Cash-flow based valuation of insurance liabilities

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The problem

- Consider an insurance portfolio with aggregate claims c_t payable at time $t = 1, \dots, T$.
- Our aim is to value the current liabilities so any future additions to the insurance portfolio are ignored.
- We assume that the liabilities amortize in finite time and that the last claim will be paid at time T.
- After paying the claims c_t at time t, the insurer invests the remaining wealth in financial markets over the next period $[t, t+1]$.
- A unit of cash invested at time t returns R_t at time $t+1$.
- What is the least amount of initial capital sufficient for paying out all the claims?

The problem

- Solvency II, Article 75.1(b): "liabilities shall be valued at the amount for which they could be transferred, or settled, between knowledgeable willing parties in an arms length transaction."
- Solvency II, Article 76.2: "The value of technical provisions shall correspond to the current amount insurance and reinsurance undertakings would have to pay if they were to transfer their insurance and reinsurance obligations immediately to another insurance or reinsurance undertaking."
- IAIS, Standard on the structure of regulatory capital requirements: "A total balance sheet approach should be used in the assessment of solvency to recognise the interdependence between assets, liabilities, regulatory capital requirements and capital resources and to ensure that risks are appropriately recognized."

Best estimate

When the claims $c = (c_t)_{t=1}^T$ and investment returns $R = (R_t)_{t=1}^T$ are deterministic, the problem can be written as

$$\begin{aligned} &\text{minimize} && V_0 \quad \text{over} \quad (V_t)_{t=0}^T \\ &\text{subject to} && V_t = R_t V_{t-1} - c_t \quad t = 1, \dots, T, \\ & && V_T \geq 0. \end{aligned}$$

In this deterministic model, the minimum initial wealth is given by

$$V_0 = \sum_{t=1}^T \frac{c_t}{\prod_{s=1}^t R_s}.$$

Best estimate

- If $\prod_{s=1}^{t} R_s = \exp(tY_t)$, where Y_t is the value of riskless yield curve at maturity t, the value can be expressed as

$$V_0 = \sum_{t=1}^{T} e^{-tY_t} c_t$$

- If c_t are the expectations of the future claims, this becomes the "best estimate" in Article 77.2 of Solvency II.

But, riskless yield curves are meant for valuation of deterministic cash-flows, not uncertain ones.

- For example, the "best estimate" of a European call-option is much higher than its market (or Black-Scholes) value.
- Adding a positive "risk margin" makes things worse.

Risk sensitive valuation

- From now on, both the claims c_t and the investment returns R_t will be random variables on a probability space $(\Omega, \mathcal{F}, P)$.
- The probability measure P models the views of the insurer (or a supervisor) concerning the future development of the underlying risk factors.
- We are still dealing with only one asset.

Risk sensitive valuation

The valuation problem can now be written as

$$\begin{aligned} &\text{minimize} && V_0 \quad \text{over} \quad V \in \mathcal{N} \\ &\text{subject to} && V_t = R_t V_{t-1} - c_t \quad t = 1, \dots, T, \quad P\text{-a.s.} \\ & && V_T \in \mathcal{A}, \end{aligned}$$

where

- $\mathcal{N}$ is the set of adapted processes (the value of V_t depends only on information observed by time t),
- $\mathcal{A}$ is a set of random variables that the decision maker views as acceptable terminal positions.

Risk sensitive valuation

- superhedging: $\mathcal{A} = \{V \mid V \geq 0 \; P\text{-a.s.}\}$.
- quantile hedging: $\mathcal{A} = \{V \mid P(V \geq 0) \leq \delta\}$.
- zero utility principle: $\mathcal{A} = \{V \mid Eu(V) \geq u(0)\}$, where u is a utility function.
- acceptable hedging: $\mathcal{A} = \{V \mid \rho(V) \leq 0\}$, where ρ is a risk measure. This covers e.g.
  - all the above examples
  - Conditional Value at Risk.

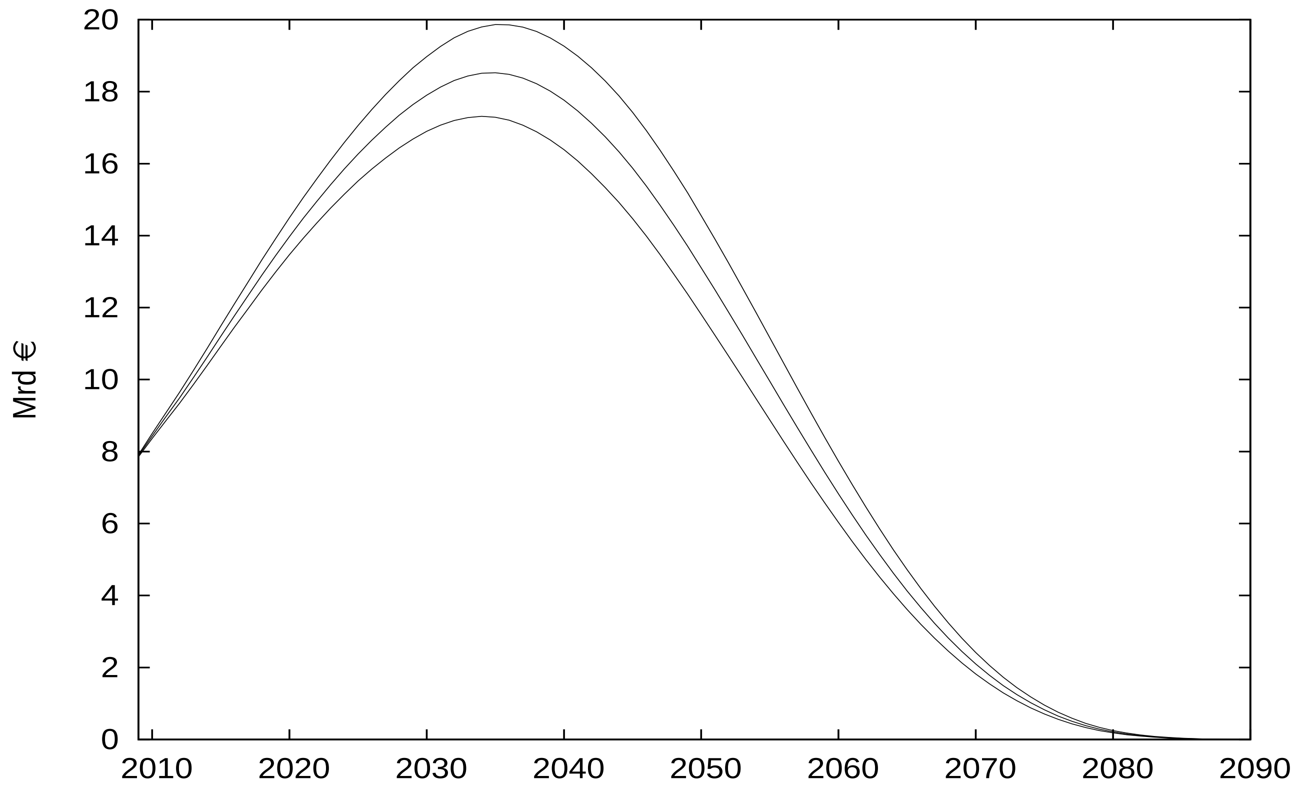
In general, analytical solutions to the pricing problem are not available (even in this one asset model) but for many choices of $\mathcal{A}$ it can be solved numerically using integration quadratures and a simple line search.

Case study

- Consider the insurance portfolio of the Finnish private sector occupational pension system.
- The yearly claims c_t consist of aggregate old age, disability and unemployment pension benefits that have accrued by the end of 2008 and become payable during year t.
- The claims depend e.g. on mortality and the wage and consumer price indices.

Case study

Figure 1: Evolution of yearly claims.



Case study

- We model the investment returns by

$$\ln R_t = \mu + \sigma\varepsilon_t,$$

where ε_t are iid standard normal and the parameters μ and σ are chosen so that the annualized logarithmic returns have a mean and standard deviation of 6%.

- We will use the acceptance sets

$$\mathcal{A} = \{V \mid \rho(V) \leq 0\},$$

where ρ is either the Value at Risk or the Conditional Value at Risk with varying confidence levels.

Case study

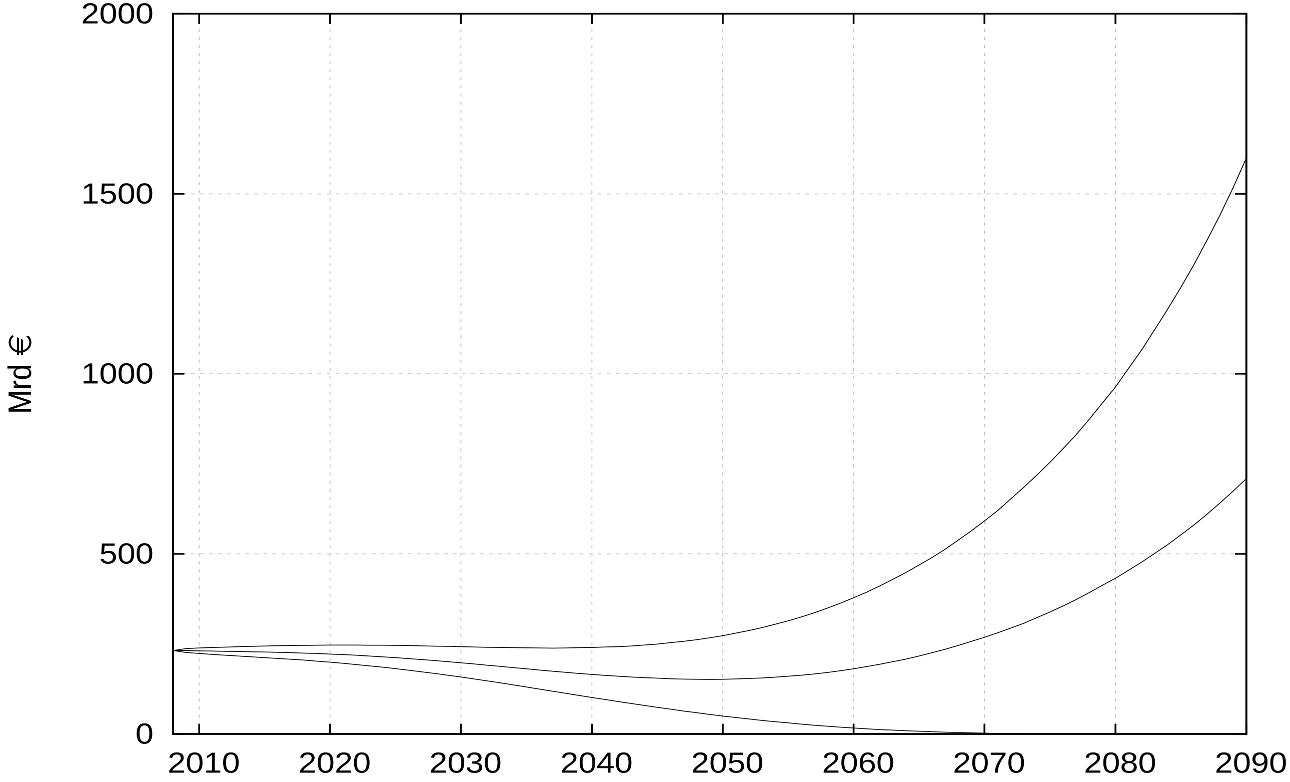
	Confidence level				
	95%	90%	85%	80%	66%
$V@R$	289	271	259	250	232
$CV@R$	305	288	276	268	252

Table 1: Pension liability in billion euros.

	Confidence level				
	95%	90%	85%	80%	66%
$V@R$	24.9	26.6	27.9	28.9	31.1
$CV@R$	23.6	25.1	26.1	26.7	28.7

Table 2: Solvency ratios.

Figure 2: The development of 34%, 50%- and 66%-quantiles of V_t when the initial capital corresponds to $V@R_{66\%}$.



Market consistent valuation

- When there are multiple investment opportunities, it may be possible to reduce the required initial capital by adapting the investment strategy to the liabilities.
- For example, the Black-Scholes formula gives the initial capital required for a strategy that replicates the claim in a complete market model.
- In reality, riskless hedging is often prohibitively expensive so one may be willing to trade off some safety for the possibility of profits.
- The construction of appropriate hedging strategies for insurance liabilities is one of the most important tasks of an insurance company.

Market consistent valuation

- Assume a finite set J of investment classes (bonds, equities, real estate, ...).
- Denote by $R_{t,j}$ the total return on class $j \in J$ over period $[t-1, t]$.
- Let $h_{t,j}$ be the amount of wealth invested in class $j \in J$ at the beginning of period t.
- The portfolio $h_t = (h_{t,j})_{j \in J}$ depends only on the information observed by time t.

The budget constraint becomes

$$\sum_{j \in J} h_{t,j} + c_t \leq \sum_{j \in J} R_{t,j} h_{t-1,j} \quad P\text{-a.s. } t = 1, \dots, T.$$

Market consistent valuation

The valuation problem can be written as

$$\begin{aligned}
\text{minimize} \quad & \sum_{j\in J} h_{0,j} \qquad \text{over} \quad h \in \mathcal{N} \\
\text{subject to} \quad & \sum_{j\in J} h_{t,j} + c_t \le \sum_{j\in J} R_{t,j} h_{t-1,j} \quad t = 1,\dots,T, \\
& h_{t,j} \ge 0 \quad j \in J \setminus \{0\}, \\
& \sum_{j\in J} h_{T,j} \in \mathcal{A},
\end{aligned}$$

where $\mathcal{N}$ denotes the investment strategies adapted to the available information. Analytical solutions are not available, in general, but efficient numerical techniques exist.

Market consistent valuation

The above valuation framework

- extends actuarial premium principles by incorporating the possibility of dynamic trading.
- extends superreplication principles of financial mathematics by incorporating more reasonable risk tolerances.
- expresses the liability value as the sum of
  - market value of the "replicating portfolio" h_0
  - value of the unhedged part $\sum_{j\in J} h_{T,j}$ (the residual terminal wealth) in terms of the risk measure

$$\rho(V_T) = \inf\{\alpha \in \mathbb{R} \,|\, V_T + \alpha \in \mathcal{A}\}.$$

- can be used internally and/or for regulatory purposes, depending on whose views the probability measure P and the acceptance set $\mathcal{A}$ are based on.

Case study

Table 3: The asset classes and some of the quantiles of their annualized returns

	5%	50%	95%
Money market	2.9	3.6	4.4
Bonds	-0.6	4.4	10.8
Nordic equities	-26.8	7.8	58.2
European equities	-17.9	6.7	38.6
US equities	-19.7	6.7	41.7
Asian equities	-22.9	7.7	50.6
Real estate	-17.4	6.2	36.5

Case study

- We evaluated the liabilities using 529 different dynamic investment strategies.
- The strategies are based on the buy and hold, fixed proportion and constant proportion portfolio insurance rules with varying parameters.
- All strategies were modified to accommodate for claim payments and the portfolio constraints.
- In addition to these 529 strategies, we evaluated the liabilities using a strategy that diversifies the initial capital among the different strategies.
- The diversification was optimized numerically using integration quadratures and optimization techniques.

Case study

Table 4: Capital requirements with varying investment strategies and risk tolerances for CV@R.

	Confidence level				
	95%	90%	85%	80%	66%
Best basis	296	284	273	261	239
Optimized	288	271	254	236	202

Table 5: Corresponding solvency ratios.

	Confidence level				
	95%	90%	85%	80%	66%
Best basis	24.3	25.4	26.4	27.6	30.1
Optimized	25.0	26.6	28.3	30.5	35.6

Summary

- The value of liabilities depends essentially on the following subjective factors:
  1. **probability distribution**,
  2. **risk preferences**,
  3. **hedging strategy**.
- Asset management is an integral part of valuation: pricing without hedging is meaningless.
- When implemented properly, financial and actuarial valuation principles coincide: call options and pension liabilities can be priced with the same techniques.
- Coming up with good investment strategies is one of the most important functions of an insurance company: better the strategy, lower the price (or higher the profits).

References

1. Hilli, Koivu and Pennanen, Cash-flow based valuation of pension liabilities, European Actuarial Journal, 2011.
2. Hilli, Koivu and Pennanen, Optimal construction of a fund of funds. European Actuarial Journal, 2011.
3. Koivu and Pennanen, Galerkin methods in dynamic stochastic programming. Optimization, 2010.
4. Pennanen, Superhedging in illiquid markets. Mathematical Finance, 2011.