Policyholder Exercise Behavior for Variable Annuities including Guaranteed Minimum Withdrawal Benefits[1]

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Overview

Motivation

- Variable Annuities:
  - Popular long-term investment vehicles
  - Tax-deferred growth
  - Investment evolves according to underlying (risky) portfolio
  - Uncertain payout
- Guaranteed Minimum (Death / Income / Accumulation / Withdrawal) Benefits
  - Insurers offer guaranteed payments
  - Policyholders can purchase security
  - Similar to (combination of) financial options

Motivation

- Withdrawal uncertainty
  - Could mitigate or intensify insurer's exposure to investment and/or mortality risks
  - Interactions non-trivial
  - Affects pricing and risk management
- Insurers in trouble
  - Disintermediation in 1970s
  - Equitable Life closed to new business in 2000
  - The Hartford accepted $3.4*B* in TARP money in June 2009 after losing $2.75*B* in 2008, hurt by investment losses and the cost of VA guarantees

Risk-Neutral Valuation Approach

- Used in actuarial literature to price variety of options:
  - Milevsky and Posner (2001): GMDB
  - Ulm (2006): “Real option to transfer”
  - Zaglauer and Bauer (2008): Participating life insurance contracts
- To analyze withdrawal behavior for GMWBs:
  - Milevsky and Salisbury (2006)
  - Bauer, Kling and Russ (2008)
  - Optimal stopping problem, akin to pricing American put option
  - Exercise / Withdraw if exercise value exceeds continuation value
  - Worst-case scenario, calculate correct upper bound
- VA market incomplete: cannot sell – or repurchase – policy at its risk-neutral value
  - Withdrawing means giving up possible guarantees and tax benefits

The Model

- Consider withdrawal decisions in life-cycle model with outside investment
- PH maximizes expected lifetime utility
  - Consumption and bequests
  - Initial wealth W_0
  - Annual (deterministic) income I_t
- Invests P_0 in VA with finite maturity T, remainder in outside account
  - Includes GMWB, possibly other guarantees
  - Return-of-investment guarantees
  - Other types possible, at cost of larger state space
  - All guarantee accounts identical to benefits base, G_t
  - Annual guarantee fee ϕ as percentage of concurrent account value

The Model

- VAs grow tax-deferred
  - Withdrawals taxed on last-in first-out basis
  - Early withdrawal tax (10%) if PH withdraws prior to age 59.5
- Restrict all actions to policy anniversary dates
  - Four state variables
    - ⋆ VA account X_t^-
    - ⋆ Outside account A_t^-
    - ⋆ Benefits base G_t
    - ⋆ Tax base H_t
  - Three choice variables
    - ⋆ Withdrawal amount w_t
    - ⋆ Consumption C_t
    - ⋆ Risk allocation in outside account ν_t

Bellman Equation

$$V_t(y_t) = \max_{C_t, w_t, \nu_t} u_C(C_t) + e^{-\beta} \cdot E_t\left[q_{x+t} \cdot u_B(b_{t+1}|S_{t+1}) + p_{x+t} \cdot V_{t+1}(y_{t+1}|S_{t+1})\right], \quad (1)$$

$$\begin{aligned}
y_t &\equiv (A_t^-, X_t^-, G_t, H_t), \\
X_t^+ &= \left(X_t^- - w_t\right)^+, \\
A_t^+ &= A_t^- + I_t + w_t - C_t - \text{fee}_I - \text{fee}_G - \text{taxes}, \\
\text{fee}_I &= s \cdot \max\left\{w_t - \min(g_t^W, G_t^W)\right\}, \\
\text{fee}_G &= s^g \cdot (w_t - \text{fee}_I) \cdot \mathbb{I}_{\{x+t<59.5\}}, \\
\text{taxes} &= \tau \cdot \min\{w_t - \text{fee}_I - \text{fee}_G, (X_t^- - H_t)^+\}, \\
G_{t+1} &= \begin{cases} (G_t - w)^+ & : w \le g_t^W \\ \left(\min\left\{G_t - w\,,\ G_t \cdot \frac{X_t^+}{X_t^-}\right\}\right)^+ & : w > g_t^W \end{cases}, \\
H_{t+1} &= H_t - \left(w_t - \left(X_t^- - H_t\right)^+\right)^+, \\
A_{t+1}^- &= A_t^+ \cdot \left[\nu_t \cdot \frac{S_{t+1}}{S_t} - \kappa \cdot \left(\nu_t \cdot \frac{S_{t+1}}{S_t} - 1\right)^+\right], \\
X_{t+1}^- &= X_t^+ \cdot e^{-\phi} \cdot \left[\nu^X \cdot \frac{S_{t+1}}{S_t}\right], \\
b_{t+1} &= A_{t+1}^- + \max\{X_{t+1}^-, G_{t+1}^D\}, \\
\nu_t &\ge 0\,,\ \textstyle\sum_i \nu_t(i) = 1,
\end{aligned}$$

Implementation in a Black-Scholes Framework

- Solve by recursive dynamic programming:

  (I) *Create appropriate state space grid*

  (II) *For* $t = T$*: for all grid points* $(A, X, G^{\cdot}, H)$*, compute* $V_T^-(A, X, G^{\cdot}, H)$*.*

  (III) *For* $t = T - 1, T - 2, \ldots, 1$*: Given* V_{t+1}^-*, calculate* $V_t^-(A, X, G^{\cdot}, H)$ *recursively for each* $(A, X, G^{\cdot}, H)$ *on the grid using an approximation of the integral in* (1)

  - *Discretize return space and evaluate via Green's function*
  - *Gauss-Hermite quadrature*

  (IV) *For* $t = 0$*: For the given starting values* $A_0 = W_0 - P_0$*,* $X_0 = P_0$*,* $G_0^{\cdot} = G_1^{\cdot} = P_0$ *and* $H_0 = H_1 = P_0$*, compute* $V_0^-(W_0 - P_0, P_0, P_0, P_0)$ *recursively from Equation* (1)

Parameter Assumptions

- Policyholder is 55 years old, $T = 15$ years to maturity
- $P_0 = 100K$; $W_0 = 2 \cdot P_0 = 200K$; $I_t = 40K$
- CRRA($\gamma = 3$) utilities; $B = 1$; $\beta = r$
- $\tau = 30\%$, $\kappa = 15\%$
- Guarantee fee $\phi = 50$ bps
- Surrender fee $s = 5\%$,

$$g_t^W = \begin{cases} 0 & : t \leq 5 \\ 20,000 & : t > 5 \end{cases}$$

- $r = 4\%$, $\mu = 8\%$, $\sigma = 15\%$
  - Merton ratio: $\frac{\mu - r}{\sigma^2 \cdot \gamma} = \frac{0.08 - 0.04}{0.15^2 \cdot 3} \approx 0.5926$
- $\nu^X = 100\%$ equity exposure in VA

Results

Results

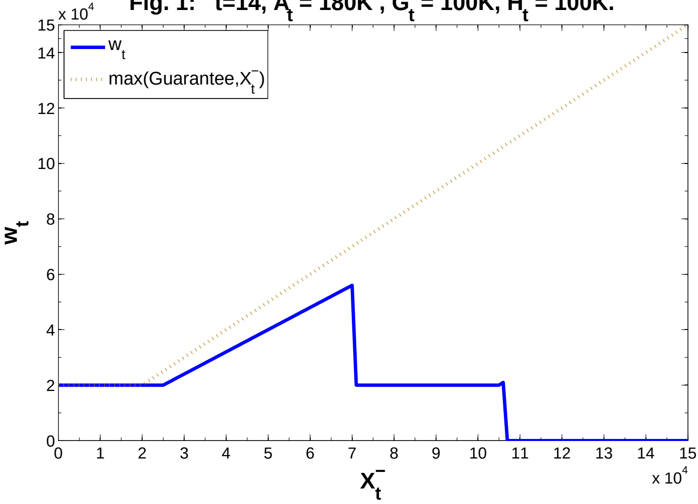
Withdrawal Behavior

- Little withdrawal activity (approx. 12K per PH on average)
  - No withdrawals during accumulation period
  - No premature withdrawals in 67% of cases
  - PH empties guarantee account in 6% of cases
  - $<$ 1% chance of excessive withdrawal during contract phase
- Two main reasons to withdraw prematurely:
  - VA account below tax base (approx. 7K on average)
    - Nuanced patterns
    - Interaction of in-the-moneyness of guarantee, tax considerations and excess withdrawal charge
  - VA account much greater than outside account (approx. 5K on average)
    - To reduce overall risk exposure (Merton ratio)

Results

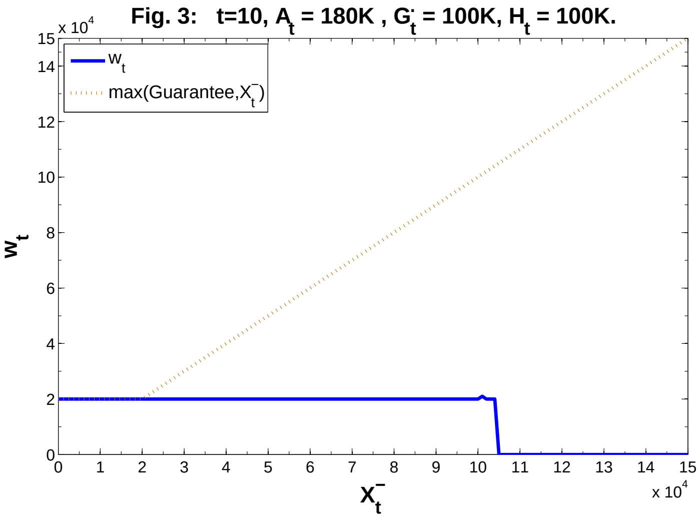
Withdrawal Behavior

Fig. 1: t=14, A_t = 180K , G_t = 100K, H_t = 100K.



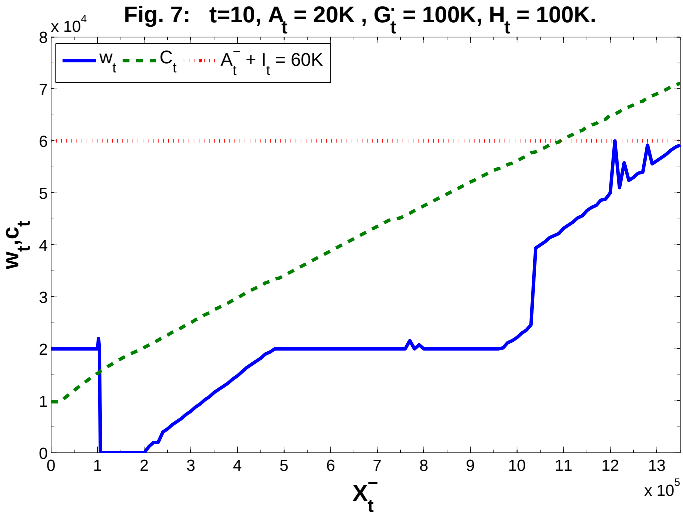
Results

Withdrawal Behavior



Results

Withdrawal Behavior



Results

Pricing and Sensitivities

- Guarantee fee of $\phi = 50$ bps sufficient to cover expected costs
- In-the-moneyness appears to be OK proxy for pricing
  - Different source to withdrawals
- Eliminating excess withdrawal fee increases net profits (win-win)

	Base Case	w/d if $X_t^- \leq G_t$	$s = 0\%$
$\mathbb{E}^{\mathbb{Q}}$[Fees]	$6,252$	$5,925$	$5,907$
$\mathbb{E}^{\mathbb{Q}}$[Excess-Fee]	19	0	0
$\mathbb{E}^{\mathbb{Q}}$[Guarantee]	$4,558$	$4,761$	$2,136$
%(Guarantee > 0)	24.34%	33.97%	31.94%
$\mathbb{E}$[agg. w/d]	$12,084$	$14,180$	$16,374$
$\mathbb{E}$[agg. w/d & $t \leq 6$]	0	0	$4,374$
$\mathbb{E}$[agg. w/d & $X_t^- \leq H_t$]	$6,953$	$14,119$	$6,047$
$\mathbb{E}$[agg. w/d & $X_t^- > H_t$]	$5,030$	0	$5,816$

Pricing and Sensitivities

- Withdrawal patterns highly sensitive to volatility
- Considering taxes important

	BC: $\sigma = 15\%$	$\sigma = 20\%$	$\sigma = 25\%$	No Taxes
$\mathbb{E}^{\mathbb{Q}}$[Fees]	6,252	6,047	5,152	4,734
$\mathbb{E}^{\mathbb{Q}}$[Excess-Fee]	19	62	1,006	64
$\mathbb{E}^{\mathbb{Q}}$[Guarantee]	4,558	8,384	11,533	4,746
%(Guarantee $>$ 0)	24.34%	36.48%	45.57%	28.16%
$\mathbb{E}$[agg. w/d]	12,084	29,914	85,879	88,791
$\mathbb{E}$[agg. w/d & $t \leq 6$]	0	54	13,356	82
$\mathbb{E}$[agg. w/d & $X_t^- \leq H_t$]	6,953	17,615	27,877	19,500
$\mathbb{E}$[agg. w/d & $X_t^- > H_t$]	5,030	10,674	31,221	69,033

Conclusions and Future Research

Conclusions and Future Research

- Develop lifetime-utility model to analyze withdrawal behavior for VA with guarantees
- Numerically solve policyholder's decision making problem in Black-Scholes framework
  - Return-of-investment GMWB
- Infrequent withdrawals
- PH withdraws when VA account is below tax base
  - Interaction of in-the-moneyness of guarantee, tax considerations and excess w/d fee
- PH withdraws when VA account is large
  - To lower overall risk exposure

Conclusions and Future Research

- Extend policyholder environment
  - Unemployment Risk
  - Subjective mortalities
- Withdrawal patterns highly sensitive w.r.t. volatility σ
  - Stochastic volatility framework
- Alternatives to EUT
  - Epstein-Zin preferences
  - Correlation Aversion

THANK YOU!