods

Constituents of the market portfolio

- The market portfolio was assumed to comprise:
  - listed equities included in the all-share index on the JSE Securities Exchange;
  - zero-coupon conventional bonds with maturities of 3, 10 and 20 years; and
  - with effect from 31/12/2002, zero-coupon inflation-linked bonds with the same maturities
- According to Maitland (2001), almost all the variability in the yields on bonds may be be explained by the first three principal components of the yield curve, and therefore by those on three well-dispersed zero-coupon bonds
- Therefore the reduction of the bond portfolio to maturities of 3, 10 and 20 years therefore results in no material loss in generality

The following variables need to be defined

The value of R_{it}, the return on component *i* of the market portfolio during quarter *t* was determined from relevant spot rates for $i \in I^{+}$, $t \in [1,\ldots,152]$ and for $i \in I^{-}$, $t \in [153,\ldots,185]$

Where I^{-} comprises:

- listed equities included in the all-share index on the JSE Securities Exchange; and
- zero-coupon conventional bonds with maturities of 3, 10 and 20 years;

And I^{+} comprises:

- listed equities as above;
- zero-coupon conventional bonds with maturities as above; and
- zero-coupon inflation-linked bonds with the same maturities

For $t \in [153,\ldots,185]$ the excess return on component *i* and the market portfolio *M* during quarter *t* was determined as:

$r_{it} = R_{it} - R_{Ft}$ and $r_{Mt} = R_{Mt} - R_{Ft}$ respectively

where R_{Ft} is the return during quarter *t* on an inflation-linked bond maturing at the end of that quarter

The return on the market portfolio during quarter *t* was determined using the market capitalisation m^{*}_{it} of component *i* of the market portfolio at time *t*, as:

$$R_{Mt} = \begin{cases} \dfrac{\sum_{i \in I^{-}} m^{*}_{i,t-1} R_{it}}{\sum_{i \in I^{-}} m^{*}_{i,t-1}} & \text{for } t \in [1,\ldots,152]; \\[2ex] \dfrac{\sum_{i \in I^{+}} m^{*}_{i,t-1} R_{it}}{\sum_{i \in I^{+}} m^{*}_{i,t-1}} & \text{for } t \in [153,\ldots,185]. \end{cases}$$

The methodology used to perform tests of the zero-beta version of the CAPM can be explained by the following formulae

Prior Betas for each component *i* for each quarter $t = 21, \ldots, 185$ were determined using five years of quarterly prior returns were used to give the prior beta as:

$$\hat{\beta}_{it} = \frac{\hat{\sigma}_{\mathrm{iMt}}}{\hat{\sigma}_{\mathrm{MMt}}}$$

In-period betas and in-period returns were determined for each component *i*, for each calendar year as:

$$\hat{\beta}_{i[Y]} = \frac{\hat{\sigma}_{\mathrm{iM[Y]}}}{\hat{\sigma}_{\mathrm{MM[Y]}}} \qquad R_{i[Y]} = \sum_{t \in [Y]} R_{it}.$$

A further investigation was carried out using in-period betas for each sub-period $[p] \in \{[1,33],[34,85],[86,100],[101,152],[153,185]\}$ The in-period beta and in-period return for each component *i* in sub-period [*p*] and was estimated as:

$$\hat{\beta}_{i[p]} = \frac{\hat{\sigma}_{\mathrm{iM}[p]}}{\hat{\sigma}_{\mathrm{MM}[p]}} \qquad R_{i[p]} = \frac{1}{q_{[p]}} \sum_{t \in [p]} R_{it}.$$

A further investigation was carried out using all sub-periods combined. For this purpose, for each component *i*, the in-period beta for all sub-periods combined was estimated as:

$$\hat{\beta}_{i} = \frac{\hat{\sigma}_{\mathrm{iM}}}{\hat{\sigma}_{\mathrm{MM}}} \qquad R_{i} = \frac{1}{185} \sum_{t=1}^{185} R_{it}.$$

The methodology used to perform tests of the standard version of the CAPM can be explained by the following formulae

As for the zero-beta version, the investigation was first carried out by calculating a prior beta for each component *i* for each quarter $t = 173, \ldots, 185$ using five years of quarterly prior real returns

$$\hat{\beta}_{it} = \frac{\hat{\sigma}_{\text{iMt}}}{\hat{\sigma}_{\text{MMt}}}$$

In-period betas and in-period returns were determined for each component *i*, for each calendar year $Y = 2003, \ldots, 2010$ as:

$$\hat{\beta}_{i[Y]} = \frac{\hat{\sigma}_{\text{iM[Y]}}}{\hat{\sigma}_{\text{MM[Y]}}} \qquad r_{i[Y]} = \sum_{t \in Y} r_{it}$$

A further investigation was carried out using all quarters combined. For this purpose, for each asset class *i*, in-period beta and in-period return for all quarters $t = 153, \ldots, 185$ combined was calculated as:

$$\hat{\beta}_i = \frac{\hat{\sigma}_{\text{iM}}}{\hat{\sigma}_{\text{MM}}} \qquad r_i = \frac{1}{33} \sum_{\forall t} r_{it}$$

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Null hypothesis for the zero-beta version of the CAPM

Return-generating process

- It is not necessary to refer to the risk-free asset
- In order to test the zero-beta version of the CAPM, it is necessary to translate the ex-ante parameters of an equilibrium model into ex-post realisations. For that purpose it is necessary to assume the validity of some return-generating function:

$$R_{it} = \gamma_{0t} + \beta_{it}\gamma_{1t} + \varepsilon_{it}$$

Test

- This serves as the null hypothesis for the zero-beta version of the CAPM, for a given quarter t

Zero-Beta version of the CAPM

Tests using prior betas

Tests of the explanatory power of the CAPM using prior betas

The return for each component *i* of the market portfolio was regressed, for each quarter where $t = 21,\ldots,185$ and $Y = 1969,\ldots,2010$ against the corresponding prior beta

$$R_{it} = \gamma_{0[Y]} + \gamma_{1[Y]}\hat{\beta}_{it} + \varepsilon_{it}$$

The return for each component *i* was regressed, for each sub-period [*p*] , against the corresponding prior beta estimate

$$R_{it} = \gamma_{0[p]} + \gamma_{1[p]}\hat{\beta}_{it} + \varepsilon_{it}$$

The return on each component *i* for each quarter *t* for , $t = 21,\ldots,185$ against the corresponding prior beta estimate

$$R_{it} = \gamma_0 + \gamma_1\hat{\beta}_{it} + \varepsilon_{it}$$

Tests for non-linearity using prior betas

The return for each component *i* of the market portfolio was regressed, for each quarter where $t = 21,\ldots,185$ and $Y = 1969,\ldots,2010$ against beta and beta squared

$$R_{it} = \gamma_{0[Y]} + \gamma_{1[Y]}\hat{\beta}_{it} + \gamma_{2[Y]}\hat{\beta}_{it}^2 + \varepsilon_{it}$$

The return for each component *i* was regressed, for each sub-period [*p*] , against beta and beta squared

$$R_{it} = \gamma_{0[p]} + \gamma_{1[p]}\hat{\beta}_{it} + \gamma_{2[p]}\hat{\beta}_{it}^2 + \varepsilon_{it}$$

The return on each component *i* for each quarter *t* for , $t = 21,\ldots,185$ against the corresponding prior beta estimate

$$R_{it} = \gamma_0 + \gamma_1\hat{\beta}_{it} + \gamma_2\hat{\beta}_{it}^2 + \varepsilon_{it}$$

Zero-Beta version of the CAPM

Tests using in-period betas

Tests of the explanatory power of the CAPM using in-period betas

The in-period return for each component *i* of the market portfolio was regressed, for each calendar year *Y* where $Y = 1965, \ldots, 2010$ against the corresponding in-period beta

$$R_{i[Y]} = \gamma_{0[Y]} + \gamma_{1[Y]}\widehat{\beta}_{i[Y]} + \varepsilon_{i[Y]}$$

The in-period return for each component *i* was regressed, for each sub-period [*p*] , against the corresponding in-period beta estimate

$$R_{i[p]} = \gamma_{0[p]} + \gamma_{1[p]}\widehat{\beta}_{it} + \varepsilon_{i[p]}$$

The in-period return on each component *i* for each quarter *t* for
$t = 1,\ldots,185$ against the corresponding in-period beta estimate

$$R_i = \gamma_0 + \gamma_1\widehat{\beta}_i + \varepsilon_i$$

Tests for non-linearity using in-period betas

The in-period return for each component *i* of the market portfolio was regressed, for each calendar year *Y* where $Y = 1965, \ldots, 2010$ against beta and beta squared

$$R_{i[Y]} = \gamma_{0[Y]} + \gamma_{1[Y]}\widehat{\beta}_{i[Y]} + \gamma_{2[Y]}\widehat{\beta}^2_{i[Y]} + \varepsilon_{i[Y]}$$

The return for each component *i* was regressed, for each sub-period [*p*] , against beta and beta squared

$$R_{i[p]} = \gamma_{0[p]} + \gamma_{1[p]}\widehat{\beta}_{i[p]} + \gamma_{2[p]}\widehat{\beta}^2_{i[p]} + \varepsilon_{i[p]}$$

The in-period return on each component *i* for each quarter *t* for ,
$t = 1,\ldots,185$ against the corresponding prior beta estimate

$$R_i = \gamma_0 + \gamma_1\widehat{\beta}_i + \gamma_2\widehat{\beta}_i^2 + \varepsilon_i$$

The following results were observed for the test of the explanatory power and non-linearity of the zero-beta version of the CAPM

Prior betas

- For the tests of the explanatory power of the CAPM, the zero-beta version of the CAPM predicts that $\gamma_0 > 0$ and $\gamma_1 > 0$
- The null hypothesis is that $\gamma_0 = 0$, $\gamma_1 = 0$ and the alternative hypothesis is that $\gamma_0 < 0$, $\gamma_1 < 0$
- The statistics for each test were analysed both on sub-periods and annually but more importantly of the all the years considered the number of years during which, one or both of the parameters were outside of their 95% confidence limits were analysed
- The CAPM must therefore be rejected on the grounds of these tests
- However the relationship was found to be linear

In-period betas

- Using in-period betas it was found that the explanatory power of the CAPM was higher
- Period [153,185] showed significant results
- Considered annually, the CAPM is rejected on the grounds of these tests (described above)
- However the relationship was found to be linear

Null hypothesis for the standard version of the CAPM

Return-generating process

- In order to test the standard version of the CAPM, it is necessary to translate the ex-ante parameters of an equilibrium model into ex-post realisations. For that purpose it is necessary to assume the validity of some return-generating function:

$$r_{it} = \beta_{it} r_{Mt} + \varepsilon_{it}$$

Test

- This serves as the null hypothesis for the zero-beta version of the CAPM, for a given quarter t

Standard version of the CAPM

Tests using prior betas

Tests of the explanatory power of the CAPM using prior betas

The return for each component *i* of the market portfolio was regressed, for each quarter where $t = 173,\ldots,185$ and $Y = 2007,\ldots,2010$ against the corresponding prior beta

$$r_{it} = \gamma_{0[Y]} + \gamma_{1[Y]}\hat{\beta}_{it} + \varepsilon_{it}$$

The return on each component *i* for each quarter *t* for , $t = 173,\ldots,185$ against the corresponding prior beta estimate

$$r_{it} = \gamma_0 + \gamma_1\hat{\beta}_{it} + \varepsilon_{it}$$

Tests for non-linearity using prior betas

The return for each component *i* of the market portfolio was regressed, for each quarter where $t = 173,\ldots,185$ and $Y = 2007,\ldots,2010$ against beta and beta squared

$$r_{it} = \gamma_{0[Y]} + \gamma_{1[Y]}\hat{\beta}_{it} + \gamma_{2[Y]}\hat{\beta}_{it}^2 + \varepsilon_{it}$$

The return on each component *i* for each quarter *t* for , $t = 173,\ldots,185$ against the corresponding prior beta estimate

$$r_{it} = \gamma_0 + \gamma_1\hat{\beta}_{it} + \gamma_2\hat{\beta}_{it}^2 + \varepsilon_{it}$$

Standard version of the CAPM

Tests using in-period betas

Tests of the explanatory power of the CAPM using in-period betas

The in-period return for each component *i* of the market portfolio was regressed, for each calendar year *Y* where $Y = 2003, \ldots, 2010$ against the corresponding in-period beta

$$r_{i[Y]} = \gamma_{0[Y]} + \gamma_{1[Y]} \widehat{\beta}_{i[Y]} + \varepsilon_{i[Y]}$$

The in-period return on each component *i* for each quarter *t* for $t = 153, \ldots, 185$ against the corresponding in-period beta estimate

$$r_i = \gamma_0 + \gamma_1 \widehat{\beta}_i + \varepsilon_i$$

Tests for non-linearity using in-period betas

The in-period return for each component *i* of the market portfolio was regressed, for each calendar year *Y* where $Y = 2003, \ldots, 2010$ against beta and beta squared

$$r_{i[Y]} = \gamma_{0[Y]} + \gamma_{1[Y]} \widehat{\beta}_{i[Y]} + \gamma_{2[Y]} \widehat{\beta}^2_{i[Y]} + \varepsilon_{i[Y]}$$

The in-period return on each component *i* for each quarter *t* for , $t = 153, \ldots, 185$ against the corresponding prior beta estimate

$$r_i = \gamma_0 + \gamma_1 \widehat{\beta}_i + \gamma_2 \widehat{\beta}_i^2 + \varepsilon_i$$

The following results were observed for the test of the explanatory power and non-linearity of the standard version of the CAPM

Prior betas

- Standard version of the CAPM could not be rejected
- Tests of its linearity could also not be rejected
- However, the explanatory power of the CAPM was low

In-period betas

- Using in-period betas it was found that the standard CAPM must be rejected for the period [153,185]

Parametric and non-parametric tests were also performed on in-period betas

	Parametric test of the SML using in-period betas	Non-parametric test of the SML using in-period betas
Zero-beta version	Rejected	Not rejected
Standard version		Not Rejected

As per the previous study, the Hotelling's test in Shanken (1985) for the residuals of the zero-beta version of the CAPM was applied to those periods for which it was possible to do so. This comprised the sub-periods [1,33] and [101,152], as well as all sub-periods combined

As per the previous study, this test is a Chi-squared test based on the error terms for each component *i* for each calendar year *Y* considered

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Conclusion

Parametric Tests

- Parametric tests show that both the zero-beta and standard versions of the CAPM must be rejected for in-period betas and that the zero-beta version must also be rejected for prior betas
- However, for prior betas the standard version of the CAPM could not be rejected
- It is counter-intuitive that the tests should be failed for in-period betas and yet passed for prior betas: one would expect that prior knowledge of in-period betas would enhance the effectiveness of the CAPM

Non-parametric Test

- On the other hand, non-parametric tests of the residuals could not be rejected, suggesting that the assumption of normality needs to be relaxed in tests of the CAPM in this market
- It may be, though, that the non-parametric tests used in this paper were not sufficiently powerful to reject the CAPM

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