A COMPLETE MODEL OF GENERAL DYNAMIC IMMUNIZATION

Dr. D. Iván Iturricastillo Plazaola

Dr. D. J. Iñaki De La Peña Esteban

Dr. D. Rafael Moreno Ruiz

Dr. D. Eduardo Trigo Martínez

UNIVERSIDAD DE MÁLAGA

eman ta zabal zazu

Universidad del País Vasco   Euskal Herriko Unibertsitatea

INDEX
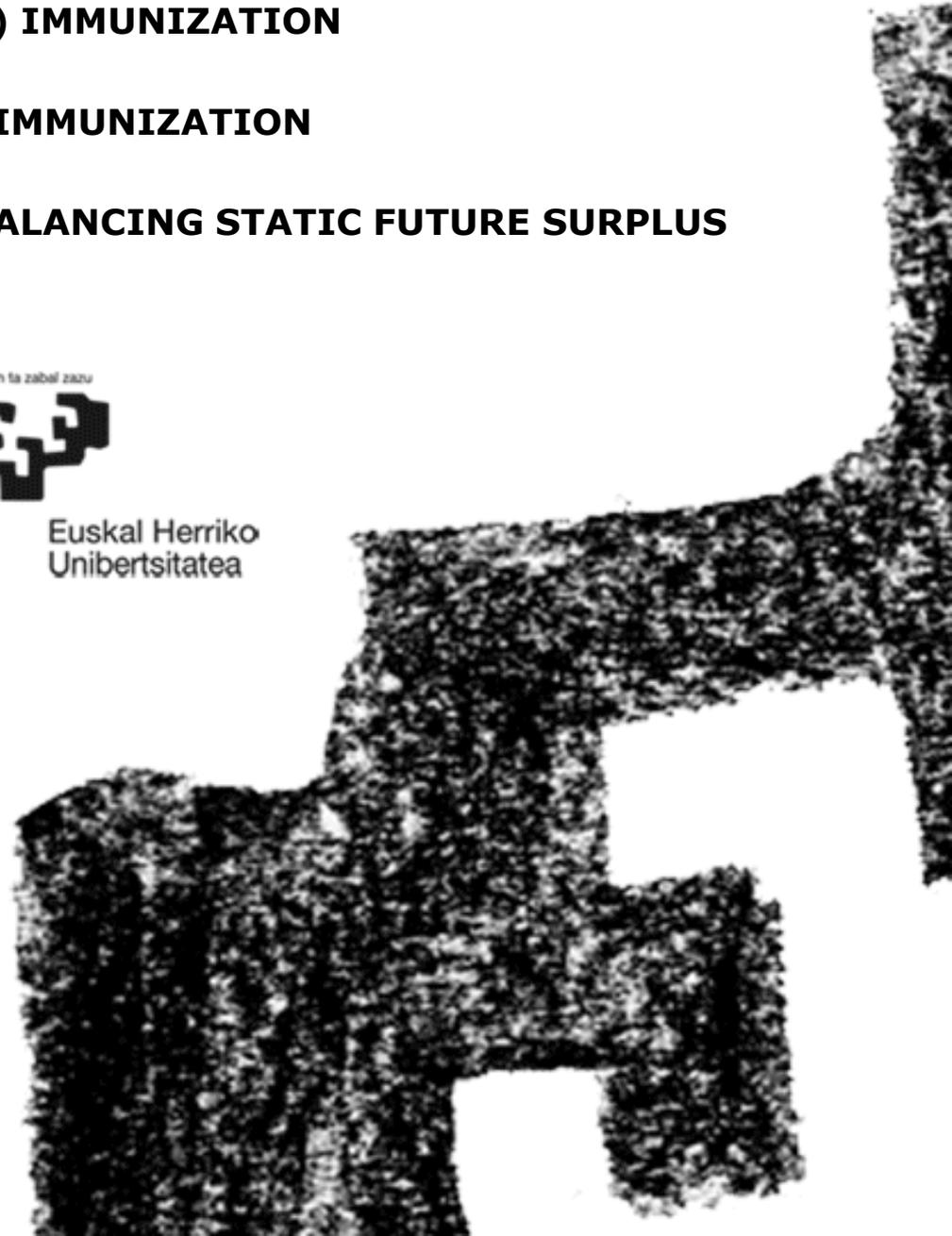
1. INTRODUCTION

INTRODUCTION

Immunization tries to eliminate the interest rate risk balancing the positions of the assets and the liabilities of the portfolio.

This technique has been studied for very long…
And always has been said that one of the problems it can have is that this tech asks for continuous rebalancings to mantain the porfolio immunized.

It has been asked for…

…Dynamic immunization.

But...

What´s dynamic?

To take care every day?

To run day after day…?

To run second by second…?

Or, maybe not?

Maybe is to think for the future?

Before it comes…!

Iturricastillo & De la Peña (2003) have found that a specific immunizing strategy has no need of rebalancing while the assumptions are fullfiled.

So, only has to been supervised if they are fullfiled!

They have called it dynamic...

This founding has been completed with some published works cited in the bibliography.

The basic assumptions of the whole model are:

1. The interest rate curve will evolve following the Rational Expectations Hypothesis (REH), ie the forward rates (implied by the initial curve) are the spot rates that there will be in the future.
This would be the scenario that is considered, therefore, the no displacement scenario of the curve of interest.

2. So, the shifts in the interest rate curve would be the shifts happened by comparing the current curve with the curve described by the forwad rates for any moment.
These movements are supposed to be parallel.
If both assumptions are verified, Iturricastillo (2007) has shown that the future forward interest curves will face also an almost parallel shift whenever a previous parallel shift happens.

So, if the model presented in this paper is followed, the only thing that should be monitored by the portfolio manager, with regard to interest rate risk, is if the interest rates curve is further than is reasonable from the one provided by the REH or a parallel one, in which case it is the only time that there would be a reason to consider to rebalance before the planned moment.

For this reason, the immunization risk must be controlled.

The immunization risk can be measured and, therefore, controlled, using the RIA.

The RIA has been found to be a better way to measure this risk than the classic M^2 , and even than the M^A.

[The RIA was first presented by Iturricastillo (2007) and developed by Iturricastillo & De la Peña.]

The RIA measures the medium term between the outflows and the inflows that compensates them:

$$RIA = \frac{\sum\limits_{h=0}^{n}\left|\sum\limits_{j=1}^{h}\left(F_j - L_j\right)\cdot\left(1 + {}_j i_0\right)^{-j}\right|}{\sum\limits_{h=1}^{n} F_h \cdot \left(1 + {}_t i_0\right)^{-h}} \cdot \frac{1}{k}$$

F_j : Inflow at t
L_j : Outflow at t
${}_j i_0$: Interest (Spot) for the period (0,0+j)
k : Number of periods considered inside each year

Why general?

1. **The presented model for any of the last two immunizations are a generalization of the model presented for the classical case**.

2. By the way, **the proposed model is a generalization of the model presented by Redington (1952).**

2. NO REBALANCING DYNAMIC IMMUNIZATION

NRD IMMUNIZATION

At the t moments…

…are committed outflows of Lt face value.

The objective is…

…to invest in a portfolio that guarantees this payments against interest rates shifts,…

…with no need of rebalancing.

The strategy to get it is the Horizon Matching,

that is,

an initial period of Absolute Matching, and

a Duration Matching for the whole portfolio…

NRD IMMUNIZATION

...with the following conditions:

1. **Current values of asset and liability equals**.
2. **Modified Duration of asset and liability equals**.
3. **Modified Convexity of asset higher than the one of the liability**.
4. **All Past Net Cash Flows are zero**. (That is, the no rebalancing period is a part of –or all- the absolute matching period.)
5. **During the prescribed period of no rebalancing, the RIA is less than the limit set by management**.

During the period of absolute matching, this strategy will guarantee that the portfolio is kept immunized by itself while the assumptions are verified.

NRD IMMUNIZATION

An example

It will be shown the theoretical evolution of two portfolios immunized with an Horizon Matching and with three years of Absolute Matching period.

In both portfolios the liabilities are the same, but the inflows have been established in one with a lower risk of immunization than the other (Table 2).

In both portfolios are kept the no rebalancing dynamic immunization conditions.

1. In the beginning we will use the interest spot rates of the Spanish public debt at December 30, 2008, and after that date it will be strictly applied the REH.

2. Subsequently, it will be analysed the effects if we apply the interest spot rates of the Spanish public debt at June 30, 2010, ie one and a half year later, curve that is not exactly parallel to the expected one.

NRD IMMUNIZATION

Table 2: Assets and Liabilities of each portfolio

Term (years)	Liabilities	Porfolio 1 (lower risk)	Porfolio 2 (higher risk)
0,5			
1	1.000.000,00 €	1.000.000,00 €	1.000.000,00 €
1,5			
2	750.000,00 €	750.000,00 €	750.000,00 €
2,5			
3	1.000.000,00 €	1.000.000,00 €	1.000.000,00 €
3,5		1.198.211,11 €	**6.665.084,51 €**
4	2.000.000,00 €	908.499,50 €	48.287,46 €
4,5		826.234,42 €	65.377,49 €
5	2.500.000,00 €	796.073,93 €	88.729,32 €
5,5		851.557,94 €	111.800,80 €
6	1.500.000,00 €	877.725,68 €	133.514,06 €
6,5		930.142,10 €	75.514,72 €
7	1.000.000,00 €	1.003.117,61 €	80.376,96 €
7,5		1.091.307,59 €	84.979,14 €
8	3.500.000,00 €	1.355.171,23 €	89.603,43 €
8,5		1.063.389,57 €	74.485,73 €
9	2.500.000,00 €	739.807,84 €	60.614,39 €
9,5		529.747,07 €	48.526,08 €
10	1.000.000,00 €	770.761,47 €	39.188,03 €
10,5		1.016.470,53 €	29.964,02 €
11	2.000.000,00 €	695.925,50 €	19.897,41 €
11,5		1.386.986,98 €	**8.917.560,28 €**

NRD IMMUNIZATION

Table 3: Main figures in the initial stage and in his subsequent theoretical evolution of the Portfolio 1 (lower risk)

Moment	Net Current Value	Current Value of the Asset	DMA - DML	CXMA - CXML	RIA
0	0,00 €	14.485.543,95 €	**0,000**	**0,071**	0,297
0,5	0,00 €	14.637.977,86 €	**0,000**	**0,102**	0,297
1	0,00 €	13.855.871,30 €	**0,000**	**0,144**	0,319
1,5	0,00 €	14.048.197,04 €	**0,000**	**0,179**	0,319
2	0,00 €	13.612.331,02 €	**0,000**	**0,229**	0,336
2,5	0,00 €	13.767.801,68 €	**0,000**	**0,268**	0,336
3	0,00 €	12.925.856,35 €	**0,000**	**0,333**	0,362

NRD IMMUNIZATION

Table 4: Main figures in the initial stage and in his subsequent theoretical evolution of the Portfolio 2 (higher risk)

Moment	Net Current Value	Current Value of the Asset	DMA - DML	CXMA - CXML	RIA
0	0,00 €	14.485.543,95 €	**0,000**	**0,300**	1,615
0,5	0,00 €	14.637.977,86 €	**0,000**	**0,647**	1,615
1	0,00 €	13.855.871,30 €	**0,000**	**1,070**	1,732
1,5	0,00 €	14.048.197,04 €	**0,000**	**1,453**	1,732
2	0,00 €	13.612.331,02 €	**0,000**	**1,952**	1,827
2,5	0,00 €	13.767.801,68 €	**0,000**	**2,373**	1,827
3	0,00 €	12.925.856,35 €	**0,000**	**3,016**	1,969

NRD IMMUNIZATION

In this tables is shown that while the assumptions are fullfiled, the immunization conditions are mantained by themselves...

So: there is no need of rebalancing.

The RIA tell us that the second portfolio has a clearly higher immunization risk.

NRD IMMUNIZATION

In the following is analyzed:

1. What happens to these portfolios if they have not been rebalanced before June 30, 2010,

2. And how would they evolve theoretically, if the portfolios are left unchanged, even the rates suffer a displacement that is not strictly parallel to the expected one.

3. Which is the first thing to check!

…Is the shift parallel to the expected rates?

In the next table it is observed that most rates have had a shift of a 1%, more or less,

but there are plenty that change in a fairly distant figure to the 1% (green)

and there is a value that differs very clearly (in yellow).

So… it has not been parallel!

But, it has not been either a shift that has had a clear effect, making higher or smaller the slope, for example, so it is **"quite parallel".**

NRD IMMUNIZATION

Table 5:
Expected
rates
(forwards
of the
previous
curve) vs
current
spot rates
at June 30,
2010

Term	Expecteds (forwards)	Spot rates	Difference
0,5	4,52223%	1,20342%	3,31881%
1	3,40379%	2,20604%	1,19774%
1,5	3,03762%	2,46666%	0,57097%
2	3,98834%	2,36687%	1,62146%
2,5	4,61859%	2,31254%	2,30605%
3	4,34125%	2,36229%	1,97896%
3,5	4,14908%	2,70337%	1,44571%
4	3,94207%	2,81352%	1,12856%
4,5	3,96432%	2,84166%	1,12266%
5	3,99092%	3,20620%	0,78472%
5,5	4,02103%	3,56850%	0,45252%
6	4,05409%	3,65795%	0,39615%
6,5	3,86242%	3,69478%	0,16763%
7	4,46871%	3,96032%	0,50839%
7,5	5,05144%	3,95205%	1,09939%
8	5,40252%	4,25591%	1,14661%
8,5	5,15370%	4,18674%	0,96696%
9	4,93319%	4,06183%	0,87135%
9,5	5,30906%	4,21106%	1,09801%
10	5,22276%	4,35022%	0,87253%

The question is whether and how this shift affects the portfolio at that time…

…and from this moment on, if no rebalance is done.

NRD IMMUNIZATION

Table 6: Main figures at the time of the shift and the subsequent theoretical evolution of the Portfolio 1 (lower risk)

Moment	Net Current Value	Current Value of the Asset	*DMA - DML*	*CXMA - CXML*	*RIA*
1,5	**-26.244,57 €**	14.687.673,63 €	**0,004**	**0,216**	0,323
2	-26.402,02 €	14.025.786,82 €	**0,005**	**0,271**	0,342
2,5	-26.823,54 €	14.249.715,95 €	**0,005**	**0,314**	0,342
3	-27.221,58 €	13.461.171,91 €	**0,005**	**0,386**	0,369

After the moment of the **shift, the portfolio**:
1. Suffers a loss of 0.18% of the current value of the asset.
2. **Is reasonably well immunized and the RIA will remain limited** if the assumptions are met in the following.

NRD IMMUNIZATION

Table 7: Main figures at the time of the shift and the subsequent theoretical evolution of the Portfolio 2 (higher risk)

Moment	Net Current Value	Current Value of the Asset	DMA - DML	CXMA - CXML	RIA
1,5	**55.440,56 €**	14.769.358,76 €	**0,069**	**2,690**	1,715
2	55.773,15 €	14.107.961,99 €	**0,072**	**3,168**	1,808
2,5	56.663,60 €	14.333.203,09 €	**0,072**	**3,533**	1,810
3	57.504,45 €	13.545.897,94 €	**0,076**	**4,184**	1,946

After the moment of the **shift the portfolio**:
1. "suffers" a (higher) gain in net present value of 0.38%.
2. **is <u>still</u> reasonably well immunized and the RIA is <u>still</u> controlled.** (but not so well)

So…

…if a Horizon Matching is set

with a smart RIA limitation,

the interest rate risk of the portfolio could be

reasonably well controlled.

3. NO REBALANCING DYNAMIC SURPLUS´ IMMUNIZATION

NRD SURPLUS´ IMMUNIZATION

Lets assume a portfolio where...

The asset value is bigger than the value of the liabilities and the objective is to invest immunizing the value of the surplus against interest rates shifts with no rebalancings.

The strategy to get this goal is the Horizon Matching with the following conditions:

1. The difference between the current values of asset and liability is the surplus to be guaranteed.

That is, **the surplus to guarantee has to exist**.

2. The relationship between the Modified Duration of asset and the one of liability should be the reverse of the relationship between the values of assets and liabilities.

$$DM_A = DM_L*L/A$$

3. The relationship between the Modified Convexity of Assets and the Modified Convexity of Liabilities must be at least equal to the reverse of the relationship between the values of assets and liabilities.

$$CXM_A \geq CXM_L*L/A$$

4. Surplus invested at sight.

5. All Past Net Cash Flows are zero.

6. The initial RIA does not exceed PFFx times the maximum RIA set by the management.

Where *x* is the time wanted to be rebalance free, except for unforeseen reasons.

In other words:

During the prescribed period of no rebalancing, the RIA is less than the limit set by management.

This strategy could be understood as the sum of two portfolios:
1.one that follows the strategy set for the case with no surplus, and
2.the other involving the investment of the surplus at sight.
The first portfolio will be immunized, as seen before.
The second has no interest rate risk because is invested at sight, but may involve the loss of investment opportunities.

So…

There are some very short term investments

whose profitability should be examined seriously,

because they are interest risk-free investments.

NRD SURPLUS´ IMMUNIZATION

In the example is observed that:

1. **immunization is maintained while the assumptions are met**, that

2. the immunization risk increases moderately (as shown in the Iberian Congress 2011), depending on the weight of the past zero net cash flows, and that

3. after a non parallel shift the portfolio evolves in a way similar that the one shown in the NRD Immunization.

4. NO REBALANCING DYNAMIC SURPLUS / ASSET RATIO´S IMMUNIZATION

NRD SURPLUS / ASSET RATIO´S IMMUNIZATION

Lets assume a portfolio where...

The asset value is bigger than the value of the liabilities and the objective is to invest the portfolio immunizing the the value of the surplus / asset ratio against interest rates shifts with no rebalancings.

The strategy to get this goal is the Horizon Matching with the following conditions:
1. Surplus / Asset ratio > 1.
2. No difference of Modified Duration of asset and liability.
3. Possitive difference of Modified Convexity of asset and liability.
4. Net Cash Flow of Assets and liabilities in proportion to their initial current values, in the no rebalancing period.
5. During the prescribed period of no rebalancing, the RIA is less than the limit set by management.

NRD SURPLUS / ASSET RATIO´S IMMUNIZATION

In the example is observed that:

1. **immunization of this ratio is maintained while the assumptions are met**, that

2. the immunization risk increases moderately (as shown in the Iberian Congress 2011), depending on the weight of the past zero net cash flows, and that

3. after a non parallel shift the portfolio evolves in a way similar that the one shown in the NRD Immunization.

Even it is obvious...

...in the example...

...the shift has provoked that the net current value has grown in 416,021.42 €, which represents a 1.88% of the current value.

But, this is not the parameter we are trying to immunize, and it can not be immunized different parameters at once.

Thus, if the ratio is immunized, the current value will not.

In this case it results in a profit, but it could have been resulted in a loss.

5. AN ALTERNATIVE TO THE NRD SURPLUS´ IMMUNIZATION: THE REBALANCING STATIC FUTURE SURPLUS IMMUNIZATION

It has been shown how to create a portfolio that does not require rebalancing while the <u>surplu</u>s is permanently <u>immunized</u>.

The *problem* with this strategy is that the surplus is invested at sight and this entails, almost surely, a loss of profitability.

If this situation is considered unsatisfactory, the alternative could be the one set out by several authors…

To immunize looking forward!

That is, to leave **not correctly immunized** the portfolio at the beginning, **but with the expectation that it will be in the future**.

This strategy, along with the possibility of higher returns, has the following disadvantages:

1. In the initial stage the portfolio is not immunized.
So, **the loss could be important**, if the company is forced to rebalance too soon, **even if the shift is parallel to the expected curve.**

2. The immunization can only be guaranteed at a particular moment.
From that moment on, it has to be applied the strategy shown in this paper, or has to be applied again the same strategy for a later moment, because, otherwise, the portfolio will be unbalanced by itself.

Therefore, it would be needed to immunize from a preset moment to a another preset moment.

3. **The period during which the portfolio is not perfectly immunized, can only be shortened reducing the time between each rebalancing.**

   Which would provoke higher commissions costs, therefore offsetting the extra return that was intended to obtain.

4. If at the appointed moment to rebalance / to immunize again, there is a financial earthquake, the alternatives would be to rebalance in the heart of the storm or to wait it to abate, knowing that the portfolio would become unbalanced by itself.

   It may even happen that if we have to wait too long, the portfolio began to have nonzero net cash flows which should be funded or reinvested in the middle of the financial earthquake.

So…

The objective is to invest in a portfolio that immunizes the surplus value against interest rates shifts without sacrificing the higher returns that could be get if the surplus is not invested at sight.

The strategy to get it could be the Horizon Matching, with the following conditions:

1. The surplus to guarantee has to exist.
2. On the future moment in which the surplus has to be immunized, the relationship between the Modified Duration of asset and the one of liability should be the reverse of the relationship between the values of assets and liabilities.

$$DM_A = DM_L * L/A$$

3. On the future moment in which the surplus has to be immunized, the relationship between the Modified Convexity of Assets and the Modified Convexity of Liabilities must be at least equal to the reverse of the relationship between the values of assets and liabilities.

$$CXM_A \geq CXM_L * L/A$$

4. All Past Net Cash Flows are zero till the future moment in which the surplus is to be immunized.
(It could be not zero, but in this case, there is an human intervention in between. So, it won´t be comparable.)

5. On the future moment in which the surplus is to be immunized, the RIA must be smaller than limit set by management.

So…

Having the same liabilities…

The investment is adapted to the above conditions.

And the surplus will be allocated after the absolute matching period…

In the example is observed that immunization is achieved in the preset moment (moment 3), if the assumptions are met, but at the beginning the immunization risk is not moderate.

After a year and a half, which is when the previous checks have been performed, that risk has already been reduced by a half.

REBALANCING STATIC FUTURE SURPLUS´ IMMUNIZATION

<u>After this 1,5 years</u>, when are taken into account the new interest rates…

…**<u>the net present value changed to 1,07% from the expected.</u>**

<u>Although in this case is an increase, it could have been the other way, so it is a sign of the increased risk that is run.</u>

In this moment, the conditions of immunization vary, so even waiting until the scheduled moment (the third year) immunization will no be perfect, except if the interest rate curve get back to where it was expected to be –or to a parallel one-.

(The latter should not be discarded a priori…)

To sum up, this strategy could generate some additional revenue, but in exchange for assuming more risk.

6. CONCLUSIONS

1. **In this paper a practical immunizing model is shown. It can be applied with ease and does not require a continuous trading of securities, making it dynamic, at the same time that it is discreet.**

2. **This model has been presented both for the <u>classical case</u> of an immunization of a portfolio with no surplus and for the two most important immunizations showed by Kaufman and Bierwag (1985), ie, <u>surplus immunization</u> and <u>surplus / asset ratio immunization</u>.**

   These last two are a generalization of the first one.

3. **This model is a generalization of Redington´s classical immunization.**

4. **In this paper is shown an example of the alternative strategies of a pseudo dynamic immunization applied to the sole case in what these alternatives may have really a trade-off between profitability, costs and extra risks: Surplus immunization.**

Thank you very much for your attention
Merci beaucoup par son attention
Muchas gracias por su atención

ivan.iturricastillo@ehu.es

jinaki.delapena@ehu.es

moreno@uma.es

etrigom@uma.es